Abstracts of talks

KNOTS in WASHINGTON IX;
The Conference on Knot Theory and its Ramifications

September 24-25 , 1999, at the George Washington University
Organizers:
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Friday, September 24, Time: 2:30 PM
Speaker: Witold Rosicki, (Gdańsk University)
Title: Coloring and other Quandle invariants of surfaces in 4-manifolds
Place: Funger Hall Room 307
Coffee and cookies will be served in Funger 307 before the talk
(about 11:30 we will go with the speaker for lunch)
Abstract:
The n-coloring is a very useful simple invariant in classical knot theory. We can define analogous invariant for the surfaces embedded in 4-space. We define "bridges" for surfaces (after S.Carter and M.Saito):
Definition. If $K : F \to R^4$ is an embedding and p is a projection, such that the composition $p \circ K$ is in general position, then the lower decker set $A = \{x \in F \ : \ exist \ y \in F, \ p \circ K(x) = p \circ K(y) \ and \ K(x) \ is \ the \ lower \ point\}$ separates the surface into regions (bridges), connected components of $K(F) - A$. The connected components of the set of (exactly) double points are called edge-crossings. They are edges of the graph A.
Theorem. Let $K : F \to R^4$ and $p : R^4 \to R^3$ be as above. Let n be an integer, $n > 2$. To every bridge of the projection we assign a number smaller then or equal to n. We say that such an assignment is an n-coloring iff for every edge-crossing in $p \circ K(F)$ the sum of numbers assigned to lower regions is congruent (mod n) to twice the number assigned to the upper region. Then the number of possible n-coloring is an invariant of the knotting.
If we verify that the Roseman moves preserve this number we prove the theorem. We can define the Fenn-Rourke core group for surfaces embedded in R^4 and check, in the same way, that it is invariant of the position.
I will discuss the joint work with J.H.Przytycki giving the topological interpretation of this group.

I will describe also J.S.Carter, D.Jeslovsky, S.Kamada,L.Langford and M.Saito work on coloring of surfaces by quandle and its rich consequences.

Saturday, September 25, 1999; Funger 428 – seminar room:

10:00 - 10:30am Coffee and refreshments

10:30 - 11:20 Ted Stanford (U.S. Naval Academy),
Knots and links modulo braid subgroups
Abstract:
Since every link may be realized as the closure of a braid, it is natural to ask what sort of correspondence exists between equivalence relations on braids and equivalence relations on links. More specifically, consider a sequence of groups Q_1, Q_2, Q_3, etc, where Q_i is a quotient of the braid group B_i, and where the sequence satisfies some kind of compatibility condition with the standard inclusions from B_i to B_{i+1}. For any such sequence Q_i, an induced equivalence relation on links is obtained. n-equivalence of knots, dual to Vassiliev invariants of order $< n$, arises this way, but so do other interesting equivalence relations which do not factor through n-equivalence for any n. I will present some elementary results and examples.

11:30 - 11:55 Mike Veve (GWU),
Open problems in billiard knots
Abstract:
While mathematical billiards have been studied quite extensively for many years the same cannot be said for billiard knots. Billiard knots are a special type mathematical billiard, namely, periodic trajectories without self-intersections inside some billiard room (a billiard room is 3-manifold inside R^3 with a piecewise smooth boundary).

Some of the open problems involving billiard knots that will be discussed are:

1) Classify the billiard knots of a specific room (or a family of rooms) according to knot type.

2) Is there a billiard room such that every knot type is equivalent to some billiard knot in this room? If it exists, what are its properties?

3) By a simple construction, one can show that for any tame knot there is a billiard room which contains it as a billiard knot. Usually this room will not be convex. For any tame knot, is there a convex room that contains a billiard knot that is equivalent to the tame knot?

12:05 - 12:30 Mark Kidwell (U.S. Naval Academy),
Solution to a conjecture of Stoimenow about plane curves.
Abstract:
A prime curve γ in the plane with distinct endpoints, no nugatory crossings and all crossings transverse has crossing number c. Stoimenow introduced a new quantity, the endpoint distance d: it is the minimum intersection number of a curve γ' with the same endpoints as γ with γ . Stoimenow proved that $d \leq max(3, c-3)$ and conjectured that $d \leq Floor(c/2)$. We will prove this conjecture, and give some applications to bridges in knot diagrams.

