The Role of Transactional Sex in Spreading HIV in Nigeria

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ABSTRACT. The sex industry has been implicated in the spread of HIV across the world. In this article we propose two models, the first consisting of two core groups of interacting heterosexual populations, and the second one with the two core groups linked to the general population. The core groups in the first model consist of male truck drivers and female sex workers. The resulting sexual activities between these two groups is responsible for HIV/AIDS fast spread in these two groups. These high-risk groups influence the disease spread in the general population and a simplified version of such dynamics is described in our second model. We explore the potential impact of high levels of infection on the general population. Furthermore, we show potential results of prophylaxis.

1. Introduction

The spread of HIV has reached an epidemic proportion in sub-Saharan Africa and Asia [**8**], in particular, in Nigeria [**9**]. In 2001, an estimate of the number of adults and children who died of AIDS in Nigeria was 170,000 [**8**]. The sex industry is identified as a major factor in fast spreading of the disease. In Nigeria studies [**13**] have identified major contributors to this spread: long distance truck drivers, commercial motorcycle riders and the uniformed services who are the primary clients of female sex workers the group which is also a contributor because of the exposure to multiple partners.

Our main objective is to look at the impact of core group behavior on HIV prevelence in the general population, Efforts at understanding the dynamics of the HIV/AIDS epidemic within risk groups have targeted two core populations, namely, the truck drivers and female sex workers, see [**20**], [**10**], [**14**], or [**19**]. For instance, I.O. Orubuloye et al.[**20**] concluded that occupation demands of truck drivers and itinerant market sex workers in Nigeria along the Ilorin-Ibadan-Lagos highway had resulted in a network of multiple partners.that resulted in these occupations being of particularly high-risk with respect to the spread of HIV/AIDS. The typical scenario is truck driver spend many nights away from home and frequently make use of the services of the female sex workers in stop-over towns near major transportation routes. Although sex workers are often subject to great deal of stigma and exploitation, the industry has continued to thrive because of extreme poverty

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and falling standard of living in Nigeria. Thus, the female sex workers enter the profession out of necessity and only quit when they can. On the other hand, truck driving is a lucrative profession which unskilled males are willing to take, when such opportunities present themselves because of enormous material and monetary benefit associated with it by local standards. Although the truck drivers are engaged in legitimate businesses, they are often seen as a "bridge" population, or the one through which HIV reaches the larger population, particularly to people who are considered at lower risk. Most of the truck drivers have wives or other sexual partners in their communities who are thus at risk of HIV infection by the truckers.

In 1991 a study of truck drivers' sexual cultures was conducted in which the truck drivers reported an average of 6.3 current sexual partners (sex workers), 12 sexual partners during the previous year and 25 partners besides their wife during a lifetime [**20**]. A similar study of truck drivers between 1999 and 2001 [**19**] found out that the prevalence of HIV infection among truck drivers in Nigerian transit towns was 54 percent as compared to 17 percent in the non-transit towns. Studies of an area along a major highway in Uganda have found an HIV prevalence of 35 percent among truck drivers, and 37 percent of truckers estimated having more than 50 female sexual partners during their lifetimes [**27**].

Another important issue is an insignificant prevention of the disease. Sunmora 2005 [**26**] investigated sexual practices and barriers to condom use among truck drivers in Nigeria and concluded that the use of condom among the truck drivers was only 9 percent, though about 70 percent of them knew about the importance of condoms as an HIV preventative measure. On the other hand, condom usage is generally acceptable by female sex workers [**21**], but their clients sometimes insist on non usage, thus placing the sex workers as well as their clients at risk of contracting HIV.

In the general population prevalences are estimated to range from 3.5 to 8 percent in the adult population (ages 15-49) with the average holding at about 5 percent between 1999 and 2003. The rates among men and women is approximately the same among this age group[**8**]. The rates of use of commercial sex workers among adult men ranges from 8 to 11% at least once per year compared to reported truck drivers' extramarrital and casual sex (which we assume to be mostly with sex workers due to logistics) of 72-92% [**18**], [**23**]. For sub-Saharan Africa the fraction of the female population that are sex workers varies an order of magnitude between lows in the city to highs in rural towns that are along major transportation routes, but on average the figure is about 1-2% for an entire country [**18**]. The fraction of population that are truck drivers is about twice this or 2 to 4% [**18**], [**20**], [**21**], and [**23**]

In this study, we first model how the role of transactional sex affects the dynamics of HIV within the two core groups. In the second part we look more closely at the interactions between the core groups and the general population and make some observations about the strength of "network" connections between the core groups and the general population.

The paper is constructed as follows: in the next section we formulate the main model, and we fully investigate the stability of the endemic equilbrium in section 3. In Section 4 we study the dependence of the basic reproductive number and the endemic equilibrium on given parameters, in particular, for the latter we project the impact of condom use on the prevalence of disease in the subpopulations and show that there is a multiplicitive effect of prophylaxis for the general population. Section 5 connects the core and non-core populations. We assume that the risk of HIV infection in the general population is directly tied in to core prevalence. We reformulate the equations for infection in the general population (under specified conditions) in terms of just the core groups. This approach was selected since the parametrization of the model for the general population requires data that is not available. Section 6 describes the data we have gathered and parameter estimation we use. We make some rough calculations to estimate the dilution of infectivity due to the limited circle of contacts possible between even highly sexually active individuals. Last section contains conclusions, discussion and future work.

2. Core Group Model

We start with considering a simplified scenario in which the two core groups are experiencing the epidemic (we show that this corresponds to data gathered, refer to Section 6). The model considers the two core groups only - male transportation workers (truck drivers) and female sex workers. Individuals removed from both groups due to retirement, natural mortality (or other factors), and AIDS are considered to be replaced immediately through the recruitment of new members from the general population. In other words, we assume that the economic conditions are such that the supply of workers exceeds the demand.

The following assumptions are made in this model:

- Sex other than between truck drivers and sex workers is not considered.
- No condom use by the truck drivers or female sex workers.
- Random mixing between the two groups.
- Transmission rates are constant over the life of the disease.
- Recrutment to the core groups is of uninfected persons only.
- Each of the core groups have losses due to AIDS but the populations size of both core groups remain fixed, since each truck driver and sex worker gets replaced immediately. We assume that the AIDS cases are highly symptomatic so that progression from HIV to AIDS removes individuals from the sexually active core groups.

Let S_m and S_f denote the number of susceptible truck drivers and female sex workers respectively, and let I_m and I_f be the number of infected male truck drivers and females sex workers, respectively. Both new truck drivers and new sex workers are recruited at the same rate as they are lost, this assumes that the supply of available workers is always adequate and there is a minimal time required to train a person. The losses from the system are due to natural mortality (μ's) retirement (ρ's) and removal or death due to progression from HIV to AIDS(γ's). We assume that the onset of AIDS results in a person being unable to continue working as a truck driver or sex worker. New infections are caused exclusively by heterosexual contacts between susceptible core group members and infected core group members (the rate of infection given by the β's). The following system of differential equations describes HIV dynamics in this simplified two-sex mixing core

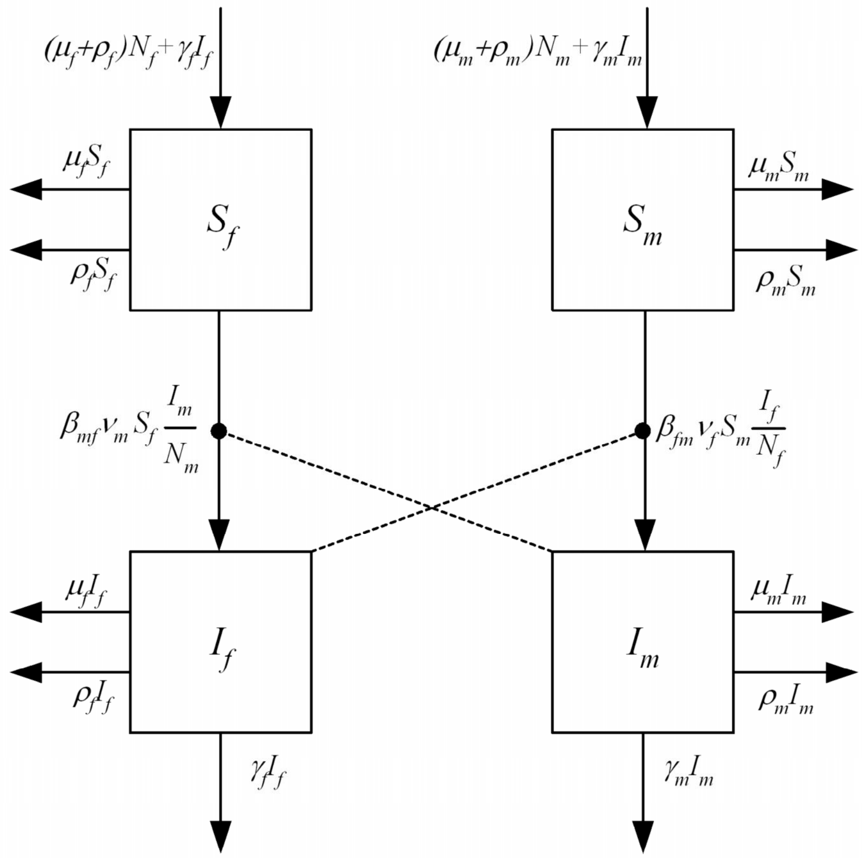

FIGURE 1. A compartmental diagram for the system of equations in (2.1).

population.

$$
\begin{aligned}
\dot{S}_f &= ((\mu_f + \rho_f)\, N_f + \gamma_f I_f) - (\mu_f + \rho_f)\, S_f - \beta_{mf}\nu_m \tfrac{I_m}{N_m} S_f, \\
\dot{S}_m &= (\mu_m + \rho_m)\, N_m + \gamma_m I_m - (\mu_m + \rho_m)\, S_m - \beta_{fm}\nu_f \tfrac{I_f}{N_f} S_m, \\
\dot{I}_f &= \beta_{mf}\nu_m \tfrac{I_m}{N_m} S_f - (\gamma_f + \mu_f + \rho_f) I_f, \\
\dot{I}_m &= \beta_{fm}\nu_f \tfrac{I_f}{N_f} S_m - (\gamma_m + \mu_m + \rho_m) I_m.
\end{aligned}
\tag{2.1}
$$

Here, the total populations of truck drivers is $N_m = S_m + I_m$ and commercial sex workers is $N_f = S_f + I_f$. The parameters used in system (2.1) are described in

Table 1.

