MATHEMATICAL RELIABILITY:
AN EXPOSITORY PERSPECTIVE

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Edited by
REFIK SOYER
Department of Management Science
The George Washington University
Washington, DC 20052

THOMAS A. MAZZUCHI
Department of Engineering Management and Systems Engineering
The George Washington University
Washington, DC 20052

NOZER D. SINGPURWALLA
Department of Statistics
The George Washington University
Washington, DC 20052

Kluwer Academic Publishers
Boston/Dordrecht/London

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Preface

Background:

Sometime during 1998 - perhaps even earlier - I was contacted by Professor Fred S. Hillier of Stanford University to orchestrate a contribution in reliability for the sub-series on "Advancing-the-State-of-the-Art" of the series in Operations Research and Management Science that he was editing for Kluwer. Being a friend of Fred, and having benefited from his classic treatise on Operations Research with the late Professor Gerald Lieberman (also a much missed friend), this request was one that could not be denied, despite the enormity of the task that was proposed. A solution was therefore found. It was to invite two other colleagues, both active researchers in reliability, and both much larger in physical stature than myself, to co-orchestrate and to co-edit the said contribution. Thus, evolved this triage of Editors, each taking responsibility for the several chores that lay ahead, from conceptualizing the content, cajoling the contributors, and most important, meeting the stylistic requirements of the publisher. This latter task was admiringly and patiently accomplished by Refik Soyer who has now earned the distinction of being the lead Editor.

The Choice of Topics:

Research in reliability has been categorized as having one of the following two orientations: engineering or mathematical (to include probability and statistics). For a series in Operations Research and Management Science, the former is clearly not suitable. With respect to the latter, care was taken to ensure that the topics covered would be such that the underlying mathematics was motivated by a real need, be it in the context of engineering or in biomedicine (i.e. survival analysis). Topics that purport to do mathematics for the sake of just doing mathematics with reliability and survival analysis as a camouflage, were deliberately excluded. We feel that the era of this style of research in reliability has long been over. Consideration was given to more advanced theoretical approaches and novel applications of reliability to ensure that topics having a futuristic impact be specifically included. Topics like finance, forensics, information, and orthopedics, as well as the more traditional reliability topics were purposefully undertaken to make this collection different from the

existing books in reliability. We think that we may have succeeded.

The Selection of Contributors:

Our aim was to ensure adequate coverage of the topics we thought were both important and futuristic. With this in mind, a list of topics was developed and potential contributors - those we felt were the most influential researchers in their respective areas - were contacted. A majority of those that were contacted obliged by responding in the affirmative. We were gratified by this response, especially since the scholarship and the stature of these respondents were outstanding. A few of those who were contacted politely refused on grounds that they were either over committed or had nothing to say that could be viewed as being new and state-of-the-art. We appreciate their candor and modesty.

We were fortunate in the fact that a majority of those who responded in the affirmative, delivered their entry within the time constraints. The final culprit who held up the works, ended up being myself; may my papers linger on a referee's desk until doomsday ! However, I was an alternate, since the original nominee excused himself but after much delay.

Review and Dedication:

Since our selection of contributors was based on their expertise and contributions to their areas of specialty, we did not feel it necessary, nor appropriate, to subject their entries to a review. After all, what we wanted from each is a summarization of the developments in their assigned topic - not their latest research. This has resulted in some unevenness of prose - a small price to pay for not hampering the style of the contributor(s).

Finally, all along the development of this project, from its inception to conclusion, we had in mind dedicating this work as Festschrift for Professor Richard E. Barlow in celebration of his 70-th birthday in 2001. Dick's contributions to reliability are legendary. More important, he has been an inspiration, a friend, and a mentor to me, and this is the least that I can do to reciprocate. The fact that Dick was one of the handful who choose not to respond to our invitation, has not deterred us from dedicating this work to him. Happy 70-th Dick from all of the contributors, and wishing for many more to come.

NOZER D. SINGPURWALLA

A Festschrift for Richard E. Barlow on the occasion of his 70-th Birthday.

Contributing Authors

Elja Arjas, Rolf Nevanlinna Institute, University of Helsinki, Finland.

Madhuchhanda Bhattacharjee, Rolf Nevanlinna Institute, University of Helsinki, Finland.

