Nuclear Physics: Conventions

The Natural System of Units is particularly popular in Nuclear and High-Energy Physics since as many fundamental constants as possible have as simple a value as possible (see [MM]!).

Set the speed of light and Planck’s quantum to \( c = \hbar = 1 \). This expresses velocities in units of \( c \), and actions and angular momenta in units of \( \hbar \). Then, only one fundamental unit remains, namely either an energy- or a length-scale. Time-scales have the same units as length-scales. We also set Boltzmann’s constant \( k_B = 1 \), so energy and temperature have the same units. Now one only memorises a handful of numbers. [Setting Newton’s gravitational constant \( G_N = 1 \) eliminates any dimension-ful unit – only String Theorists do that.]

Electrodynamic Units: The Rationalised Heaviside-Lorentz system will be used throughout. Formally, it can be obtained from the SI system by setting the dielectric constant and permeability of the vacuum to \( \epsilon_0 = \frac{1}{\mu_0 c^2} = 1 \). The system is uniquely determined by any two of the fundamental equations which contain \( \vec{E} \) and a combination of \( \vec{E} \) and \( \vec{B} \). More on systems, units and dimensions e.g. in [MM].

Charges \( Q = Ze \) are measured in units \( Z \) of the elementary charge \( e > 0 \); electron charge \( -e < 0 \).

Lagrangian: \( \mathcal{L}_{\text{elmag}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \implies \) Maxwell’s equations: \( \partial_{\mu} F^{\mu\nu} = j^\nu \)

Lorentz force: \( \vec{F}_L = Z e [\vec{E} + \vec{\beta} \times \vec{B}] \); Coulomb’s law: \( \Phi(r) = \frac{Z e}{4\pi r} \)

“Restoring” SI units from “natural units”: Multiply by \( c^n h^\beta k_B^\gamma \epsilon_0^\delta \) and determine the exponents such that the proper SI unit remains, using \( [c] \): [m s^{-1}], \([h]\): [kg m^2 s^{-1}], \([k_B]\): [m^2 kg s^{-2} K^{-1}] and \([\epsilon_0]\): [CV^2 m^{-1}]. For example: \( E = m \implies E = m c^2 h B^2 \epsilon_0 \), and you have to convert kg m^2/s^2 into kg, i.e. add two powers of m/s, so that \( \alpha = 2, \beta = \gamma = \delta = 0 \).

Conventions Relativity: Einstein Summation Convention; “East-coast” metric (+ − − −):

\[
A^2 \equiv A^\mu A_\mu := (A^0)^2 - \vec{A}^2. \text{ Velocity } \beta, \text{ Lorentz factor } \gamma = (1 - \beta^2)^{-1/2}.
\]

Conventions QFT: “Bjørken/Drell”: [HM, PS] – close to [HH], but fermion norms different:

Quantised complex scalar: \( \Phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left[ a(\vec{k}) e^{-i k \cdot x} + b^\dagger(\vec{k}) e^{i k \cdot x} \right] \) with \( E_k := k^0 = +\sqrt{k^2 + m^2} \)

Minimal substitution in QED: \( D^\mu = \partial^\mu + iZe A^\mu; \) in non-Abelian gauge theories (QCD, . . . ): \( D^\mu = \partial^\mu - ig A^\mu \).

\( \gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 = \gamma_5; 2mP_+ := \sum_{s=\pm} u_s(p) \bar{u}_s(p) = \bar{p} + m, \ -2mP_- := \sum_{s=\pm} v_s(p) \bar{v}_s(p) = p - m \implies P_+ + P_- = 1 \)

Elastic cross section (our convention) in cm:\n
\[
\frac{d\sigma}{dt} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2; \text{ lab, } m = 0: \frac{d\sigma}{dt} = \frac{1}{64\pi^2 M^2} \left( \frac{E'}{E} \right)^2 |\mathcal{M}|^2
\]

Decay of particle with mass \( M \) (cm, our convention): \( \Gamma [A \rightarrow B(\vec{p}') + C] = \frac{|\vec{p}'|}{8\pi M^2} \int d\Omega |\mathcal{M}|^2 \)

More cross section formulae & conventions in “Summary Electron Scattering Cross Sections” below.