Symbol	Description
β_{fm}	Rate at which female sex workers infect truck drivers per susceptible truck driver
β_{mf}	Rate at which truck drivers infect commercial sex workers per susceptible sex worker
μ_m	natural mortality rate of truck drivers
μ_f	natural mortality rate of sex workers
ρ_m	retirement rate of truck drivers
ρ_f	retirement rate of sex workers
ψ_m	turnover rate of uninfected truck drivers ($= \mu_m + \rho_m$)
ψ_f	turnover rate of uninfected sex workers ($= \mu_f + \rho_f$)
γ_m	Rate at which infected men progress to AIDS
γ_f	Rate at which infected women progress to AIDS
ν_f	the fraction of adult females that are sex workers
ν_m	the fraction of adult males that are truck drivers

Table 1. Model parameters.

Since we assume a constant number of workers in both groups ($N_f =$const. and $N_m =$const.), the above equations can be reduced to the following model

$$\begin{aligned} \dot{I}_f &= \beta_{mf}\nu_m \tfrac{I_m}{N_m}\left(N_f - I_f\right) - (\gamma_f + \psi_f)I_f, \\ \dot{I}_m &= \beta_{fm}\nu_f \tfrac{I_f}{N_f}\left(N_m - I_m\right) - (\gamma_m + \psi_m)I_m. \end{aligned} \tag{2.2}$$

Using the assumption that the population size is constant, we reformulate 2.2 in terms of proportions of the respective core groups that are infected. Therefore, let $X = \frac{I_f}{N_f}$ and $Y = \frac{I_m}{N_m}$, and substituting ψ for $\mu + \rho$ we have

$$\begin{aligned} \dot{X} &= \beta_{mf}\nu_m Y\left(1 - X\right) - (\gamma_f + \psi_f)X, \\ \dot{Y} &= \beta_{fm}\nu_f X\left(1 - Y\right) - (\gamma_m + \psi_m)Y. \end{aligned} \tag{2.3}$$

This system (2.3) is analyzed in the next section.

3. Core model Analysis

There are two equilibria of system (2.3): a disease free equilibrium $E_0 = (0,0)$ and an endemic equilibrium E_1 with $X > 0$ and $Y > 0$. We compute the basic reproduction number using the next generation operator at the disease free equilibrium $E_0 = (0,0)$ (e.g., [**4**] or [**7**]). We find that

$$R_0 = \sqrt{\frac{\beta_{mf}\beta_{fm}\nu_m\nu_f}{(\gamma_m + \psi_m)(\gamma_f + \psi_f)}}. \tag{3.1}$$

The basic reproduction number R_0 represents the number of secondary infections generated by a typical individual from the core population of susceptibles S_f and S_m at the disease free (E_0) equilibrium. The local and global stability of the disease free equilbrium (E_0) is thoroughly investigated in [**14**] for a structurally similar model and they show that it is globally stable for $R_0 < 1$. Here we analyze the stability of the endemic equilibrium(E_1).

The endemic equilibrium $E_1 = (\tilde{X}, \tilde{Y})$ is

$$\tilde{X} = \frac{\beta_{mf}\beta_{fm}\nu_m\nu_f - (\gamma_m + \psi_m)(\gamma_f + \psi_f)}{\beta_{fm}\nu_f(\beta_{mf}\nu_m + \gamma_f + \psi_f)}, \tag{3.2a}$$

$$\tilde{Y} = \frac{\beta_{mf}\beta_{fm}\nu_m\nu_f - (\gamma_m + \psi_m)(\gamma_f + \psi_f)}{\beta_{mf}\nu_m(\beta_{fm}\nu_f + \gamma_m + \psi_m)}. \tag{3.2b}$$

The values of $\tilde{I}_f$, $\tilde{S}_f$ $\tilde{I}_m$, and $\tilde{S}_m$ are given by

$$\begin{aligned} \tilde{I}_f &= \tilde{X}N_f, \quad \text{and} \quad \tilde{S}_f = \left(1 - \tilde{X}\right)N_f, \\ \tilde{I}_m &= \tilde{Y}N_m, \quad \text{and} \quad \tilde{S}_m = \left(1 - \tilde{Y}\right)N_m. \end{aligned} \tag{3.3}$$

Linearized Jacobian of (2.3) at E_1 is

$$\mathbf{J} = \begin{bmatrix} -\frac{\beta_{fm}\nu_f(\beta_{mf}\nu_m+\gamma_f+\psi_f)}{\beta_{fm}\nu_f+\gamma_m+\psi_m} & \frac{\beta_{mf}\nu_m(\gamma_f+\mu_f)(\beta_{fm}\nu_f+\gamma_m+\psi_m)}{\beta_{fm}\nu_f(\beta_{mf}\nu_m+\gamma_f+\psi_f)} \\ \frac{\beta_{fm}\nu_f(\gamma_m+\psi_m)(\beta_{mf}\nu_m+\gamma_f+\psi_f)}{\beta_{mf}\nu_m(\beta_{fm}\nu_f+\gamma_m+\psi_m)} & -\frac{\beta_{mf}\nu_m(\beta_{fm}\nu_f+\gamma_m+\psi_m)}{\beta_{mf}\nu_m+\gamma_f+\psi_f} \end{bmatrix}.$$

Denote by r_{mf} the number of infections an infected male truck driver causes in females in a disease-free population before the first infected female is removed from the sexually active population (e.g., by progression to AIDS, or death, etc.), and by r_{fm} the corresponding value for infections caused by females. That is,

$$r_{mf} = \frac{\beta_{mf}\nu_m}{\gamma_f + \psi_f} \quad \text{and} \quad r_{fm} = \frac{\beta_{fm}\nu_f}{\gamma_m + \psi_m}. \tag{3.4}$$

Note that this is not strictly analogous to the biological definition of the basic reproductive number since the basis of comparison is not the time before the infecting agent has recovered but the time before the first *infected* has recovered. Observe that the basic reproductive number is the geometric mean of the above values $R_0 = \sqrt{r_{mf} \cdot r_{fm}}$. Make the substitutions for $\beta_{mf}\nu_f$ and $\beta_{fm}\nu_m$

$$\begin{aligned} \dot{X} &= (\gamma_f + \psi_f)\left(\tfrac{R_0^2}{r_{fm}}Y(1 - X) - X\right), \\ \dot{Y} &= (\gamma_m + \psi_m)(r_{fm}X(1 - Y) - Y). \end{aligned}$$

Rewriting the endemic equilibrium value $\tilde{X}$ and $\tilde{Y}$ in terms of r_{mf} and r_{fm}, we obtain

$$\begin{aligned} \tilde{X} &= \tfrac{r_{mf}r_{fm}-1}{r_{fm}(r_{mf}+1)}, \\ \tilde{Y} &= \tfrac{r_{mf}r_{fm}-1}{r_{mf}(r_{fm}+1)}. \end{aligned} \tag{3.5}$$

The eigenvalues of the Jacobian always have negative real parts when the determinant of $\mathbf{J}$ is positive. Observe that

$$\begin{aligned} \operatorname{trace}(\mathbf{J}) &= -\left(\frac{\beta_{mf}\nu_m(\beta_{fm}\nu_f + \gamma_m + \psi_m)^2 + \beta_{fm}\nu_f(\beta_{mf}\nu_m + \gamma_f + \psi_f)^2}{(\beta_{mf}\nu_m + \gamma_f + \psi_f)(\beta_{fm}\nu_f + \gamma_m + \psi_m)}\right) \\ &= -\left(\beta_{fm}\nu_f\left(\frac{r_{mf}(r_{fm}+1)}{r_{fm}(r_{mf}+1)}\right) + \beta_{mf}\nu_m\left(\frac{r_{fm}(r_{mf}+1)}{r_{mf}(r_{fm}+1)}\right)\right) \end{aligned}$$

is always negative, and since

$$\begin{aligned}\det(\mathbf{J}) &= \beta_{mf}\beta_{fm}\nu_m\nu_f - (\gamma_f+\psi_f)(\gamma_m+\psi_m)\\ &= \beta_{mf}\beta_{fm}\nu_m\nu_f\left(\frac{r_{mf}r_{fm}-1}{r_{mf}r_{fm}}\right)\\ &= \frac{\beta_{mf}\beta_{fm}\nu_m\nu_f}{R_0^2}\left(R_0^2-1\right),\end{aligned}$$

the endemic equilibrium is locally stable when $R_0 > 1$. Also there are no complex eigenvalues (and thus, no periodic solutions or spiral sinks). For there to be complex eigenvalues

$$(\text{trace}(\mathbf{J}))^2 < 4\det(\mathbf{J}),$$

which is

$$\left(\frac{\beta_{fm}\nu_f r_{mf}(r_{fm}+1)}{r_{fm}(r_{mf}+1)} + \frac{\beta_{mf}\nu_m r_{fm}(r_{mf}+1)}{r_{mf}(r_{fm}+1)}\right)^2 < \frac{4\beta_{mf}\beta_{fm}\nu_m\nu_f(r_{mf}r_{fm}-1)}{r_{mf}r_{fm}}.$$

When $R_0 > 1$, this is equivalent to

$$\left(\frac{r_{mf}r_{fm}}{r_{mf}r_{fm}-1}\right) < \frac{4\beta_{mf}\beta_{fm}\nu_m\nu_f}{\left(\beta_{fm}\nu_f\left(\frac{r_{mf}(r_{fm}+1)}{r_{fm}(r_{mf}+1)}\right) + \beta_{mf}\nu_m\left(\frac{r_{fm}(r_{mf}+1)}{r_{mf}(r_{fm}+1)}\right)\right)^2}. \tag{3.6}$$

Furthermore, the left hand side

$$\left(\frac{r_{mf}r_{fm}}{r_{mf}r_{fm}-1}\right) = \frac{R_0^2}{R_0^2-1} > 1.$$

