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ON ACTIONS OF APROXIMABLE GROUPS IN THE ULTRAPRODUCTS OF MEASURABLE SPACES

M.A.Alekseev, E.I.Gordon (Nizhny Novgorod,Russia)

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In the paper [1] the various aspects of approximability of infinite groups by finite ones were considered. In particular, the class of groups, locally embedded into the class of finite groups (LEF groups), was investigated. By the classical results of A.I.Mal'tsev this is exactly the class of subgroups of the ultraproducts of finite groups. This makes it natural to use the Loeb spaces for the investigation of the connection between the approximability of groups and group actions in the measurable spaces.

Consider a family $\{\langle X_i, \Omega_i, \mu_i\rangle \mid i \in I\}$ of the spaces with finitely additive probability measures and a free ultrafilter $\mathcal{F}$ over 2^I which is not countably complete. Let X be the ultraproduct of X_i over $\mathcal{F}$, Ω the algebra of subsets of X which are the ultraproducts of any families of the sets $A_i \in \Omega_i$ over $\mathcal{F}$. We can define the finitely additive measure μ on Ω such that if A is the ultraproduct of A_i then $\mu(A) = \lim\limits_{\mathcal{F}} \mu_i(A_i)$. Using the ω_1-saturation of the ultraproducts over countably incomplete ultrafilters, it is easy to see that μ can be extended to the σ-additive measure μ_L on the σ-algebra Ω_L, generated by Ω. The space $\langle X, \Omega_L, \mu_L\rangle$ is called a Loeb space. It was introduced by P.Loeb in the language of nonstandard analysis and has a lot of applications in nonstandard probability theory.

The following theorem, which strengthens the theorem 3.3° of [1] (see also the theorem in [2]) is proved with the help of Loeb spaces.

Theorem 1. *Let a quasiinvariant action of an infinite group G in a measure space (X, μ) with a probability measure be such that for some q, $0 < q < 1$, for any $\varepsilon > 0$ and for any finite subset $\{g_1, \dots, g_n\}$ of G there exists such finite set $\{k_1, \dots, k_n\}$ of quasiinvariant transformations of X that $\mu\{x \in X \mid g_i x = k_i x,\ i = 1, \dots, n\} \geq 1 - \varepsilon$, the group*

$\langle k_1, \ldots, k_n \rangle$ generated by $k_1, \ldots, k_n$ is finite and for any $h \in \langle k_1, \ldots, k_n \rangle$ $\mu\{x \in X \mid h \cdot x = x\} \geq q$. Then G is locally embedded into the class of finite groups.

The idea of the prove is in consideration of an ultraproduct $\mathcal{G}$ of the groups $\langle k_1, \ldots, k_n \rangle$, which acts in a Loeb space - the ultrapower $\mathcal{X}$ of (X, μ). This action is such that $\forall \xi \in \mathcal{G} \mu_L\{\eta \in \mathcal{X} \mid \xi\eta = \eta\} \leq q$. This implies that injection $\varphi : G \to \mathcal{G}$, induced by the maps $g_i \to k_i$, is a homomorphism. Thus G is a subgroup of an ultraproduct of finite groups.

The similar considerations make it possible to prove the following theorem, which strengthens the Fölner condition for amenability.

Theorem 2. *A group G is amenable if there exists such $q > 0$ that for any finite subset $K \subseteq G$ there exists a finite subset $U \subseteq G$ which satisfies the following unequality $|\bigcap_{k \in K} kU \cap U||U|^{-1} \geq q$.*

As it was noticed in [1] there exist the examples of amenable groups which are not LEF and vice versa. So it is interesting to investigate the class (LEA) of groups, locally embedded into the class of amenable groups (the subgroups of ultraproducts of amenable groups). This question is open. The following theorem contains a new definition of approximability of infinite groups by finite ones, according to which both the amenable and LEF groups are approximable and thus the LEA groups are also approximable.

Theorem 3. *If G is a LEA group then for any $\varepsilon > 0$ and for any finite set $H \subseteq G$ there exists such $n \in \mathbb{N}$ and such injection $\varphi : H \to S_n$ that*

$$\forall h_1, h_2 \in H(h_1 \cdot h_2 \in H \to \frac{|\{m \leq n \mid \varphi(h_1) \circ \varphi(h_2)(m) \neq \varphi(h_1 \cdot h_2)(m)\}|}{n} \leq \varepsilon).$$

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Nizhny Novgorod State University, Department of mechanics and Mathematics.

Gagarin ave., 23. 603600 Nizhny Novgorod, Russia.
gordon@irina.nnov.ru