Henry W. Block, Department of Statistics, University of Pittsburgh, Pittsburgh, PA, USA.

Philip J. Boland, Department of Statistics, National University of Ireland - Dublin, Ireland.

I. Robert Chiang, School of Business Administration, University of Connecticut, Storrs, CT, USA.

Kjell A. Doksum, Department of Statistics, University of Wisconsin, Madison, WI, USA.

Nader Ebrahimi, Division of Statistics, Northern Illinois University, DeKalb, IL, USA.

Luis A. Escobar, Louisiana State University, Baton Rouge, LA, USA.

Michael S. Hamada, Group D-1, Statistical Sciences, Los Alamos National Laboratory, Los Alamos, NM, USA.

Myles Hollander, Department of Statistics, Florida State University, Tallahassee, FL, USA.

Lancelot F. James, Department of Information Systems and Management, The Hong Kong University of Science and Technology, Hong Kong.

Uwe Jensen, Department of Stochastics, University of Ulm, Germany.

Lynn Kuo, Department of Statistics, University of Connecticut, Storrs, CT, USA.

Way Kuo, Texas A&M University, College Station, TX, USA.

James D. Lynch, Department of Statistics, University of South Carolina, Columbia, SC, USA.

Nicholas J. Lynn, Canadian Imperial Bank of Commerce, London, UK.

Harry F. Martz, Group D-1, Statistical Sciences, Los Alamos National Laboratory, Los Alamos, NM, USA.

Vijay N. Nair, University of Michigan, Ann Arbor, MI, USA.

Süleyman Özekici, Department of Industrial Engineering, Koç University, Turkey.

William J. Padgett, Department of Statistics, University of South Carolina, Columbia, SC, USA.

Edsel A. Peña, Department of Statistics, University of South Carolina, Columbia, SC, USA.

Rajendra Prasad, KBSI, College Station, TX, USA.

C. Shane Reese, Group D-1, Statistical Sciences, Los Alamos National Laboratory, Los Alamos, NM, USA.

Francisco J. Samaniego, Department of Statistitics, University of California - Davis, Davis, CA, USA.

Thomas H. Savits, Department of Statistics, University of Pittsburgh, Pittsburgh, PA, USA.

Nozer D. Singpurwalla, Department of Statistics, The George Washington University, Washington, DC, USA.

Ehsan S. Soofi, School of Business Administration, University of Wisconsin-Milwaukee, Milwaukee, WI, USA.

Refik Soyer, Department of Management Science, The George Washington University, Washington, DC, USA.

Fabio Spizzichino, Department of Mathematics, University La Sapienza Rome, Italy.

Andrew W. Swift, Mathematical Sciences Department, Worcester Polytechnic Institute, Worcester, MA, USA.

Alyson G. Wilson, Group D-1, Statistical Sciences, Los Alamos National Laboratory, Los Alamos, NM, USA.

Simon P. Wilson, Department of Statistics, Trinity College, Ireland.

Foreword

The entries in this volume have been categorized into seven parts, each part emphasizing a theme that in our judgment seems poised for the future development of reliability as an academic discipline with relevance. The seven parts: are *Networks and Systems*; *Recurrent Events*; *Information and Design*; *The Failure Rate Function and Burn-in*; *Software Reliability and Random Environments*; *Reliability in Composites and Orthopedics*, and *Reliability in Finance and Forensics*. Embedded within the above are some of the other currently active topics such as causality, cascading, exchangeability, expert testimony, hierarchical modeling, optimization and survival analysis. Collectively, these when linked with utility theory constitute the science base of risk analysis.

Part I on *Networks and Systems* consists of three entries each striking a unique and different chord. Boland and Samaniego introduce the notion of the "signature" of a system. The term signature (or imprint), resonates well with engineering wherein it is used to describe the characteristics of rotating machinery vis a vis its vibration. Boland and Samaniego use their notion to characterize the manner in which a system is put together, irrespective of the inherent quality of each member of the system. They make connections between their notion and the notions used in computer science. Their treatment of the topic is exhaustive; it promises to generate added interest in the notion of signatures. The second paper by Kuo and Prasad is in some sense unique among all other entries because it brings into the picture the role of optimization in reliability. Since mathematical optimization is a core discipline of operations research, Kuo and Prasad's entry is noteworthy on two counts. It exposes reliability theorists to the relevance of optimization in system design, and it makes this volume's inclusion in a series in Operations Research and Management Science germane. The third paper by Swift summarizes some of the more recent work in assessing the reliability of systems from a statistical point of view. Such work, motivated by the more recent concerns of infrastructure protection, entails aspects of hierarchical modeling, computations via the Markov chain Monte Carlo, notions of interdependence (causal and cascading failures) and the use of neural nets for reliability assessment. To whet the appetite of probability theorists, Swift caps his entry by including in it the pitfalls of not paying

