Homework 04

You are suggested to finish all questions prior to Quiz 04.

1. Find out $\|A\|_2$ if $A \in \mathbb{R}^{n\times n}$ is an orthogonal matrix.
2. Show that $\rho(A) \leq \|A\|$ for any norm $\|\cdot\|$. Here $\rho(A)$ represents the spectral radius of A, which is defined as the largest eigenvalue in absolute value of A.
3. If $\|\delta I\| < 1$, show that $I + \delta I$ is invertible, and
$$\|(I + \delta I)^{-1}\| \leq \frac{1}{1 - \|\delta I\|}.$$
4. Show that $\kappa(A) \geq 1$ for any $A \in \mathbb{R}^{n\times n}$.
5. Find out $\kappa_2(A)$ if $A \in \mathbb{R}^{n\times n}$ is an orthogonal matrix.
6. Perform conditioning analysis for matrix inversion for a matrix $A \in \mathbb{R}^{n\times n}$.