Intermediate Microeconomics
Fall 2015

Problem Set 9
Due Lecture 11 in class on paper

1. GLS Chapter 9, Question 13

See charts for all subsections of this question at the end of the problem set.

(a) Calculate the profit-maximizing price and quantity of fairy dust. Calculate the seller's profit.

The monopolist finds the profit maximizing Q by setting $MR = MC$. We can use the formula given in the notes to find $MR = 100 - 2Q$.

Setting $MR = MC$, we can solve

$$\begin{aligned} MR &= MC \\ 100 - 2Q &= 20 \\ 80 &= 2Q \\ Q &= 40 \end{aligned}$$

To find the price, find what the maximum consumers are willing to pay at this quantity. This means plug in the Q you just found into the demand curve: $P = 100 - Q = 100 - 40 = 60$.

Find the profit by subtracting total cost from total revenue. Note that because marginal cost is constant, $ATC = MC$, so we can write:

$$\begin{aligned} \pi &= TR - TC \\ &= PQ - MC(Q) \\ &= 60(40) - 20(40) \\ &= 40(40) \\ &= 1,600 \end{aligned}$$

(b) Suppose the new inverse demand curve is $P = 200 - 2Q$. Find the new profit-maximizing Q and P.

Follow the steps as in (a). We can use the formula given in the notes to find $MR = 200 - 4Q$.

Setting $MR = MC$, we can solve

$$\begin{aligned} MR &= MC \\ 200 - 4Q &= 20 \\ 180 &= 4Q \\ Q &= 45 \end{aligned}$$

To find the price, find what the maximum consumers are willing to pay at this quantity. This means plug in the Q you just found into the demand curve: $P = 200 - 2Q = 200 - 2(45) = 110$.

Find the profit by subtracting total cost from total revenue.

$$\begin{aligned} \pi &= TR - TC \\ &= PQ - MC(Q) \\ &= 110(45) - 20(45) \\ &= 90(45) \\ &= 4,050 \end{aligned}$$

(c) Suppose that the new inverse demand curve is $P = 100 - 0.5Q$. Find the new profit-maximizing P and Q.

Follow the steps as in (a). We can use the formula given in the notes to find $MR = 100 - Q$.

Setting $MR = MC$, we can solve

$$\begin{aligned} MR &= MC \\ 100 - Q &= 20 \\ 80 &= Q \\ Q &= 80 \end{aligned}$$

To find the price, find what the maximum consumers are willing to pay at this quantity. This means plug in the Q you just found into the demand curve: $P = 100 - 0.5Q = 100 - 0.5(80) = 60$.

Find the profit by subtracting total cost from total revenue.

$$\begin{aligned}\pi &= TR - TC \\ &= PQ - MC(Q) \\ &= 60(80) - 20(80) \\ &= 40(80) \\ &= 3,200\end{aligned}$$

(d) Suppose that the new inverse demand curve is $P = 150 - Q$. Find the new profit-maximizing P and Q.

Follow the steps as in (a). We can use the formula given in the notes to find $MR = 150 - 2Q$.

Setting $MR = MC$, we can solve

$$\begin{aligned}MR &= MC \\ 150 - 2Q &= 20 \\ 130 &= 2Q \\ Q &= 65\end{aligned}$$

To find the price, find what the maximum consumers are willing to pay at this quantity. This means plug in the Q you just found into the demand curve: $P = 150 - Q = 150 - 65 = 85$.

Find the profit by subtracting total cost from total revenue.

$$\begin{aligned}\pi &= TR - TC \\ &= PQ - MC(Q) \\ &= 85(65) - 20(65) \\ &= 65(65) \\ &= 4,225\end{aligned}$$

(e) When demand increases, profits for monopolistic suppliers always increase. Individually, P and Q both don't need to increase, but total profits do increase.

2. GLS Chapter 9, Question 15

(a) Find the competitive price and quantity.

The competitive P and Q are where $P = MC$, or where the MC line intersects the demand curve. However, we don't know the precise function for the demand curve in this problem.