attention to Borel's paradox which can arise naturally in the context of system reliability assessment.

Part II on *Recurrent Events* consists of three entries, one emphasizing an engineering scenario, another the biomedical scenario and the third a matter of foundations. Arjas and Bhattacharjee demonstrate the importance of hierarchical modeling as a way to borrow strength when dealing with heterogeneous data. They motivate their work by starting with a real life example involving valve failures in a nuclear plant and analyze the ensuing data using the Markov chain Monte Carlo approach in a Bayesian context. Their analyses show how modern statistical techniques when coupled with sophisticated computational approaches can lead to useful practical insights. The second paper by Doksum and James pertains to the use of a class of priors originally proposed by Doksum. These priors are called neutral to the right and they have gained popularity in Bayesian inference. Doksum and James do a Bayesian analysis of Barlow's total time on test transform and we are fortunate to receive this contribution. The third paper by Pena and Hollander is both archival and state-of-the-art. The authors introduce a general class of models for the treatment of recurrent event data that arises in a variety of contexts: health sciences, engineering, economics and sociology. The models are able to incorporate the effects of interventions, accumulations and concomitance. The list of references is exhaustive and the material is expository enough for any novice to benefit. It offers the Bayesians a new window of opportunity for research in an area of investigation that is very general.

Part III on *Information and Design* consists of three entries two of which share a common theme. The aim of failure data analysis, irrespective of whether the data arises from a designed life-testing experiment or retrospectively from the field, is to gain information or knowledge. The latter enables one to make meaningful predictions about future lifetimes. Discrimination, entropy, and information are the three legs on which the notion of "quantified knowledge" rests. In the first entry, Ebrahimi and Soofi provide an authoritative synopsis of the above triage with a focus on how it relates to reliability and life-testing. The entry is rich in examples and almost complete vis a vis coverage; an exception is the topic of how to design experiments for extracting the maximum amount of information that one possibly can. All the same, Ebrahimi and Soofi 's entry should motivate researchers in reliability to consider incorporating information theoretic ideas in reliability analysis; this entry provides a valuable service. The second entry by Nair, Escobar and Hamada pertains to the design of experiments for gathering performance data, with a view towards enhancing reliability. This point of view, popularized by Taguchi, advocates an active philosophy in the sense that the aim of reliability analysis should be to improve performance, not to merely report observed performance - the passive view. Notions of accelerated testing, degradation analysis, robustness and censoring are embodied in

the context of the theme of the entry. The third entry by Wilson, Reese, Hamada and Martz has a futuristic motif. It pertains to the fusion of information about lifetimes that arises from two different sources: physical experiments and computer simulations. The latter is necessitated by either cost and time constraints or by the impossibility of conducting physical tests. For example, the inability to test nuclear weapons due to test ban treaties. The authors' aim is achieved by three modern technologies: hierarchical modeling, Bayesian pooling and Markov chain Monte Carlo. The entry is both state-of-the-art and futuristic; it reinforces the idea that new research and new paradigms are often driven by new problems.

