Quiz 04

1. Find the Choleksy decomposition $A = LL^T$ for the following matrix:

$$A = \begin{bmatrix} 4 & 2 & -2 \\ 2 & 10 & 5 \\ -2 & 5 & 6 \end{bmatrix},$$

and verify it by matrix multiplication.

$A = LL^T$,

first column of L: $L = \begin{bmatrix} 2 & & \\ 1 & & \\ -1 & & \end{bmatrix}$

$$\begin{bmatrix} 10 & 5 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 \\ -1 \end{bmatrix} [1, -1]$$

$$= \begin{bmatrix} 10 & 5 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 6 & 5 \end{bmatrix}$$

then second column of L:

$$L = \begin{bmatrix} 2 & & \\ 1 & 3 & \\ -1 & 2 & \end{bmatrix}$$

$5 - (2)(2)^T = 1$, so third column of L:

$$L = \begin{bmatrix} 2 & & \\ 1 & 3 & \\ -1 & 2 & 1 \end{bmatrix}$$

2. Show that $\kappa(A) \geq 1$ for any norm $\|\cdot\|$.

$$A \cdot A^{-1} = I$$

$$\Rightarrow \|A \cdot A^{-1}\| = \|I\| = 1$$

$$\Rightarrow 1 = \|A \cdot A^{-1}\| \leq \|A\| \cdot \|A^{-1}\| = K(A).$$