Nuclear Physics: Some Oft-Overlooked Bare Essentials

Know these by heart! Physicists spend a lot of time solving complicated problems, so we want to always start with an idea of the result. We have ideas when we are hiking, cycling, in the shower, etc., and usually not on our desk. We discuss them with colleagues on the blackboard, and we cannot waste their and our time with looking stuff up. Therefore, we need to be able to do calculations without computers, books or calculators, i.e. in our head or with a piece of scrap paper and a dull pencil.

Here a list of numbers most commonly used for estimates, back-of-the-envelope calculations, etc. of the Nuclear Physics tool-chest. You need them for that purpose but they are often overlooked. You should know the following by heart when woken up at night. The list is not exhaustive, not meant to be relevant and may not be useful. Your mileage may vary. It’s better to know too much than not enough. You should check these values, how they come about, and their limitations. Trivialities are not included, like most “Essentials” from Mathematical Methods [MM] (dimensional estimates, guesstimates, Natural System of Units). If you know more things worth remembering, let me know!

This is the list of often-overlooked numbers, not of the minimum necessary – the minimum is much larger (including names, spins, charges, masses of all fundamental particles, etc.).

Please turn over.
approximate: number rounded for easier memorising – it suffices to know the first significant figure.

correspondences are correct only in the natural system of units.

**Quantity**  | **Value**
--- | ---
speed of light in vacuum  | \( c := 2.99792458 \times 10^8 \text{ m s}^{-1} \) (def.) \( \approx 3 \times 10^8 \text{ m s}^{-1} \)
energy electron gains when accelerated by 1 V  | 1 electron Volt (eV) \( \approx 1.6 \times 10^{-19} \text{ Joule} = \frac{1}{\text{C}} \) J
conversion factor  | \( 1 \text{ J} \approx 6 \times 10^{18} \text{ eV} \)
typ. subatomic length-scale (proton/neutron size)  | \( h \alpha \approx 1 \text{ fermi (femtometre, fm)} = 10^{-15} \text{ m} \)
conversion factor energy – length  | \( h \alpha \approx 1 = 197.327 \ldots \text{ MeV fm} \approx 200 \text{ MeV fm} \)
conversion factor distance – time (nat. units)  | \( 1 \text{ fm} = 1\frac{\text{fm}}{c} \approx \frac{1}{3} \times 10^{-23} \text{ s} \)
conversion factor elmg.: fine-structure constant  | \( \alpha = \frac{e^2}{4\pi} = \frac{e^2}{4\pi \epsilon_0 h c} \approx \frac{1}{137} \) (no units!)
conversion factor energy – temperature: \( E = k_B T \)  | \( 1 \text{ eV} \approx 11600 \text{ Kelvin}, \quad 300 \text{ K} \approx \frac{1}{40} \text{ eV} \)
“classical” electron radius  | \( r_e = \frac{\alpha}{m_e c^2} = \frac{e^2}{4\pi m_e} \) \approx \frac{1}{6} \times 10^{-23} \text{ g} \)

**Masses**  | Conversion factor: atomic unit \( 1 \text{ u} = \text{mass}^{12} \text{C atom} = \frac{12}{\text{mass}^{12} \text{C atom}} = \frac{12}{\text{mass}^{12} \text{C atom}} = \frac{12}{6.022 \times 10^{23}} \approx \frac{1}{6} \times 10^{-23} \text{ g} \)
eddyron  | \( m_e \approx 511 \text{ keV} \)
muon  | \( m_e \approx 110 \text{ MeV} \approx 200 \text{ m}_e \)
nucleon  | \( M_N \approx 940 \text{ MeV} \approx 1800 \text{ m}_e \approx 1 \text{ GeV} \approx 1 \text{ u} \)
proton  | \( M_p \approx 938 \text{ MeV} \)
neutron  | \( M_n \approx 940 \text{ MeV} \)
kaon  | \( m_K \approx 500 \text{ MeV} \)
pion  | \( m_\pi \approx 140 \text{ MeV} \approx \frac{1}{7} M_N \)
mesons  | \( m_\rho \approx 700 \text{ MeV} \)
Higgs boson  | \( M_H \approx 125 \text{ GeV} \)
W boson  | \( M_W \approx 80 \text{ GeV} \)
Z boson  | \( M_Z \approx 90 \text{ GeV} \)

**Scattering**  | nuclear cross-section unit: \( 1 \text{ barn} = 100 \text{ fm}^2 = (10 \text{ fm})^2 = 10^{-28} \text{ m}^2 \approx \frac{1}{400 \text{ MeV}^2} \)
geometric scattering: \( \sigma_{\text{geometric}} = 4\pi a^2 \).