Part IV on the *Failure Rate Function and Burn-in* consists of two entries the first being a prelude to the second. The entry by Block and Savits addresses the fundamental question upon whose answer depends the need for the second entry by Jensen and Spizzichino. The notion of the failure rate function is perhaps unique to reliability and survival analysis. Indeed statistical reliability can be said to owe its existence to the notion of failure rate. Engineers often claim that components and systems exhibit a failure rate function whose shape is like that of a bath-tub. The decreasing form of the failure rate function is intriguing; specifically, is the decrease of the failure rate due to some natural phenomenon or is it the manifestation of something else, like a mixture (be it physical or be it psychological). A knowledge of the form of the failure rate function is useful for commissioning an item to service. This is the theme of the second entry by Jensen and Spizzichino. In the first entry, Block and Savits provide an overview of the various forms of the failure rate function that can occur due to mixing - irrespective of what causes the mixture. The treatment of Block and Savits tend to be mathematical (but not necessarily technical); however, their entry here is expository and relaxed. This entry embodies the view that the good mathematics of reliability theory should be driven by a genuine need. The second entry by Jensen and Spizzichino exploits the kind of results that the first entry can produce, in order to address the question of how much one should test an item (i.e. the notion of "burn-in") prior to commissioning it for use. This entry explores several ramifications of the problem and the material - which tends to be technically sophisticated - embodies the notion of utilities (via costs) - Bayesian decision making under uncertainty and sequential control theory; aspects of Operations Research and Management Science.

Part V on *Software Reliability and Random Environments* pertains to an issue that is currently important and will continue to be so. As systems become more and more software driven and software dependent, unreliable software is the critical component of a system. The first entry by Chiang and Kuo uses some of the notions and ideas that are useful in reliability, to manage the software development process. This is noteworthy on two counts: the first is that it has often been claimed by experienced software engineers that it is the process that

produces a piece of software that ensures its reliability - not just the innate abilities of programmers to produce error-free codes; the second is that by using system reliability data to manage the process, Chiang and Kuo put into practice Taguchi's philosophy of reliability techniques playing an active role for producing quality software. There is a parallel between this entry and that of Jensen and Spizzichino vis a vis optimum time to "burn-in" and optimum time to release software. Hopefully, these entries will provide some synergy between the said topics. The second entry by Özekici and Soyer pertains to a generic topic in reliability - be it hardware or be it software - namely the manner in which the effects of a random environment can be treated in the context of assessing survivability. The entry, albeit focussed on the context of software, provides an overview of the several modern approaches - mostly based on stochastic process theory linked with Bayesian methodology - that are used in the context mentioned above.

Parts VI and VII pertain to some new and important avenues of application of reliability, namely, *composite materials*, *orthopedics*, *finance* and *forensics*. Of these, finance and orthopedics seem to be most intriguing, and composite materials the most crucial. Lynch and Padgett provide an overview of the recent work on the strength of fibre bundles that they have been doing over the past few years. With the increased emphasis on infrastructure protection and the use of composite materials, this type of research has an added urgency. Their entry pulls together several related topics (such as pooling failure data, interacting systems, Gaussian and inverse Gaussian processes and inferential issues) to develop a coherent package that should appeal to both engineers and statisticians. Their list of references will support this latter claim. Wilson and also Lynn introduce a new frontier for the application of reliability. The former focuses on a specific problem in orthopedics, namely the life-length of hip replacements, and uses a hierarchical approach in the context of a Bayesian analysis to assess lifetimes of such replacements. He illustrates the validity of his approach by considering actual data. This scenario further attests to the growing importance of hierarchical modeling in reliability analysis. Via this work we note the importance of the application of reliability theory to such burgeoning areas as biomedical engineering. Another area of importance for the application of reliability techniques is illustrated in Lynn's entry which is more on the conceptual front than on the practice front. He introduces notions in fixed income instruments - like bonds - and discusses their risk of default (i.e. failure). He then points out opportunities wherein notions of reliability and risk could come into play and discusses some possibilities. He then moves to the notion of "derivatives" and again points out scenarios wherein there could be an interplay between reliability and finance. Lynn's entry is important because it opens a new window of opportunity for the techniques of reliability. The final entry is on warranties. These bring into play the various notions of

probability (objective, logical and personal), utility and game theory, failure models indexed by multiple scales, and forecasting using leading indicators. Violations of warranty are often the cause of litigation - sometimes in millions of dollars - and the role of reliability analyst as an expert witness becomes central. Thus the label reliability in forensics.

Despite the broad coverage that we have endeavored to encompass, we are aware of the fact that there may be other topics that should have been included. In excluding these we take the blame; but then we are also quite delighted with what we have included.

Finally, we would like to take this opportunity to acknowledge the several years of support provided by The Army Research Office, and the Office of Naval Research, for sustaining our work in reliability through the George Washington University's *Institute for Reliability and Risk Analysis*.