Now on the right hand side we can rearrange to get

$$\frac{4}{\left(\sqrt{\frac{\beta_{fm}\nu_f}{\beta_{mf}\nu_m}}\left(\frac{r_{mf}(r_{fm}+1)}{r_{fm}(r_{mf}+1)}\right) + \sqrt{\frac{\beta_{mf}\nu_m}{\beta_{fm}\nu_f}}\left(\frac{r_{fm}(r_{mf}+1)}{r_{mf}(r_{fm}+1)}\right)\right)^2},$$

which is of the form $\frac{4}{(a+\frac{1}{a})^2}$, the denominator of which is a minimum (equals 4) when $a = 1$. That is

$$1 \geq \frac{4}{\left(a+\frac{1}{a}\right)^2}.$$

Thus, we always have

$$1 \geq \frac{4}{\left(\sqrt{\frac{\beta_{fm}\nu_f}{\beta_{mf}\nu_m}}\left(\frac{r_{mf}(r_{fm}+1)}{r_{fm}(r_{mf}+1)}\right) + \sqrt{\frac{\beta_{mf}\nu_m}{\beta_{fm}\nu_f}}\left(\frac{r_{fm}(r_{mf}+1)}{r_{mf}(r_{fm}+1)}\right)\right)^2}.$$

Hence,

$$\left(\frac{r_{mf}r_{fm}}{r_{mf}r_{fm}-1}\right) > 1 \geq \frac{4\beta_{mf}\beta_{fm}\nu_m\nu_f}{\left(\beta_{fm}\nu_f\left(\frac{r_{mf}(r_{fm}+1)}{r_{fm}(r_{mf}+1)}\right) + \beta_{mf}\nu_m\left(\frac{r_{fm}(r_{mf}+1)}{r_{mf}(r_{fm}+1)}\right)\right)^2},$$

which contradicts inequality (3.6), thus $(\text{trace}(\mathbf{J}))^2$ is always greater than $4\det(\mathbf{J})$ and no complex eigenvalues exist for the stable endemic equilibrium. Global stability of the endemic equilibrium can be demonstrated for $R_0 > 1$. We change coordinates of the system to put the origin at the endemic equilibrium. Let $U = 1 - X/\tilde{X}$ and $V = 1 - Y/\tilde{Y}$ then

$$X = \frac{r_{mf}r_{fm}-1}{r_{fm}(r_{mf}+1)}(1-U) \quad \text{and} \quad Y = \frac{r_{mf}r_{fm}-1}{r_{mf}(r_{fm}+1)}(1-V).$$

Making the substitution $r_{mf} = R_0^2/r_{fm}$, we obtain

$$
\begin{aligned}
\dot{U} &= (\gamma_f + \psi_f)\left((1-U) - (1-V)\left(\tfrac{\left(R_0^2 + r_{fm}\right)}{(r_{fm}+1)} - \tfrac{\left(R_0^2-1\right)(1-U)}{(r_{fm}+1)}\right)\right), \\
\dot{V} &= (\gamma_m + \psi_m)\left((1-V) - (1-U)\left(\tfrac{R_0^2(1+r_{fm})}{\left(R_0^2+r_{fm}\right)} - \tfrac{r_{fm}\left(R_0^2-1\right)(1-V)}{\left(R_0^2+r_{fm}\right)}\right)\right).
\end{aligned} \tag{3.7}
$$

$$
\begin{aligned}
L_{E_1} &= \frac{(r_{fm}+1)}{(\gamma_f+\psi_f)}U + \frac{\left(R_0^2 + r_{fm}\right)}{(\gamma_m+\psi_m)}V \\
\dot{L}_{E_1} &= -V\left(R_0^2 - 1\right)(1-U)(r_{fm}+1)
\end{aligned}
$$

For $V > 0$ then $L_{E_1} > 0$ and when $R_0 > 1 \Longrightarrow \dot{L} < 0$. For $V < 0$ we can make the substitution $r_{fm} = R_0^2/r_{mf}$ and by symmetry we have that when $U > 0$ then $L_{E_1} > 0$ and when $R_0 > 1 \Longrightarrow \dot{L}_{E_1} < 0$. When both $X > \tilde{X}$ and $Y > \tilde{Y}$ then we let $U = X/\tilde{X} - 1$ and $V = Y/\tilde{Y} - 1$ and the Lyapunov function (same as above) yields

$$\dot{L}_{E_1} = -V\left(R_0^2 - 1\right)(U+1)(r_{fm}+1)$$

Thus these facts plus the local stability gives global stability of the endemic equilibrium when $R_0 > 1$

4. Sensitivity Analysis

In this section, we carry out a sensitivity analysis on R_0. For our measure of sensitivity we use the normalized sensitivity index (elastisity) of R_0 with respect to each of the eight parameters (see [**11**] or chapter 9 in [**5**] for an excellent review of methods). This approach is a local measure of changes in R_0 when parameter values change by a small amount.

The basic reproductive number is a function of six parameters

$$R_0(\beta_{mf}, \beta_{fm}, \nu_f, \nu_m, \gamma_m, \gamma_f, \psi_m, \psi_f) = \sqrt{\frac{\beta_{mf}\beta_{fm}\nu_m\nu_f}{(\gamma_m+\psi_m)(\gamma_f+\psi_f)}}.$$

The sensitivity indices of each of the eight parameters are:

$$\frac{\beta_{fm}}{R_0}\frac{\partial R_0}{\partial \beta_{fm}} = \frac{1}{2} \quad \text{and} \quad \frac{\beta_{mf}}{R_0}\frac{\partial R_0}{\partial \beta_{mf}} = \frac{1}{2},$$

$$\frac{\nu_m}{R_0}\frac{\partial R_0}{\partial \nu_m} = \frac{1}{2} \quad \textit{and} \quad \frac{\nu_f}{R_0}\frac{\partial R_0}{\partial \nu_f} = \frac{1}{2},$$

$$\frac{\gamma_m}{R_0}\frac{\partial R_0}{\partial \gamma_m} = -\frac{1}{2}\frac{\gamma_m}{(\gamma_m+\psi_m)} = -0.3869,$$

$$\frac{\gamma_f}{R_0}\frac{\partial R_0}{\partial \gamma_f} = -\frac{1}{2}\frac{\gamma_f}{(\gamma_f+\psi_f)} = -0.3681,$$

$$\frac{\psi_m}{R_0}\frac{\partial R_0}{\partial \psi_m} = -\frac{1}{2}\frac{\psi_m}{(\gamma_m+\psi_m)} = -0.1131,$$

and

$$\frac{\psi_f}{R_0}\frac{\partial R_0}{\partial \psi_f} = -\frac{1}{2}\frac{\psi_f}{(\gamma_f+\psi_f)} = -0.1319.$$

From the above it follows that the basic reproductive number is less sensitive to changes in the rate at which the infected are being removed due to AIDS and the turnover rate than the rates of HIV transmission (in both classes).

The study of the dependence of the endemic equilibrium on HIV transmission parameters (β) will allow us to look at the impact of transmissions rate (for example, by truck drivers wearing condoms, assuming that condoms offer complete protection against infection) in reducing the HIV cases in both groups.

Suppose that a fraction s of truck drivers wear condoms while interacting with female sex workers. This implies that the HIV transmission rate will decrease by s, or equivalently, the transmission rates in both directions (from females to males and from males to females) will be reduced so that $\beta_i^{new} = k \cdot \beta_i^{old}$, where $k = 1 - s$, and $i = mf$ or fm (we use the same k for transmission in both directions). Hence, the values r_{fm} and r_{mf} will change to

$$r_{fm} \to k \cdot r_{fm}, \quad \text{and} \quad r_{mf} \to k \cdot r_{mf}. \tag{4.1}$$

Without loss of generality, consider only the truck driver group. Substituting (3.5) in (3.3) we modify the expression using (4.1), the new value $\tilde{I}_m^{new}$ of the infected truck drivers at the endemic equilibrium (assuming that a fraction k of them will not wear condoms) will be

$$\tilde{I}_m^{new} = \frac{k^2 r_{mf} r_{fm} - 1}{k r_{mf} \left(k r_{fm} + 1\right)} N_m, \tag{4.2}$$

Thus, we have defined a function $\tilde{I}_m(k)$ where $0 < k \leq 1$. In order to determine the effect from wearing condoms by truck drivers in the endemic population, we focus on the quotient

$$J_m(k) = \frac{\tilde{I}_m(k)}{\tilde{I}_m(1)}. \tag{4.3a}$$

And similarly for sex workers we have

$$J_f(k) = \frac{\tilde{I}_f(k)}{\tilde{I}_f(1)}. \tag{4.3b}$$

The left hand side in the above expression indicates the fraction of the initial endemic number of infected truck drivers. To obtain the percentage *decrease* in endemic HIV cases among truck drivers, we calculate $\mathcal{P}(s) = (1 - J(1-s)) \cdot 100\%$.

The results for the available data (Section 6) is given below. The dependence of $\mathcal{P}$ on s for truck drivers ($\mathcal{P}_m$) and sex workers ($\mathcal{P}_f$) is given in Figure 2.

This graph shows the high and low HIV prevalence estimates due to variation in transmission rates based on 95% C.I. (see more details in Section 6). Selected

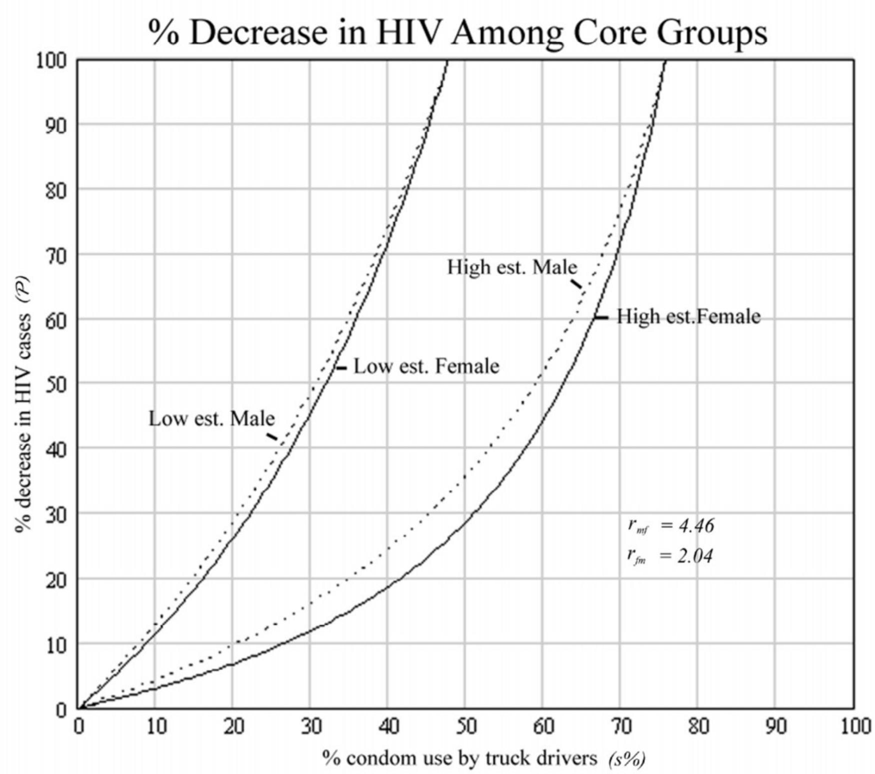

FIGURE 2. The effect of condom use by truck drivers on prevalence of HIV in the core populations.

values are given in Table 2.