12:30 - 2:00 Lunch

2:00 - 2:25 Maxim Sokolov (GWU), Dichromatic skein modules and invariants of 3-manifolds Abstract:
Deloup's invariants $\tau(M, G, q)$, essentially introduced by his teacher V. Turaev via a simple example of modular categories, are an interesting generalization of the Murakami-Ohtsuki-Okada invariants for 3-manifolds. Here G is a finite abelian group and $q : G \to Q/Z$ is a quadratic form. Deloup showed that these invariants depend only on $b_1(M)$ and the linking form of the manifold. Recently, Deloup and Gille proved that these invariants keep all the information about the linking form and $b_1(M)$. Moreover, they proved that all the information can, in fact, be recovered from two special cases: $\tau(M, Z_{p^n}, (1/p^n))$, for prime p, and $\tau(M, Z_{2^n} \times Z_{2^n}, K)$. Here $\tau(M, Z_{p^n}, (1/p^n))$ coincides with the Murakami-Ohtsuki-Okada invariant of order p^n, and K is the quadratic form defined by $K(x, y) = xy/2^n$. We showed earlier that Murakami-Ohtsuki-Okada invariants can be obtained using Lickorish's

construction on the skein module $S_2(M, q)$ (q-analog of the first homology). It turned out that $\tau(M, Z_{2^n} \times Z_{2^n}, K)$ can be similarly defined using another simple skein module which we will call the dichromatic skein module, and which turned out to be the tensor product of two copies of $S_2(M, q)$.

2:30 - 2:55 Tatsuya Tsukamoto (GWU and Waseda Univ.)
The fourth skein module, the Montesinos-Nakanishi conjecture and n-algebraic links.
Abstract:
We analyze the concept of the fourth skein module of 3-manifolds, that is a skein module based on the skein relation $b_0L_0 + b_1L_1 + b_2L_2 + b_3L_3 = 0$ (b_0, b_3 invertible) and a framing relation $L^{(1)} = aL$. We find the necessary conditions for trivial link to be linearly independent in the module. In the case of classical links (i.e. links in S^3) our skein module suggests three polynomial invariants of unoriented framed (or unframed) links. One of them generalize the Kauffman polynomial of links and another one can be used to analyze amphicheirality of links (and may work better than the Kauffman polynomial). Using the idea of mutants and rotors, we show that there are different links representing the same element in the skein module. We show also that that algebraic links (in the sense of Conway) and closed 3-braids are linear combinations of trivial links. We introduce the concept of an n-algebraic tangle (and link) and analyze the skein module for 3-algebraic links. As a byproduct we prove the Montesinos-Nakanishi 3-moves conjecture for 3-algebraic links (including 3-bridge links).
It is a joint work with Jozef Przytycki.

2:55 - 3:15 Coffee break

3:15 - 3:40 Adam Sikora (UMD),
Skein modules at the 4-th roots of unity.
Abstract:
The skein module of a manifold M is a module over a ring R with a specified element A in R. The module is generated by links in M considered up to Kauffman bracket skein relations. The connections between skein modules and quantum invariants of 3-manifolds suggest that the most interesting choice of A is a root of unity (or A treated as a

formal element). The problem of calculating the skein module of a given manifold is very difficult, and it has been solved only for $A = +1, -1$ (giving a wonderful connection with the theory of character varieties). In this talk we calculate the skein modules of rational homology spheres for the 4-th roots of 1. We also describe the skein algebra of a surface F (for $A^4 = 1$) as the SL_2-character variety of F deformed along a 2-cocycle which is the symplectic form on F.

3:45 - 4:10 Józef H.Przytycki (GWU and UMD),
Positive knots and their properties.
Abstract:
We analyze a relation $\geq$ on links defined by $L_1 \geq L_2$ iff L_2 can be obtained from L_1 by changing some positive crossings of L_1.

We prove in particular:

1. If K is a positive knot then $K \geq (5,2)$ positive torus knot unless K is a positive pretzel knot (k_1, k_2, k_3) (it generalizes the 1985 result of Cochran and Gompf).
   (a) If a positive knot has unknotting number one then it is a positive twist knot.
2. If K is a 2-almost positive knot (i.e. it has a diagram with no more then 2 negative crossings) then either
   (i) $K \geq$ right handed trefoil, or
   (ii) $K \geq$ mirror image of 6_2 knot (in Rolfsen's book notation) or
   (iii) K is a twist knot with the negative clasp.
   (a) If K is a 2-almost positive knot different then a twist knot with negative clasp then $K(1/n)$ (i.e. $1/n$ surgery on K, $n > 0$) is a homology 3-sphere that does not bound a compact, smooth homology 4-ball (it extends a result of Cochran and Gompf, and uses the recent (in 1992) result of Akbulut).
   (b) If K is a non-trivial 2-almost positive knot different than the stevedore's knot then K is not a slice knot.
   (c) If K is a non-trivial 2-almost positive knot different than the figure eight knot then K is not amphicheiral.

It is a (old 1990-1992) joint work with Kouki Taniyama.

4:15 - 4:40 Yongwu Rong (GWU)
Mutations and the Alexander polynomial
Abstract.
The Melvin-Morton conjecture, proved by Bar-Natan and Garoufalidis, states that the colored Jones polynomial of a knot determines its Alexander polynomial. A consequence of it is that the Alexander polynomial of a knot is preserved under a mutation along a genus two surface. This is, surprisingly, a new theorem, as noted by Cooper and Lickorish who also gave a classical proof for it. In this talk, we will explain how their result can be slightly more general, based on our earlier work on mutation.