

# Homework 08

9.

Let  $A = [a_1 \dots | a_n] \in \mathbb{R}^{n \times n}$  be an orthogonal matrix.

By definition of orthogonality:

$$A^T A = I.$$

On the other hand,

$$A^T A = \begin{bmatrix} a_1^T \\ \vdots \\ a_n^T \end{bmatrix} \begin{bmatrix} a_1 & \dots & a_n \end{bmatrix} = \begin{bmatrix} a_1^T a_1 & a_1^T a_2 & \dots & a_1^T a_n \\ a_2^T a_1 & a_2^T a_2 & \dots & a_2^T a_n \\ \vdots & \vdots & \ddots & \vdots \\ a_n^T a_1 & a_n^T a_2 & \dots & a_n^T a_n \end{bmatrix}$$

therefore:

$$a_j^T a_j = 1, \quad \text{for } j = 1 : n$$

$$\text{and } a_i^T a_j = 0, \quad \text{for } i \neq j.$$

which satisfies pairwise orthogonality.

13

$$P = \frac{VU^T}{\|V\|_2^2}, \quad \text{then}$$

$$(a) \quad P^2 = \frac{VU^T V^T}{\|V\|_2^4} = \frac{V(\|V\|_2^2)V^T}{\|V\|_2^4} = \frac{\|V\|_2^2}{\|V\|_2^4} VV^T = P.$$

$$(b) \quad P^T = \frac{1}{\|V\|_2^2} (VV^T)^T = \frac{1}{\|V\|_2^2} (V^T)^T V^T = P$$

$$(c) \quad Pv = \frac{V(UV)}{\|V\|_2^2} = \frac{V(\|V\|_2^2)}{\|V\|_2^2} = V$$

14.

$$H = I - 2VV^T$$

then

$$H^T = I^T - 2(VV^T)^T = I - 2(V^T)^T V^T = I - 2VV^T = H.$$