Homework 01

You are suggested to finish question 1-6 prior to Quiz 01, and the rest prior to Quiz 02.

1. Consider matrices

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 3a_{11} + a_{21} & 3a_{12} + a_{22} & 3a_{13} + a_{23} \\ -5a_{11} + a_{31} & -5a_{12} + a_{32} & -5a_{13} + a_{33} \end{bmatrix},$$

find a matrix E such that $EA = \tilde{A}$.

2. Consider matrices

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} a_{11} & 7a_{11} + a_{12} & -2a_{11} + a_{13} \\ a_{21} & 7a_{21} + a_{22} & -2a_{21} + a_{23} \\ a_{31} & 7a_{31} + a_{32} & -2a_{31} + a_{33} \end{bmatrix},$$

find a matrix E such that $AE = \tilde{A}$.

3. Consider a matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 8 & 0 & 1 & 0 \\ \pi & 0 & 0 & 1 \end{bmatrix}.$$

Find its inverse A^{-1}.

4. Consider a matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 \\ 0 & 0 & \beta & 0 & 1 \end{bmatrix},$$

where α and β are two constants. Find its inverse A^{-1}.

5. Consider two matrices

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 & 0 \\ l_{31} & 0 & 1 & 0 & 0 \\ l_{41} & 0 & 0 & 1 & 0 \\ l_{51} & 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & 1 & 0 \\ 0 & 0 & \beta & 0 & 1 \end{bmatrix}.$$

Calculate AB and BA.

6. Consider a matrix

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & 3 & 3 & 1 \\ 8 & 7 & 9 & 5 \\ 6 & 7 & 9 & 8 \end{bmatrix}.$$

Find a Frobenius matrix E to eliminate the first column of A, in other words, EA should be of form

$$A = \begin{bmatrix} \mathrm{X} & \mathrm{X} & \mathrm{X} & \mathrm{X} \\ 0 & \mathrm{X} & \mathrm{X} & \mathrm{X} \\ 0 & \mathrm{X} & \mathrm{X} & \mathrm{X} \\ 0 & \mathrm{X} & \mathrm{X} & \mathrm{X} \end{bmatrix}.$$

7. Consider matrices

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \end{bmatrix},$$

find a matrix P such that $PA = \tilde{A}$.

8. Consider a permutation π:

$$\pi : \{1, 2, 3, 4\} \to \{2, 3, 1, 4\}.$$

   (a) Find the corresponding row permutation matrix P_π^{row} and the column permutation matrix P_π^{col};
   (b) Calculate $P_\pi^{\text{row}} \cdot A$ and $A \cdot P_\pi^{\text{col}}$;
   (c) What is the relation between P_π^{row} and P_π^{col}?
   (d) Find $(P_\pi^{\text{row}})^{-1}$; How is $(P_\pi^{\text{row}})^{-1}$ related with P_π^{row} and P_π^{col}?
   (e) Find $(P_\pi^{\text{col}})^{-1}$; How is $(P_\pi^{\text{col}})^{-1}$ related with P_π^{row} and P_π^{col}?
   (f) Find the inverse of the permutation π, denoted by π^{-1};
   (g) Find $P_{\pi^{-1}}^{\text{row}}$ and $P_{\pi^{-1}}^{\text{col}}$; how are they related with P_π^{row} and P_π^{col}?

9. Consider a Frobenious matrix

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & l_{43} & 1 & 0 & 0 \\ 0 & 0 & l_{53} & 0 & 1 & 0 \\ 0 & 0 & l_{63} & 0 & 0 & 1 \end{bmatrix}.$$

Let π be a permutation:

$$\pi : \{1, 2, 3, 4, 5, 6\} \to \{1, 2, 3, 6, 4, 5\}$$

(a) Calculate $P_\pi^{\mathrm{row}} \cdot L$, and then $(P_\pi^{\mathrm{row}} \cdot L) \cdot P_\pi^{\mathrm{col}}$;

(b) What is the relation between $P_\pi^{\mathrm{row}} L P_\pi^{\mathrm{col}}$ and L?

(c) Take a different permutation

$$\gamma : \{1, 2, 3, 4, 5, 6\} \to \{1, 2, 5, 6, 4, 3\}$$

and repeat the calculation from step (a); is $P_\gamma^{\mathrm{row}} L P_\gamma^{\mathrm{col}}$ related with L in a similar way as in (b)? Explain why.