% of Truck Drivers wearing condoms	% decrease in HIV cases among Truck Drivers	% decrease in HIV cases among Sex Workers
10%	4-12%	3-11%
20%	10-27%	7-24%
30%	16-46%	12-42%
40%	25-70%	19-67%
50%	37-100%	29-100%
60%	53-100%	45-100%
70%	79-100%	74-100%
80%	100%	100%
90%	100%	100%

Table2. Effect of condom use on HIV cases

Interpretation: If 50% of truck drivers use condoms, HIV cases will be reduced among them by 37-100% and among female sex workers by 29-100%.

What percent of truck drivers would need to wear condoms in order to reduce the prevalence by half (50%)? According to data we have gathered and sensitivity analysis (Section 4), we calculate the value s when $\mathcal{P}(s)$ =32 - 59% condom usage by truck drivers will reduce HIV cases among truck drivers by half (50%); similarly, 33 - 62% condom usage among truck drivers will reduce HIV cases in female sex workers by 50%.

5. General Population model

In this section we couple the core population with the general population. The aim is to assess the impact of the core on the spread of the disease on the general population. We assume that the role of HIV infection in the general population is directly tied in to core prevalence. This assumption is plausible since the core has sexual partners in the non-core population, and thus, any measure of sexual activity and disease transmission in the core group has a direct, albeit obscure, effect on the general population. We introduce a model that connects the general population and core groups, see Figure 3. Under some plausible assumptions the parameter space can be effectively reduced. Let the disease transmission rates between the different groups be defined as in Table 3.

from ↓ \ to ⟶	females general pop.	males general pop.	females sex workers.	males transport.
females general pop.		β_{f1m1}		β_{f1m2}
males general pop.	β_{m1f1}		β_{m1f2}	
female sex workers.		β_{f2m1}		β_{f2m2}
males transport.	β_{m2f1}		β_{m2f2}	

Table 3. Infection rates disease transmission between various subpopulations.

Note that here β_{m2f2} and β_{f2m2} are equivalent to β_{mf} and β_{fm} respectively from the isolated core model. We assume that the rate of progression to AIDS is the same for males and females ($\gamma_f = \gamma_m = \gamma$). We also use the same value of μ_f for sex workers and women in the general population and the same value of μ_m for truck drivers and men in the general population.

For model including both males and female in core and non-core groups is as follows. Let S_{m1} and S_{f1} denote the number of susceptible males and females in the general population respectively, and let I_{m1} and I_{f1} be the number of infected males and females in the general population respectively. Let S_{m2} and S_{f2} denote the number of susceptible truck drivers and female sex workers respectively, and let I_{m2} and I_{f2} be the number of infected male truck drivers and females sex workers, respectively. The parameters ρ_{f2} and ρ_{m2} are the rates of retirement. We assume that the entire population is well mixed N_m and N_f are now the entire population of sexually active males and females respectively. The dynamics of the core group is the same as in system 2.1 except that there is an additional source of infection from the general population (again only heterosexual contacts are assumed). The dynamics of the general population mirrors that of the core group so that losses due to retirement from the core group are gains to the non core groups, and the uninfected non-core groups are the source of replacement

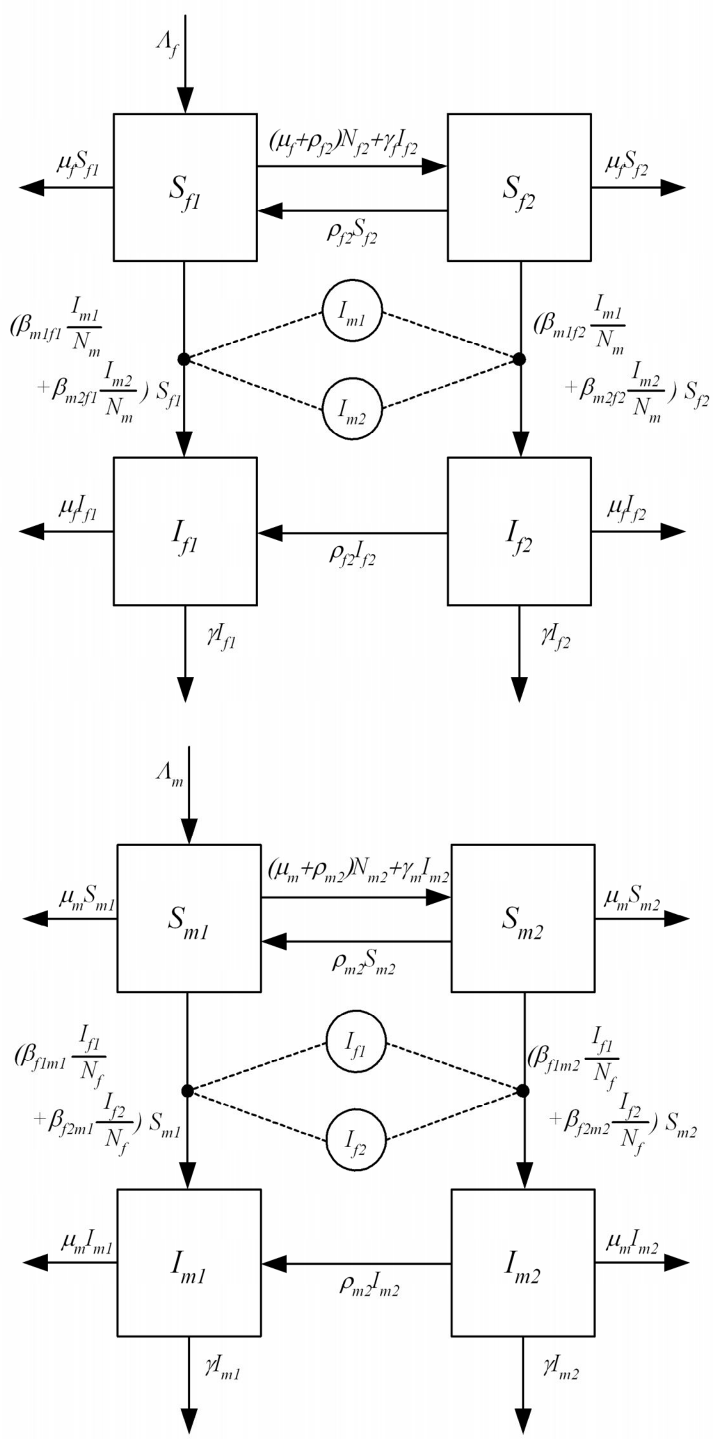

FIGURE 3. A compartmental diagram for the system of equations in (5.1).

workers. Overall recruitment to the general (sexually active) population is at constant rate Λ and consists of uninfected non-core individuals only. The following system of differential equations describes the demographic and HIV dynamics in these subpopulations.

$$
\begin{aligned}
\dot{S}_{f1} &= \Lambda_f - \left(\beta_{m1f1}\tfrac{I_{m1}}{N_m} + \beta_{m2f1}\tfrac{I_{m2}}{N_m}\right) S_{f1} + \rho_{f2} S_{f2} \\
&\qquad - \left(\left(\mu_f + \rho_{f2}\right) N_{f2} + \gamma I_{f2}\right) - \mu_f S_{f1}, \\
\dot{I}_{f1} &= \left(\beta_{m1f1}\tfrac{I_{m1}}{N_m} + \beta_{m2f1}\tfrac{I_{m2}}{N_m}\right) S_{f1} + \rho_{f2} I_{f2} - \left(\mu_f + \gamma\right) I_{f1}, \\
\dot{S}_{m1} &= \Lambda_m - \left(\beta_{f1m1}\tfrac{I_{f1}}{N_f} + \beta_{f2m1}\tfrac{I_{f2}}{N_f}\right) S_{m1} + \rho_{m2} S_{m2} \\
&\qquad - \left(\left(\mu_m + \rho_{m2}\right) N_{m2} + \gamma I_{m2}\right) - \mu_m S_{m1}, \\
\dot{I}_{m1} &= \left(\beta_{f1m1}\tfrac{I_{f1}}{N_f} + \beta_{f2m1}\tfrac{I_{f2}}{N_f}\right) S_{m1} + \rho_{m2} I_{m2} - \left(\mu_m + \gamma\right) I_{m1}, \\
\dot{S}_{f2} &= \left(\left(\mu_f + \rho_{f2}\right) N_{f2} + \gamma I_{f2}\right) - \left(\beta_{m1f2}\tfrac{I_{m1}}{N_m} + \beta_{m2f2}\tfrac{I_{m2}}{N_m}\right) S_{f2} \\
&\qquad - \left(\mu_f + \rho_{f2}\right) S_{f2}, \\
\dot{I}_{f2} &= \left(\beta_{m1f2}\tfrac{I_{m1}}{N_m} + \beta_{m2f2}\tfrac{I_{m2}}{N_m}\right) S_{f2} - \left(\mu_f + \rho_{f2} + \gamma\right) I_{f2}, \\
\dot{S}_{m2} &= \left(\left(\mu_m + \rho_{m2}\right) N_{m2} + \gamma I_{m2}\right) - \left(\beta_{f1m2}\tfrac{I_{f1}}{N_f} + \beta_{f2m2}\tfrac{I_{f2}}{N_f}\right) S_{m2} \\
&\qquad - \left(\mu_m + \rho_{m2}\right) S_{m2}, \\
\dot{I}_{m2} &= \left(\beta_{f1m2}\tfrac{I_{f1}}{N_f} + \beta_{f2m2}\tfrac{I_{f2}}{N_f}\right) S_{m2} - \left(\mu_m + \rho_{m2} + \gamma\right) I_{m2}.
\end{aligned} \tag{5.1}
$$

