3882. *Originally proposed by Mehmet Sahin; corrected version by Arkady Alt.*

Let ABC be a right angle triangle with $\angle CAB = 90^\circ$ and hypotenuse a. Let $[AD]$ be an altitude and let I_1 and I_2 be the incenters of the triangles ABD and ADC, respectively. Let ρ be the radius of the circle through the points B, I_1 and I_2 and let r be the inradius of the triangle ABC. Prove that

$$\rho = \sqrt{\frac{a^2 + 2ar + 2r^2}{2}}$$

and $\min \dfrac{\rho}{r} = \sqrt{3} + \sqrt{6}$.

3883. *Proposed by Max A. Alekseyev.*

Let a, b, c, d be positive integers such that $a + b$ and $ad + bc$ are odd. Prove that if $2^a - 3^b > 1$, then $2^a - 3^b$ does not divide $2^c + 3^d$.

3884. *Proposed by Mihai Bogdan.*

Let a, b, c and d be positive real numbers such that $a + b + c + d = k$, where $k \in (0, 8)$. Prove that:

$$\frac{a}{b^2+1} + \frac{b}{c^2+1} + \frac{c}{d^2+1} + \frac{d}{a^2+1} \geq \frac{k(8-k)}{8}.$$

When does the equality hold?

3885. *Proposed by Oai Thanh Dao.*

Let ABC be a triangle and let F be a point that lies on a circumcircle of ABC. Further, let H_a, H_b and H_c denote projections of the orthocenter H onto sides BC, AC and AB, respectively. The three circles AH_aF, BH_bF and CH_cF meet the three sides BC, AC and AB at points A_1, B_1 and C_1, respectively. Prove that the points A_1, B_1 and C_1 are collinear.

3886. *Proposed by Michel Bataille.*

Let $H_n = \sum_{k=1}^{n} \frac{1}{k}$ be the nth harmonic number and let $H_0 = 0$. Prove that for $n \geq 1$, we have

$$\sum_{k=1}^{n} (-1)^{n-k} \binom{n}{k} 2^k H_k = 2H_n - H_{\lfloor n/2 \rfloor}.$$

3887. *Proposed by Dao Hoang Viet.*

Let a, b and c be positive real numbers. Prove that

$$\frac{a^2}{bc(a^2+ab+b^2)} + \frac{b^2}{ac(b^2+bc+c^2)} + \frac{c^2}{ab(a^2+ac+c^2)} \geq \frac{9}{(a+b+c)^2}.$$