**Hierarchy of Scales**  | typ. energy  | typ. momentum  | typ. size/distance
--- | --- | --- | ---
nuclear structure  | binding: 8MeV per nucleon  | 100 keV...1MeV  | 10fm (~235U size)
few-nucleon  | deuteron: 2.2246MeV  | \( m_\pi \approx 140 \text{ MeV} \) \approx 1.5fm (Yukawa)  | \( \frac{1}{M_N} \approx 0.2\text{fm} \)
hadronic  | \( M_N, M_\rho \approx 1 \text{GeV} \)  | 1GeV (relativistic)  | \( \frac{1}{100 \text{GeV}} \approx 2 \times 10^{-3}\text{fm} \)
particle  | 100GeV Z, W masses  | 100GeV (relativistic)  |

**Interaction Scales**  | very rough – factors of 100 up or down are common
--- | --- | --- | --- | ---
range  | \( \frac{1}{M_\pi} \approx 1.4 \text{ fm} \)  | \( \frac{1}{\text{MeV}} \approx 0.2 \text{ fm} \)  | \( \frac{1}{M_{W,Z}} \approx 0.01 \text{ fm} \)  |
life time \( \tau \)  | \( 10^{-22} \text{ s} \)  | \( 10^{-23} \text{ s} \)  | \( 10^{-20} \text{ s} \)  | \( 10^{-10} \text{ s} \)  | \( 10^{-9} \text{ s} \)  |
decay width \( \Gamma \)  | 200 MeV  | 1 MeV  | \( \ll \text{eV} \)  |
cross section \( \sigma \)  | barn (NN@1MeV: 70b)  | mb  | \( \mu\text{b} \)  | \( 10^{-12} \text{ b} = 1 \text{ pb} \)  | 100 pb  |

**Miscellaneous**  | neutron lifetime: \( \tau_n \approx 880 \text{ s} \)  | hadron size \( R \approx 0.7 \text{ fm} \)  | hadronisation scale \( 1 \text{GeV} \approx 0.2 \text{ fm} \)
Weinberg mixing angle \( \sin^2 \theta_W \approx 0.22 \approx 1 - \frac{M_W^2}{M_Z^2} \)  | Fermi constant \( G_F \approx \sqrt{\frac{g^2}{8M_W^2}} \approx 1 \times 10^{-5} \text{ GeV}^{-2} \)  |
running coupling constants: \( \alpha(1 \text{ MeV}) = \frac{1}{137} \), \( \alpha(M_Z) = \frac{1}{128} \)  | \( \alpha_s(2m_\mu \approx 10 \text{ GeV}) \approx 0.2; \alpha_s(M_Z) \approx 0.118 \)  |
Summary Electron Scattering Cross Sections

Lowest-order Feynman graph

\[
\begin{align*}
\text{elastic, unipolarised, cm-frame} & \quad \frac{d\sigma}{d\Omega_{\text{cm}}} \mid_{\text{lab}} = \frac{1}{64\pi^2 s} |M|^2 \\
\text{elastic, unipolarised, lab-frame} & \quad \frac{d\sigma}{d\Omega_{\text{lab}}} \mid_{\text{lab}} = \frac{1}{64\pi^2 M^2} \left( \frac{E'}{E} \right)^2 |M|^2 \\
\text{matrix element: avg. initial, sum final spins} & \quad e^2 L_{\mu\nu} = \frac{1}{2s + 1} \sum_{s, s'} j_{s,s'}^{\mu}(k, k') j_{s,s'}^{\nu}(k, k')
\end{align*}
\]