The total subpopulations are

$$
\begin{aligned}
N_{f1} &= S_{f1} + I_{f1}, & N_{f2} &= S_{f2} + I_{f2}, \\
N_{m1} &= S_{m1} + I_{m1}, & N_{m2} &= S_{m2} + I_{m2}, \\
N_f &= N_{f1} + N_{f2}, & N_m &= N_{m1} + N_{m2}.
\end{aligned}
$$

The rates of change of the total subpopulations are

$$
\begin{aligned}
\dot{N}_{f1} &= \dot{S}_{f1} + \dot{I}_{f1} = \Lambda_f - \mu_f \left(N_{f1} + N_{f2}\right) - \gamma \left(I_{f1} + I_{f2}\right), \\
\dot{N}_{m1} &= \dot{S}_{f2} + \dot{I}_{f2} = \Lambda_m - \mu_m \left(N_{m1} + N_{m2}\right) - \gamma \left(I_{m2} + I_{m1}\right), \\
\dot{N}_{f2} &= \dot{S}_{f2} + \dot{I}_{f2} = 0, \\
\dot{N}_{m2} &= \dot{S}_{m2} + \dot{I}_{m2} = 0.
\end{aligned}
$$

Note that if the recruitment is constant (on the time scale of interest), there will be an equilibrium such that in the disease free population we have

$$
\begin{aligned}
\mu_f \left(\tilde{N}_{f1} + \tilde{N}_{f2}\right) &= \Lambda_f, \\
\mu_m \left(\tilde{N}_{m1} + \tilde{N}_{m2}\right) &= \Lambda_m.
\end{aligned}
$$

Once the endemic equilibrium is reached, we have

$$
\begin{aligned}
\mu_f \left(\tilde{N}_{f1} + \tilde{N}_{f2}\right) + \gamma \left(\tilde{I}_{f1} + \tilde{I}_{f2}\right) &= \Lambda_f, \\
\mu_m \left(\tilde{N}_{m1} + \tilde{N}_{m2}\right) + \gamma \left(\tilde{I}_{m2} + \tilde{I}_{m1}\right) &= \Lambda_m.
\end{aligned}
$$

If we assume that the general population reaches equilibrium quickly in the disease free state, and does not change much due to the disease (an appropriate assumption when the prevalence is low in the general population and the general population is large relative to the core population), then we have

$$\dot{N}_{f1} = \dot{S}_{f1} + \dot{I}_{f1} \approx 0, \qquad \dot{N}_{f2} = \dot{S}_{f2} + \dot{I}_{f2} = 0,$$
$$\dot{N}_{m1} = \dot{S}_{m1} + \dot{I}_{m1} \approx 0, \qquad \dot{N}_{m2} = \dot{S}_{m2} + \dot{I}_{m2} = 0.$$

The total subpopulations are constant and we can reduce the system to

$$\begin{aligned}
\dot{I}_{f1} &= \left(\beta_{m1f1}\tfrac{I_{m1}}{N_m} + \beta_{m2f1}\tfrac{I_{m2}}{N_m}\right)(N_{f1} - I_{f1}) - (\mu_f + \gamma)\, I_{f1} + \rho_{f2} I_{f2},\\
\dot{I}_{m1} &= \left(\beta_{f1m1}\tfrac{I_{f1}}{N_f} + \beta_{f2m1}\tfrac{I_{f2}}{N_f}\right)(N_{m1} - I_{m1}) - (\mu_m + \gamma)\, I_{m1} + \rho_{m2} I_{m2},\\
\dot{I}_{f2} &= \left(\beta_{m1f2}\tfrac{I_{m1}}{N_m} + \beta_{m2f2}\tfrac{I_{m2}}{N_m}\right)(N_{f2} - I_{f2}) - (\mu_f + \rho_{f2} + \gamma)\, I_{f2},\\
\dot{I}_{m2} &= \left(\beta_{f1m2}\tfrac{I_{f1}}{N_f} + \beta_{f2m2}\tfrac{I_{f2}}{N_f}\right)(N_{m2} - I_{m2}) - (\mu_m + \rho_{m2} + \gamma)\, I_{m2}.
\end{aligned} \tag{5.2}$$

To re-express the above as fractions of the subpopulations, we let

$$X_1 = \tfrac{I_{f1}}{N_{f1}}, \quad X_2 = \tfrac{I_{f2}}{N_{f2}}, \quad Y_1 = \tfrac{I_{m1}}{N_{m1}}, \quad \text{and} \quad Y_2 = \tfrac{I_{m2}}{N_{m2}}.$$

The proportions of the subpopulations by gender is

$$\nu_{f1} = \tfrac{N_{f1}}{N_f}, \quad \nu_{f2} = \tfrac{N_{f2}}{N_f}, \quad \nu_{m1} = \tfrac{N_{m1}}{N_m}, \quad \text{and} \quad \nu_{m2} = \tfrac{N_{m2}}{N_m}.$$

Then we have

$$\begin{aligned}
\dot{X}_1 &= (\beta_{m1f1}\nu_{m1}Y_1 + \beta_{m2f1}\nu_{m2}Y_2)(1 - X_1) - (\mu_f + \gamma)\, X_1 + \rho_{f2}\tfrac{N_{u2}}{N_{u1}} X_2,\\
\dot{Y}_1 &= (\beta_{f1m1}\nu_{f1}X_1 + \beta_{f2m1}\nu_{f2}X_2)(1 - Y_1) - (\mu_m + \gamma)\, Y_1 + \rho_{m2}\tfrac{N_{m2}}{N_{m1}} Y_{m2},\\
\dot{X}_2 &= (\beta_{m1f2}\nu_{m1}Y_1 + \beta_{m2f2}\nu_{m2}Y_2)(1 - X_2) - (\mu_f + \rho_{f2} + \gamma)\, X_2,\\
\dot{Y}_2 &= (\beta_{f1m2}\nu_{f1}X_1 + \beta_{f2m2}\nu_{f2}X_2)(1 - Y_2) - (\mu_m + \rho_{m2} + \gamma)\, Y_2.
\end{aligned} \tag{5.3}$$

We note that the terms $\rho_{f2}\frac{N_{f2}}{N_{f1}}X_2$ and $\rho_{m2}\frac{N_{m2}}{N_{m1}}Y_{m2}$ are very small for Nigeria (on the order of $10^{-4} - 10^{-3}$ whereas the smallest of the other terms are $\sim 10^{-2}$). Substituting ψ_f for $\mu_f + \rho_{f2}$ and ψ_m for $\mu_m + \rho_{m2}$, we have

$$\begin{aligned}
\dot{X}_1 &= (\beta_{m1f1}\nu_{m1}Y_1 + \beta_{m2f1}\nu_{m2}Y_2)(1 - X_1) - (\mu_f + \gamma)\, X_1\\
\dot{Y}_1 &= (\beta_{f1m1}\nu_{f1}X_1 + \beta_{f2m1}\nu_{f2}X_2)(1 - Y_1) - (\mu_m + \gamma)\, Y_1,\\
\dot{X}_2 &= (\beta_{m1f2}\nu_{m1}Y_1 + \beta_{m2f2}\nu_{m2}Y_2)(1 - X_2) - (\psi_f + \gamma)\, X_2,\\
\dot{Y}_2 &= (\beta_{f1m2}\nu_{f1}X_1 + \beta_{f2m2}\nu_{f2}X_2)(1 - Y_2) - (\psi_m + \gamma)\, Y_2.
\end{aligned} \tag{5.4}$$

Define

$$\begin{aligned}
r_{m1f1} &= \tfrac{\beta_{m1f1}N_{m1}}{(\mu_f+\gamma)N_m}, \quad & r_{f1m1} &= \tfrac{\beta_{f1m1}N_{f1}}{(\mu_m+\gamma)N_f},\\
r_{m1f2} &= \tfrac{\beta_{m1f2}N_{m1}}{(\psi_f+\gamma)N_m}, \quad & r_{f1m2} &= \tfrac{\beta_{f1m2}N_{f1}}{(\psi_m+\gamma)N_f},\\
r_{m2f1} &= \tfrac{\beta_{m2f1}N_{m2}}{(\mu_f+\gamma)N_m}, \quad & r_{f2m1} &= \tfrac{\beta_{f2m1}N_{f2}}{(\mu_m+\gamma)N_f},\\
r_{m2f2} &= \tfrac{\beta_{m2f2}N_{m2}}{(\psi_f+\gamma)N_m}, \quad & r_{f2m2} &= \tfrac{\beta_{f2m2}N_{f2}}{(\psi_m+\gamma)N_f}.
\end{aligned} \tag{5.5}$$

These parameters are interpreted as the number of new cases within each subpopulation before the first case in that subpopulation recovers (when the disease is introduced to a disease free subpopulation) times the fraction of that subpopulation (sex specific). By using the second generation operator analysis for the core population, we find the basic reproductive number for this system

$$R_0 = \sqrt{\tfrac{1}{2}\left(r_{f1m1}r_{m1f1} + r_{m1f2}r_{f2m1} + r_{f1m2}r_{m2f1} + r_{m2f2}r_{f2m2}\right)} \times\sqrt{1+\sqrt{1+\tfrac{4(r_{m1f1}r_{m2f2}-r_{m1f2}r_{m2f1})(r_{f1m2}r_{f2m1}-r_{f2m2}r_{f1m1})}{(r_{f1m1}r_{m1f1}+r_{m1f2}r_{f2m1}+r_{f1m2}r_{m2f1}+r_{m2f2}r_{f2m2})^2}}}. \tag{5.6}$$

The endemic equilibrium is the solution of the four equations

$$\begin{aligned} \tfrac{\tilde{X}_1}{1-\tilde{X}_1} &= r_{m1f1}\tilde{Y}_1 + r_{m2f1}\tilde{Y}_2, \\ \tfrac{\tilde{Y}_1}{1-\tilde{Y}_1} &= r_{f1m1}\tilde{X}_1 + r_{f2m1}\tilde{X}_2, \\ \tfrac{\tilde{X}_2}{1-\tilde{X}_2} &= r_{m1f2}\tilde{Y}_1 + r_{m2f2}\tilde{Y}_2, \\ \tfrac{\tilde{Y}_2}{1-\tilde{Y}_2} &= r_{f1m2}\tilde{X}_1 + r_{f2m2}\tilde{X}_2. \end{aligned} \tag{5.7}$$

