

## Homework 04

1. Find out  $\|A\|_2$  if  $A$  is orthogonal.

$A$  is orthogonal  $\Rightarrow AA^T = A^TA = I$ ,

then  $\|A\|_2 = \sqrt{\rho(A^TA)} = \sqrt{\rho(I)} = 1$ .

2. Let  $\lambda$  be an eigenvalue of  $A$  and  $x$  be a corresponding eigenvector, then

$$Ax = \lambda x$$

$$\Rightarrow \|Ax\| = \|\lambda x\| = |\lambda| \|x\|$$

$$\Rightarrow |\lambda| \|x\| = \|Ax\| \leq \|A\| \|x\|$$

Since  $x \neq 0$ , we have

$$|\lambda| \leq \|A\|$$

take maximum on two sides, we have

$$\max_{\text{all } \lambda} |\lambda| \leq \|A\|$$

$$\Rightarrow \rho(A) \leq \|A\|$$

3. Omitted.

4.  $I = AA^{-1} \Rightarrow \|I\| = \|AA^{-1}\| \leq \|A\| \|A^{-1}\| = K(A)$   
 $\Rightarrow 1 \leq K(A)$

5. Let  $A$  be an orthogonal matrix, then  $A^TA = I$ .

$K_2(A) = \|A\|_2 \cdot \|A^{-1}\|_2$ . from conclusion of Question 1,

the 2-norm of an orthogonal matrix is always 1,

and notice that  $A$  and  $A^\dagger$  are both orthogonal.  
therefore,

$$K(A) = \|A\|_2 \cdot \|A^\dagger\|_2 = 1.$$

6 Omitted.