However, you know two points on the demand curve: $(0, 1)$ and $(100, 0)$. From these you know that the slope is $-1/100$. And you also know that the y-intercept (or P-intercept) is 1. Given these two facts, you can write the demand curve as $P = 1 - (1/100)Q$.

Now you can return to $P = MC$, or $1 - (1/100)Q = 0.10$, which implies that $0.01Q = .90$, and $Q = 90$.

You can check this: when $Q = 90$, it must be that $P = 1 - (1/100)(90) = 1 - 0.9 = 0.1$.

(b) What are CS and PS in the competitive market?

Consumer surplus is the area below the demand curve and above the market price. Here is is a triangle 90 units wide, and $1 - 0.10 = 0.9$ tall. So $CS = (1/2)(90)(0.9) = 40.5$.

Producer surplus is the area above the supply curve and below the market price. In a competitive industry, the supply curve is the marginal cost curve, which determines the price and there is no producer surplus.

(c) What at the monopoly P and Q?

Set $MR = MC$ to find the monopolist's optimal Q. Calculate $MR = 1 - (1/50)Q$, so that

$$\begin{aligned} MR &= MC \\ 1 - (1/50)Q &= 0.10 \\ 0.90 &= (1/50)Q \\ Q_{\text{monopoly}} &= 45 \end{aligned}$$

At this Q, consumers are willing to pay $P_{\text{monopoly}} = 1 - (1/100)(45) = 0.55$.

(d) Calculate CS and PS under the monopoly.

Consumer surplus is the area below the demand curve and above the price. The width of this triangle is Q_{monopoly}, and the height is $1 - P_{\text{monopoly}} = 0.45$. Therefore, the area of the triangle is $CS = (1/2)(45)(0.45) = 10.125$.

Producer surplus is the area above the MC curve and below market (monopoly) price. This area is a rectangle of width $Q_{\text{monopoly}} = 45$ and height $0.55 - 0.10 = 0.45$, so $PS = 45(0.45) = 20.25$.

(e) How big is the deadweight loss of monopoly?

One way to calculate the deadweight loss is to let TS denote total surplus, and calcu-

late the change in total surplus: $DWL = TS_{\text{comp}} - TS_{\text{monopoly}}$. This yields $DWL = (40.5 + 0) - (10.125 + 20.25) = 10.125$.

This is entirely equivalent to finding the area of the triangle between the MC and demand curve for trades that do not take place. The width of this triangle is $90 - 45 = 45$, and the height is the same as the producer surplus rectangle, or 0.35. Therefore, the area is $DWL = (1/2)(45)(0.45) = 10.125$.

3. GLS Chapter 9, Question 18

(a) Is the firm a natural monopoly?

Yes, over the range that consumers demand, average total cost (ATC) is always declining.

(b) Will the firm earn a profit if not subject to regulation?

Left to its own devices, the firm will set Q such that $MR = MC$. This yields production at $Q = 500$. Looking at the chart, we can see profits in the rectangle from 0 to 500 on the x axis, and from 0 to where $Q = 500$ intersects with demand, which is the maximum price the firm can charge at this quantity.

The firm's costs at $Q = 500$ are the rectangle of width 500, and height where $Q = 500$ intersects the $LATC$ curve.

The revenue rectangle is bigger than the cost rectangle. Therefore, the firm makes positive profits.

(c) If the govern requires firms to charge no more than MC, what are P and Q? Is there a problem with this scheme?

If the firm can charge no more than marginal cost, it would choose to produce $Q = 1000$ and $P = 10$.

However, at this level of production, the firm cannot cover its costs, since $LATC$ are far above LMC at $Q = 1000$. The firm would exit the market.

(d) Suppose the firm is not allowed to charge more than $LATC$. What are P and Q? What is the problem with this scheme?