Since the fraction of the general population that is infected with HIV is about 0.05 in Nigeria, we assume that the fractions of the general population that are infected $\left(\tilde{X}_1 \text{and } \tilde{Y}_1\right)$ are small, and thus,

$$\begin{aligned} \tilde{X}_2 &\approx \tfrac{r_{f2m2}r_{m2f2}-1}{r_{f2m2}(r_{m2f2}+1)}, \\ \tilde{Y}_2 &\approx \tfrac{r_{f2m2}r_{m2f2}-1}{r_{m2f2}(r_{f2m2}+1)}, \end{aligned} \tag{5.8}$$

which is the same equilibrium as in the analysis of the isolated core groups. Using these values for a quasi-steady state analysis gives us the following expression for the general population equations

$$\begin{aligned} \dot{X}_1 &\approx (\mu_f + \gamma)\left(\left(r_{m1f1}Y_1 + r_{m2f1}\tilde{Y}_2\right)(1-X_1) - X_1\right), \\ \dot{Y}_1 &\approx (\mu_m + \gamma)\left(\left(r_{f1m1}X_1 + r_{f2m1}\tilde{X}_2\right)(1-Y_1) - Y_1\right). \end{aligned} \tag{5.9}$$

Assumption of a low disease level in the general population gives

$$\begin{aligned} \dot{X}_1 &\approx (\mu_f + \gamma)\left(r_{m2f1}\tilde{Y}_2(1-X_1) - X_1\right), \\ \dot{Y}_1 &\approx (\mu_m + \gamma)\left(r_{f2m1}\tilde{X}_2(1-Y_1) - Y_1\right). \end{aligned} \tag{5.10}$$

Integrating (5.10), we obtain

$$\begin{aligned} \int \tfrac{dX_1}{r_{m2f1}\tilde{Y}_2-\left(r_{m2f1}\tilde{Y}_2+1\right)X_1} &\approx \int (\mu_f+\gamma)\,dt, \\ \int \tfrac{dY_1}{r_{f2m1}\tilde{X}_2-\left(r_{f2m1}\tilde{X}_2+1\right)Y_1} &\approx \int (\mu_m+\gamma)\,dt. \end{aligned} \tag{5.11}$$

The equations in (5.11) have solutions

$$\begin{aligned} X_1 &\approx \tfrac{r_{m2f1}\tilde{Y}_2}{\left(r_{m2f1}\tilde{Y}_2+1\right)} + \left(X_1(t_0) - \tfrac{r_{m2f1}\tilde{Y}_2}{\left(r_{m2f1}\tilde{Y}_2+1\right)}\right)e^{-\left(r_{m2f1}\tilde{Y}_2+1\right)(\mu_f+\gamma)t}, \\ Y_1 &\approx \tfrac{r_{f2m1}\tilde{X}_2}{\left(r_{f2m1}\tilde{X}_2+1\right)} + \left(Y_1(t_0) - \tfrac{r_{f2m1}\tilde{X}_2}{\left(r_{f2m1}\tilde{X}_2+1\right)}\right)e^{-\left(r_{f2m1}\tilde{X}_2+1\right)(\mu_m+\gamma)t}. \end{aligned} \tag{5.12}$$

Putting these values back into the original equations and letting the core groups vary with time, we obtain

(5.13)
$$\begin{aligned}\dot{X}_1 &\approx (\mu_f+\gamma)\Bigg[\Bigg(r_{m1f1}\Bigg[\tfrac{r_{f2m1}X_2}{r_{f2m1}X_2+1}+\cdots\\ &\quad+\Bigg(X_1(t_0)-\tfrac{r_{f2m1}X_2}{r_{f2m1}X_2+1}\Bigg)e^{-(r_{f2m1}X_2+1)(\mu_{m1}+\gamma)t}\Bigg]+r_{m2f1}Y_2\Bigg)(1-X_1)-X_1\Bigg],\\ \dot{Y}_1 &\approx (\mu_m+\gamma)\Bigg[\Bigg(r_{f1m1}\Bigg[\tfrac{r_{m2f1}Y_2}{(r_{m2f1}Y_2+1)}+\cdots\\ &\quad+\Bigg(Y_1(t_0)-\tfrac{r_{m2f1}Y_2}{(r_{m2f1}Y_2+1)}\Bigg)e^{-(r_{m2f1}Y_2+1)(\mu_{f1}+\gamma)t}\Bigg]+r_{f2m1}X_2\Bigg)(1-Y_1)-Y_1\Bigg].\end{aligned}$$

The general population dynamics are now expressed as a function of the core group. A further simplification of the above equations is

$$\begin{aligned}\dot{X}_1 &\approx (\mu_f+\gamma)\left(\left(r_{m1f1}\left(\tfrac{r_{f2m1}}{r_{f2m1}X_2+1}\right)X_2+r_{m2f1}Y_2\right)(1-X_1)-X_1\right),\\ \dot{Y}_1 &\approx (\mu_m+\gamma)\left(\left(r_{f1m1}\left(\tfrac{r_{m2f1}}{r_{m2f1}Y_2+1}\right)Y_2+r_{f2m1}X_2\right)(1-Y_1)-Y_1\right).\end{aligned} \tag{5.14}$$

The terms $\left(\dfrac{r_{f2m1}}{r_{f2m1}X_2+1}\right)$ and $\left(\dfrac{r_{m2f1}}{r_{m2f1}Y_2+1}\right)$ can be regarded as the "strength" of the network coupling the core group of one sex with the non-core group of the *same* sex. Hence, for the endemic equilibrium we have approximately

$$\tilde{X}_1 \approx \frac{r_{m2f1}\tilde{Y}_2\left(1+r_{f2m1}\tilde{X}_2\right)+r_{m1f1}r_{f2m1}\tilde{X}_2}{\left(1+r_{m2f1}\tilde{Y}_2\right)\left(1+r_{f2m1}\tilde{X}_2\right)+r_{m1f1}r_{f2m1}\tilde{X}_2}, \tag{5.15a}$$

$$\tilde{Y}_1 \approx \frac{r_{f2m1}\tilde{X}_2\left(1+r_{m2f1}\tilde{Y}_2\right)+r_{f1m1}r_{m2f1}\tilde{Y}_2}{\left(1+r_{f2m1}\tilde{X}_2\right)\left(1+r_{m2f1}\tilde{Y}_2\right)+r_{f1m1}r_{m2f1}\tilde{Y}_2}. \tag{5.15b}$$

We define the relative potential force of infection[1]

$$\phi_{mf}=\frac{r_{m1f1}}{r_{m2f1}}=\frac{\beta_{m1f1}N_{m1}}{\beta_{m2f1}N_{m2}},\quad\text{and}\quad \phi_{fm}=\frac{r_{f1m1}}{r_{f2m1}}=\frac{\beta_{f1m1}N_{f1}}{\beta_{f2m1}N_{f2}}. \tag{5.16}$$

Thus,

$$\tilde{X}_1 \approx \frac{r_{m2f1}\tilde{Y}_2\left(1+r_{f2m1}\tilde{X}_2\right)+\phi_{mf}r_{m2f1}r_{f2m1}\tilde{X}_2}{\left(1+r_{m2f1}\tilde{Y}_2\right)\left(1+r_{f2m1}\tilde{X}_2\right)+\phi_{mf}r_{m2f1}r_{f2m1}\tilde{X}_2}, \tag{5.17a}$$

$$\tilde{Y}_1 \approx \frac{r_{f2m1}\tilde{X}_2\left(1+r_{m2f1}\tilde{Y}_2\right)+\phi_{fm}r_{f2m1}r_{m2f1}\tilde{Y}_2}{\left(1+r_{f2m1}\tilde{X}_2\right)\left(1+r_{m2f1}\tilde{Y}_2\right)+\phi_{fm}r_{f2m1}r_{m2f1}\tilde{Y}_2}. \tag{5.17b}$$

---

[1] These parameters suggest a measure that can be used as an index of the degree that a core group has influence on the general population. In this case we would define the indicies $\varkappa_m = \frac{(\mu_m+\gamma_m)\beta_{m2f1}N_{m2}}{(\psi_m+\gamma_m)\beta_{m1f1}N_{m1}}$ and $\varkappa_f = \frac{(\mu_f+\gamma_f)\beta_{f2m1}N_{f2}}{(\psi_f+\gamma_f)\beta_{f1m1}N_{f1}}$. These are the relative reproductive numbers at the potential maximum number of infecteds for the respective subpopulations. But see [**16**] for other ideas with respect to vector bourne diseases (e.g. vectorial capacity).

Assuming that ϕ_{mf} and ϕ_{fm} are small (much less than 1), we get

$$\tilde{X}_1 \approx \frac{r_{m2f1}\tilde{Y}_2}{1+r_{m2f1}\tilde{Y}_2}, \tag{5.18a}$$

$$\tilde{Y}_1 \approx \frac{r_{f2m1}\tilde{X}_2}{1+r_{f2m1}\tilde{X}_2}. \tag{5.18b}$$

When the relative rates of infections are very small, we obtain

$$\tilde{X}_1 \approx r_{m2f1}\tilde{Y}_2, \tag{5.19a}$$

$$\tilde{Y}_1 \approx r_{f2m1}\tilde{X}_2. \tag{5.19b}$$

Prevention study. We look at the effects of condom use by truck drivers on decreasing HIV prevalence in the general population. In this case we make the following substitutions $r_{m2f2} \to kr_{m2f2}$, $r_{f2m2} \to kr_{f2m2}$, and $r_{m2f1} \to kr_{m2f1}$. Here, all rates of infection involving truck drivers are multiplied by the factor k. We substitute equation (4.2) in the modified equation (5.19a) to obtain

$$\tilde{X}_1^{new}(k) \approx r_{m2f1}\frac{k^3 r_{m2f2}r_{f2m2}-1}{kr_{m2f2}(kr_{f2m2}+1)} \tag{5.20}$$

Again, in order to determine the effect from wearing condoms by truck drivers in the general population, we focus on the quotient

$$J_{f1}(k) = \frac{\tilde{X}_1^{new}(k)}{\tilde{X}_1^{new}(1)} = kJ_{m2}(k). \tag{5.21}$$

where $J_{m2}(k)$ is the same as from the core group model 4.3a. The left hand side in the above expression indicates the fraction of the initial endemic number of infected women in the general population. To obtain the percentage *decrease* in endemic HIV cases among truck drivers, we calculate $\mathcal{P}_{f1}(s) = (1-J_{f1}(1-s))\cdot 100\%$. We find that there is a substantial decrease in the prevalence of HIV in general population women beyond that of the decrease in the prevalence among sex workers (compare Figure 4 with Figure 2). For males in the general population we have

$$\tilde{Y}_1^{new}(k) \approx r_{f1m2}\frac{k^2 r_{m2f2}r_{f2m2}-1}{kr_{f2m2}(kr_{m2f2}+1)} \tag{5.22}$$

In this case we obtain the quotient

$$J_{m1}(k) = \frac{\tilde{Y}_1^{new}(k))}{\tilde{Y}_1^{new}(k))} = J_{f2}(k). \tag{5.23}$$

where $J_{f2}(k)$ is the same as from the core group model 4.3b. The reduction in the male general population is the same as for female sex workers since that is the main source of disease in this group. However, without non-core males using condoms there is no additional effect of reduction in the transmission of HIV from sex workers to their clients, the only effect is that of reducing the probability of encountering an infected sex worker.