\( e^{\pm} \) Coulomb scattering on infinitely heavy, composite spin-0 target: (s = \frac{1}{2}; M, S = 0)

\[
\begin{align*}
\text{target: } & \quad \frac{d\sigma}{d\Omega_{\text{lab}}} \mid_{\text{lab}} = \left( \frac{Z\alpha}{2E\sin^2 \frac{\theta}{2}} \right)^2 \cos^2 \frac{\theta}{2} \times |F(q^2)|^2 \\
\text{normalisation: } & \quad F(0) = 1; \text{ charge radius: } \langle r^2 \rangle = -\frac{4}{3} F(q^2) |q|^2
\end{align*}
\]

\( e^{\pm} \) full electromagnetic scattering on composite spin-0 target: (s = \frac{1}{2}; M finite, S = 0)

\[
\begin{align*}
\text{target: } & \quad \frac{d\sigma}{d\Omega_{\text{lab}}} \mid_{\text{lab}} = \left( \frac{Z\alpha}{2E\sin^2 \frac{\theta}{2}} \right)^2 \cos^2 \frac{\theta}{2} \times |F(q^2)|^2 \\
\text{Mott: } & \quad \frac{d\sigma}{d\Omega_{\text{lab}}} \mid_{\text{lab}} = \left[ 1 - \frac{q^2}{2M^2} \tan^2 \frac{\theta}{2} \right] |F(q^2)|^2 \\
\text{back-scattering by helicity transfer} & \quad W_{\mu\nu} = Z^2 (p + p')^\mu (p + p')^\nu |F(q^2)|^2
\end{align*}
\]

\( e^{\pm} \mu^- \rightarrow e^{\pm} \mu^+ \) scattering on massive spin-\( \frac{1}{2} \) target without structure: (s = \frac{1}{2}; M, S = \frac{1}{2})

\[
\begin{align*}
\text{target: } & \quad \frac{d\sigma}{d\Omega_{\text{lab}}} \mid_{\text{lab}} = \left( \frac{Z\alpha}{2E\sin^2 \frac{\theta}{2}} \right)^2 \cos^2 \frac{\theta}{2} \times \left[ F_1(q^2) + \tau F_2(q^2) \right] \\
& \quad \text{back-scattering by helicity transfer} \\
& \quad W_{\mu\nu} = Z^2 (p + p')^\mu (p + p')^\nu |F(q^2)|^2
\end{align*}
\]

\( e^{\pm} \) on composite, massive spin-\( \frac{1}{2} \) target: form factors \( F_1(q^2) \): Dirac; \( F_2(q^2) \): Pauli (s = \frac{1}{2}; M, S = \frac{1}{2})

\[
\begin{align*}
\text{target: } & \quad \frac{d\sigma}{d\Omega_{\text{lab}}} \mid_{\text{lab}} = \left( \frac{Z\alpha}{2E\sin^2 \frac{\theta}{2}} \right)^2 \cos^2 \frac{\theta}{2} \times \left[ F_1(q^2) + \tau F_2(q^2) \right] \\
& \quad \text{anomalous mag. term, } F_2(0) = \kappa = G_M(0) - Z; \\
& \quad \kappa \text{ anomalous mag. moment, } \mu = Z + \kappa \text{ mag. moment]
\end{align*}
\]

\( e^{\pm} \) inelastic, inclusive scattering: (s = \frac{1}{2}; M, S = \text{any [sic!]})

\[
\begin{align*}
\text{inelasticity measure: Bjorken-}x & \quad := -\frac{q^2}{2p\cdot q} = \frac{Q^2}{2M^2} \in [0; 1]; \text{ elastic: } x = 1, \text{ i.e. } F_{1,2}(q^2, x) \propto \delta(\nu + \frac{Q^2}{2M}) \\
\text{Most general elmag. hadronic ME (symmetric } \mu \leftrightarrow \nu, \text{ charge conservation; any spin [sic!]}) & \quad W_{\mu\nu} = F_1(q^2, x) \left[ q^\mu q^\nu q^2 - g_{\mu\nu} \right] + F_2(q^2, x) \frac{M}{M^2} \left[ q^\mu - \frac{p^\mu - q^\mu}{q^2} \right] \left[ q^\nu - \frac{p^\nu - q^\nu}{q^2} \right]
\end{align*}
\]

Structure functions \( F_1, F_2 \) are not the Dirac, Pauli FFs of Eq. (1.7.5)!