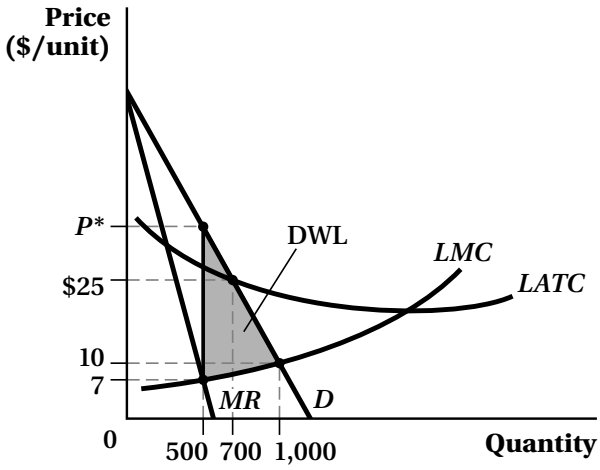
If the firm charges $LATC$, it will produce $Q = 700$, and sell at $P = 25$.

This still yields a deadweight loss – but it might be the best that the regulator can do.

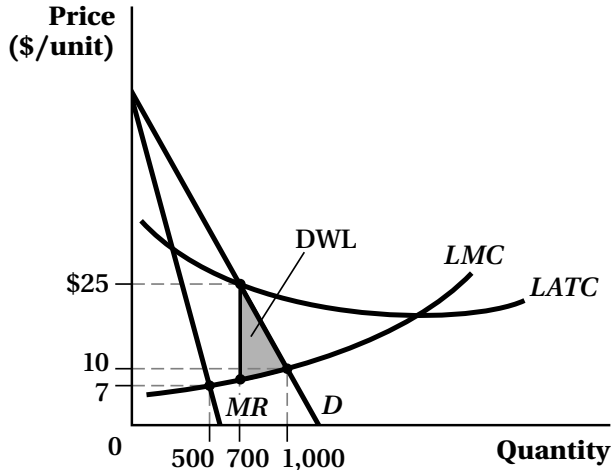
(e) Compare the DWL under each regime.

In (c) there is no deadweight loss – the firm produces Q such that $P = MC$. Of course, this firm will go out of business.

Part (b)

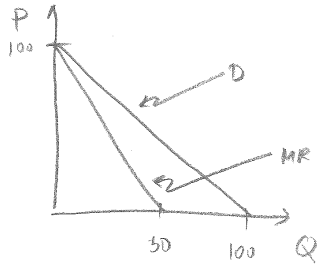


Part (d)

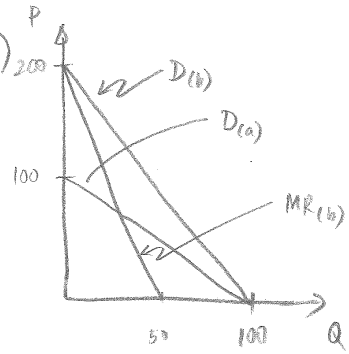


Q13

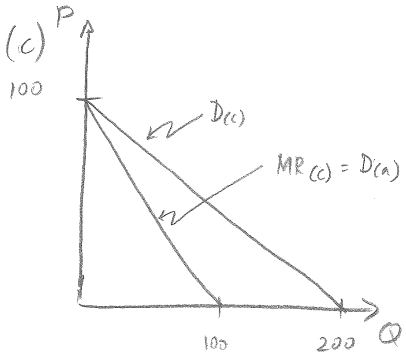
(a)


P
100
D
MR
50
100
Q

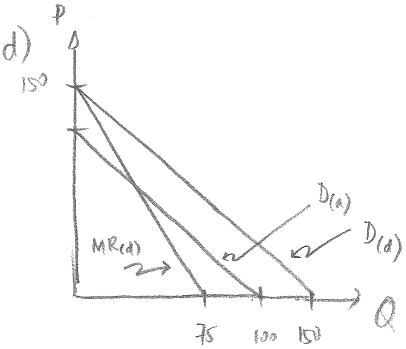
(b)


P
200
D(b)
D(a)
100
MR(b)
50
100
Q

(c)


P
100
D(c)
MR(c) = D(a)
100
200
Q

(d)


P
150
D(a)
D(d)
MR(d)
75
100
150
Q