Under the hypothesis that all men use condoms, we obtain the quotient

$$J_{m1}(k) = \frac{\tilde{Y}_1^{new}(k))}{\tilde{Y}_1^{new}(k))} = kJ_{f2}(k). \tag{5.24}$$

In this case the results are dramatic, see Figure 5.

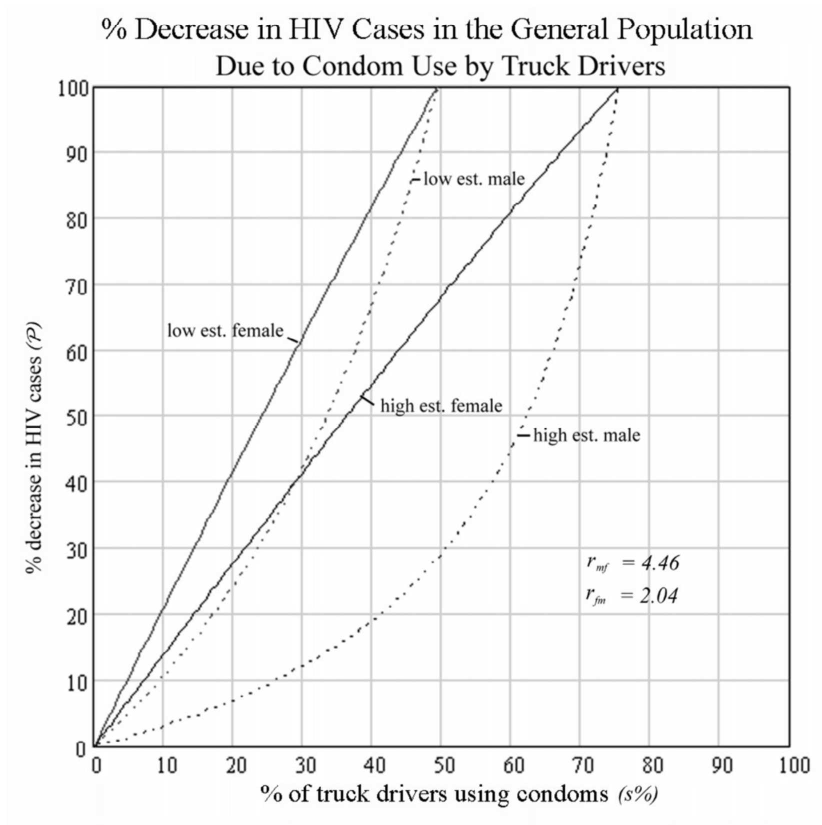

FIGURE 4. The effect of condom use by truck drivers on prevalence of HIV in the general population.

We now have a curve which is almost linear, meaning that effects of condom use on the general population are significantly greater than the effects within the core groups.

% of All men wearing condoms	% decrease in HIV cases among general pop. men	% decrease in HIV cases among general pop. women
10%	13-20%	14-21%
20%	26-40%	28-42%
30%	38-60%	41-62%
40%	51-80%	55-82%
50%	65-100%	68-100%
60%	78-100%	81-100%
70%	92-100%	94-100%
80%	100%	100%
90%	100%	100%

Table 4. Effect of condom use on HIV cases

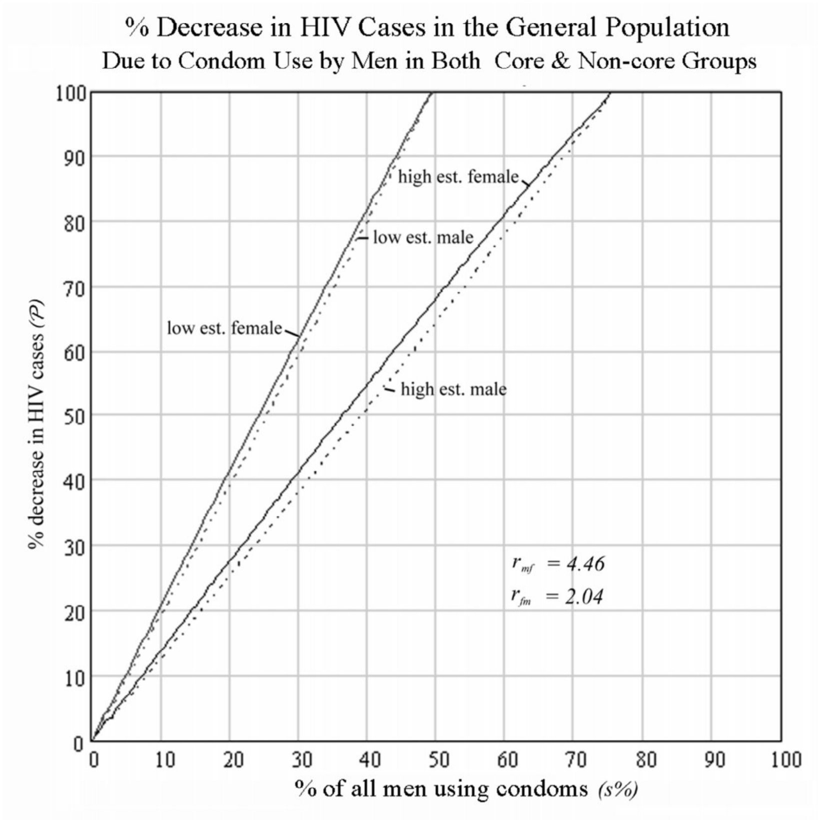

FIGURE 5. The effect of condom use by all men on prevalence of HIV in the general population.

Interpretation: If 50% of all men use condoms, HIV cases will be reduced among the men in the general population by 65 - 100% and among women by 68 - 100%.

What percent of men would need to wear condoms in order to reduce the prevalence by half (50%)? A range of 25 - 39% condom usage by all men will reduce HIV cases among men in the general population by half (50%); similarly, 24 - 36% condom usage all men will reduce HIV cases in women in the general population by half.

6. Parameters and Data Estimates

In this section we estimate the various parameters used in our model. We describe the model parameters and derivation of them. Due to unavailability of quality data our best estimates may be rather crude.

CONTACT RATES. In general the parameters β_{fm} and β_{mf} are roughly the reciprocal of the average length of time to become infectious with HIV from the time a person first becomes at risk (taken from the time entering the core group) if all encounters were with infected individuals. One approach to estimate these rates is to look at the behaviors and risks of the infection for the different groups. The rates of infectious contact, β_{fm} and β_{mf}, depend on the total number of contacts per person per year, and the probability of transmission per contact ([**18**] and [**14**]). The risks of infection from sexual transmission are quantified using epidemiological studies called "partners studies" (see [**3**]). Since most truck drivers are clients of sex workers, it is appropriate to consider the rate at which truckers are infecting the sex workers. Combining data from [**24**], [**17**] and [**15**], we obtain a range for the per contact probability of female to male transmission of $0.003-0.010$, and a range for the per contact probability of male to female transmission of $0.006-0.080$. From [**20**], [**21**] and [**9**], we gather that on average the range of contacts for a truck driver with sex workers is 3-6 times per week and the range of contacts for a female sex worker with truck drivers is 6-30 contacts per week. Calculating β as the probability of transmission per contact $\times$ number of contacts per person per time, we have

$$\begin{aligned}\beta_{fm} &= (0.003\times 3\text{contacts/week}\times 52\text{weeks/yr.},\ 0.01\times 6\text{contacts/week}\times 52\text{weeks/yr.})\\ &= \left(0.468\ \text{years}^{-1},\ 3.12\ \text{years}^{-1}\right),\end{aligned}$$

or it would take on average between 4 months to about 2 years to become infected for truck drivers if all sex workers were infected.

$$\begin{aligned}\beta_{mf} &= (0.006\times 6\text{contacts/week}\times 52\text{weeks/yr.},\ 0.08\times 30\text{contacts/week}\times 52\text{weeks/yr.})\\ &= \left(1.87\ \text{years}^{-1},\ 124.8\ \text{years}^{-1}\right),\end{aligned}$$

or it would take between 3 days and 6 months to become infected for sex workers if all truck drivers were infected. We compare this to estimates made from fitting simulation output to data on prevalence in sex workers and truck drivers.

Year	Sex Workers (Rural)	Sex Workers (Urban)	Sex Workers (Avg.)	Truck Drivers
1987	0.35%		0.35%	
1988	1.25%		1.25%	
1989		1.7%	1.7%	
1990	4.3%		4.3%	
1991		12.3%	12.3%	
1992	11.5%	9.9%	10.7%	1.6%
1993		15.4%	15.4%	
1994	21.4%	29.1%	25.2%	4%
1995	24%	33.3%	28.7%	
1996	54.7%	30.5%	42.6%	
2000				54%

Table 5. Percent prevalence in core groups (data from [**8**], [**19**] and [**9**])

We estimate the combined parameters $\beta_{fm}v_f$ and $\beta_{mf}v_m$ and use these values throughout this paper. Model output (see 6) from just the core groups (system 2.3) yielded estimates of $\beta_{fm}v_f = 0.382 \pm 0.125$ (95% CI)[2] and $\beta_{mf}v_m = 1.635 \pm 0.595$

[2]The joint model parameters$\beta_{fm}\nu_f$ and $\beta_{mf}\nu_m$ (for system 2.3) were fit to the data by minimizing the weighted sum of squared residuals between simulation output (numerical integration using Runge-Kutta 4 with a time step of 0.1) and the data. We used weights equal to number of data points available for fitting a particular differential equation, thus "sex worker" data received weights of 10 while "truck driver" data received weights of 3. These 13 data points were used to estimate the 2 joint parmeters above. The other parameters of system 2.3 were calculated as described in the sections on turnover rates and AIDS removal rates and held as constant in the

(95% CI). If all partners were infected it would take between 1/2 to 1 year for sex workers to become infectious themselves and between 2 and 4 years for truck drivers. Or at estimated year 2000 prevalence it was taking from 1 to 2 years for sex workers and 3 to $5\frac{1}{2}$ for truck drivers to become infectious. Dividing $\beta_{fm}v_f$ and $\beta_{mf}v_m$ by β_{fm} and β_{mf} we have $v_f = \frac{N_{f2}}{N_f} \approx 0.21$ and $v_m = \frac{N_{m2}}{N_m} \approx 0.025$ but both of these values are bigger than the actual estimates of the fraction that each group comprises in general population. One also would expect that the estimates from the population process (model fitting) would be lower than the rates based on individual behavior (even if averaged) since the individual based calculation doesn't take into account the limited circles of contacts that any particular individual may have even among these promiscuous groups and the hierarchy of cliques that further reduce the effective number of contacts. For example, even though a truck driver may have 3 different contacts in a week, these may be the same preferred woman (sex worker) each week. At some level there must be some base clique size where transmission rates are maximum within the clique but subsequently diluted by the limited contacts between cliques. The population level rate then would more reflect the transmissions that are occurring as a "small-world network" process. The effective population for commercial sex workers is about 5% of the actual population $\left(\frac{0.01}{0.21}\right)$, whereas the effective population size for truck drivers is about 80% of the actual population $\left(\frac{0.02}{0.025}\right)$, where the .01 and .02 are the fractions of the population that are comercial sex workers and truck drivers respectively [**18**], [**20**], [**21**], and [**23**].

Turnover rates. The rate at which truck drivers retire, die or are otherwise removed from the profession for any reason other than HIV is μ_m According to the World Health Report, 2002 [**8**], the life expectancy of a healthy Nigerian is 51 years and on average truck drivers joins the profession at 22 years [**20**]. This means that the length of the career of a healthy truck driver might be an average maximum of 29 years , however, due to any number of causes (changing jobs, moving, illness, etc.) many leave the profession before that time. Therefore, we take the average length of time spent in the profession as 20 years, and the natural turnover rate is $\psi_m = 1/20 + 1/51 = 0.07$. Similarly, we define the natural turnover rate ψ_f of female sex workers to be the reciprocal of the average time that a female sex worker spends in the profession following recruitment. Orubuloye, et al., [**20**] estimates the average age of a female sex worker at 20 years, and the average duration of time that a woman spends as a sex worker is about 4 years (we assume this average includes losses due to mortality), therefore, the natural turnover rate of female sex workers is $\psi_f = 1/4 = 0.25$. For the general population $\mu_m = \mu_f = 1/51 = 0.02$.

AIDS-related removal rates. The rate of progression to AIDS remains a contentious issue in Nigeria and other parts of Africa. Medical experts says that it takes less time to develop full blown AIDS once an individual is infected because of poor health facilities combined with high level of poverty. However, we are not aware of any research to substantiate the claim. In Hyman (1999)[**12**]an estimate of 8.6 years was assumed as a mean duration of infection. Since γ_m^{-1} and γ_f^{-1} are

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minimization algorithm. Standard errors on the parameter estimates were obtained by concurrently evaluating the time derivative of the sensitivity equations along with the model (see [**25**] and [**2**]) and using the output to estimate the covariance matrix of the model parameters following standard practice [**1**]. These errors are only errors from the fitting procedure and do not include the propagation of error from the removal rates or estimates of the data themselves.

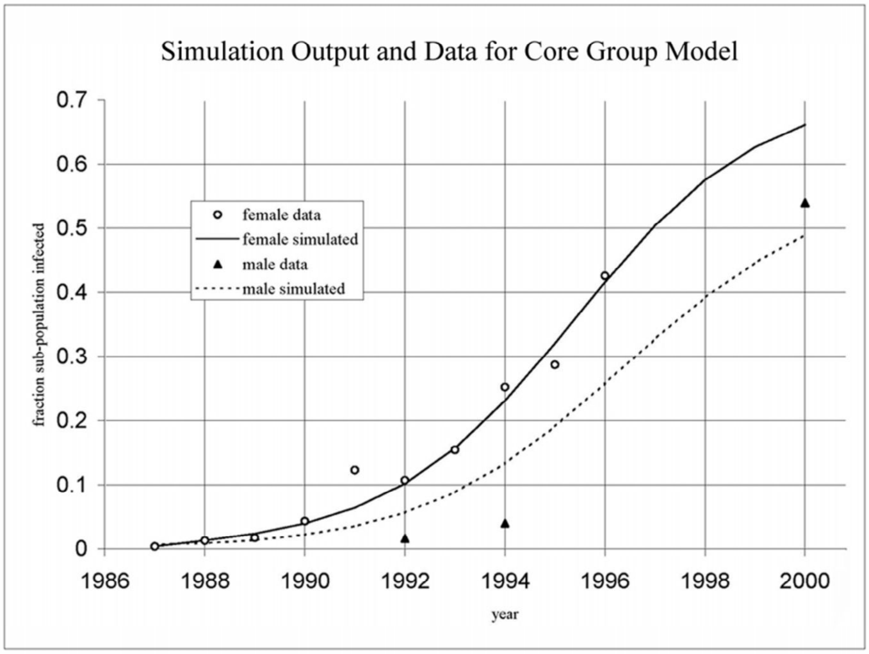

FIGURE 6. Estimate of rates of infection from fitting system (2.3) parameters to data in Table 3.

AIDS related death for truck drivers and sex workers per year, we assume that since a larger number of HIV infected cases do not get treatment, the same rate of progression to AIDS following infection for both groups which is 0.116 per year.

7. Discussion

In this paper we examine the disease dynamics of HIV in two ways; first with the core isolated from the main population, and second with both the core group and the main population linked. Our initial model takes into account the role of transactional sex in the spread of HIV in a heterosexual population consisting of two core groups: the female sex workers and the male truck drivers. These two sub-populations are then linked to the non-core general population in two ways. First, they both recruit from the non-core population. Second, they have other sexual partners who are at risk of contracting HIV that belong to the non-core population such as wives, boyfriends, girlfriends and so on.

Due to the paucity of data we relied on very rough estimates for some of the parameter values. Official sources are frequently inconsistent or at times unreliable from year to year. For instance, data on sexual activity of the core groups is thin yet there is some, on the other hand, there is almost no data for sexual activity within the main population and between the main population and the core groups. However, we do know that the level of activity is the highest within the core groups and we know that the level of infection in the general population is much lower than in the core groups. This enables us to consider a model of spreading disease in the general population (with two groups of females and males) with transmission rates relative to those in core groups. We find that even such an approach gives estimates on lowering the spread of the disease in the general population by controlling the spread in the core groups. Future work should consist of collecting better data, and refining parameter estimates. It would provide further understanding of the spread of the disease in general population and the ways to control it.

From what we were able to obtain from our analysis with available data from Nigeria, we make the following observations. First, transactional sex plays a significant role in fuelling the epidemic in the two core populations with $R_0 \approx 3.0 \pm 0.6$. In order to reverse the spread of the disease within these two core populations, it would be sufficient to concentrate towards lowering the transmission probabilities of the female sex workers and their customers β_{fm} and β_{mf}. This can be achieved if concerted efforts are made towards ensuring that more truck drivers and female sex workers use condoms. We established that if truck drivers used condoms between 30 to 60 percent of the time HIV cases among truck drivers and female sex workers would reduce by as much as 50 percent. Obviously, the more truck drivers use condoms, the more the benefits in reducing the disease will take place. These benefits increase in a nonlinear way, i.e. higher usage of condoms would result in even larger reductions in the disease than would be expected from projecting the reductions at lower levels. This indicates that a concerted effort made at education and encouragement of truck drivers to use condoms and female sex workers to insist on condom use would have increasing benefits as such a program progresses. Even more remarkable is the effect that use of condoms by just truck drivers would have on the prevalence of HIV in the non-core population of women. There is a multiplicative effect of both reducing the prevalence of HIV (in the long term) among the core groups, thereby decreasing the likelihood that a woman in the general population that has sex with a truck driver will be having it with someone who is infected, and also decreasing the likelihood that truck driver doesn't use condoms (in the case that he is infected). When all men use condoms at the same rate the same effect is seen in the non-core men by virtue of the fact that they are protected from infected women (obviously).

In addition, the removal rates γ_m, γ_f, μ_m, and μ_f which appeared in the denominator of the basic reproduction numbers account for the removal due to onset of AIDS as well as for the removal due to death of the truckers and retirement of female sex workers. Hence, an increase in the retirement of female sex workers or loss due to AIDS of both groups will result in a loss of infected individuals in the sexually active population. Furthermore, "traditional health care providers" should be brought into the health care programs to educate their clients on the risk of the disease, since they influence significantly the individuals using their services. These might also have the potential of reducing the spread of the disease on a long time scale. Modern public health measures through periodic monitoring and/or random testing could be of great benefit if infected people could be convinced to have transactions only among themselves (with little loss of income on the sex workers part). However, this seem unlikely since the perceived opportunity costs of quarantine and the stigma of advertising the disease would be high. Part of the problem is educating and even convincing (sex workers, in particular) that the non-AIDS life hazards, like having enough to eat today does not outweigh the risk from AIDS ten years from now, and that there is a future that they can look forward to that, at least, makes it worth their while to take precautions and that those precautions will not constitute a lose of earnings.

The preliminary analysis of the general population indicates that even moderate use of condoms could possibly result in substantial decreases in HIV prevalence. For example there is the possibility of essentially eliminating the disease from the general population if just 50% of the male population used condoms (with the most optimistic transmission rates) . We also note (again) that the uncertainty in the data makes this a very tenuous statement since for extremely high estimates yield only 65-68% reduction in prevalence for the same usage. However, it is noteworthy that we can make any such evaluation without the need for any additional information about the rates of infectiousness within the general population or even between the core and the non-core groups other than knowing that the prevalence (as a fraction of the population infected) in the non-core groups is substantially lower than in the core groups.

Finally, we would like to restate that our conclusions are based on a highly simplified model under the assumption that the relative size of the core is small, and levels of prevalence in the general population low. However, the fact we assumed no demographic change (over the time scale of interest) imply that over a longer time scale this last assumption would likely overestimate the current situation. Limited data, particularly on sociological parameters force us to make crude models based on rough simplifications. However, we believe that the message is clear and obvious, HIV can be contained in Nigeria.

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