I. Tools

6. Scattering and Decay of Particles

Or: How Long to Count

References: [HH; HG 10.1-2, 5.7/12; PRSZR 4; HM 4.3, 2.10, 4.4; PDG 47, 47.5, 48]
target has length $d$

typical target density for liquid/solid: $\frac{1 \text{ particle}}{\text{Ångstrom}} \approx 1 \times 10^{30} \text{m}^{-3}$

for gas: $\frac{6 \times 10^{23} \text{ particles}}{22 \text{ litres} \approx 1 \text{ mol}} \times \frac{\text{pressure}}{1 \text{ bar}} \approx \frac{1}{4} \times 10^{26} \text{m}^{-3} \times \frac{\text{pressure}}{\text{[bar]}}$
**Resonances in Quantum Mechanics**

**Classical Mechanics:** resonance frequencies reveal properties of materials.

**Electrodynamics:** Lorentz-Drude model, resonance fluorescence

**Quantum Mechanics:** interference $\rightarrow$ resonance even when no bound states.

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![Diagram of scattering and transmission resonances](image)

Figure 5.30: Scattering of a particle with energy $E$ from a one-dimensional potential well. Classically, all incident particles will be transmitted. Quantum mechanically, at small energies, the transmission coefficient $T$ is unity only at certain energies. The appearance of transmission resonances in the behavior of the transmission as a function of particle energy $E$ is shown at the right. [HG]
Describe Resonance as Creation & Decay of Unstable Particle

\[ \sigma(1 + 2 \rightarrow BC \ldots) \propto |\mathcal{M}(1 + 2 \rightarrow A^* \rightarrow BC \ldots)|^2 \]

Model as Nonrelativistic Breit-Wigner:

Collision with total cm-energy \( E_{\text{cm}} \), relative momentum \( \vec{k}_{\text{cm}} \), spins \( S_1, S_2 \).

\[ \implies \text{Produces resonance at } E_0, \text{ total decay width } \Gamma_{\text{total}}, \text{ spin } J. \]

\[ \implies \text{Decays into } A^* \rightarrow BC \ldots \text{ (final state fully specified)}. \]

\[ \sigma(1 + 2 \rightarrow A^* \rightarrow BC \ldots) = \frac{2J + 1}{(2S_1 + 1)(2S_2 + 1)} \frac{4\pi}{|\vec{k}_{\text{cm}}|^2} \frac{B_{\text{in}}^{1+2 \rightarrow A^*} B_{\text{out}}^{BC \ldots} \Gamma_{\text{total}}^2}{4 (E_{\text{cm}} - E_0)^2 + \Gamma_{\text{total}}^2 / 4} \]

\( \Gamma_{\text{total}} \): decay width into any final state: “Full Width at Half-Maximum” FWHM

\( \Gamma_{A^* \rightarrow BC\ldots} = B_{\text{out}}^{BC\ldots} \times \Gamma_{\text{total}} \): partial decay width into specific final state \( BC \ldots \)

\[ \Gamma_{\text{total}} = \sum_{\text{all finals}} \Gamma_{BC\ldots}, \sum_{\text{all finals}} B_{\text{out}}^{BC\ldots} = 1 \]

Branching Ratios: \( B_{\text{out}}^{BC\ldots} \): percentage of resonances decaying into specific final state \( BC \ldots \)

\( B_{\text{in}} = B^{1+2} \) by detailed balance: “probability” to produce \( A^* \) by colliding \( 1 + 2 \).
Be Wary of Breit-Wigner Parametrisations in Hadron Physics!

Must account for energy constraints (thresholds) in decay! $\implies$ energy-dependent width $\Gamma_{BW}(s)$

Relativistic Breit-Wigner parametrisation:

often used but not unique

$$M_{res} = \sqrt{s} \frac{\Gamma_{BW}^{\text{elastic}}(s)}{s - M_{BW}^2 + i \sqrt{s} \Gamma_{BW}^{\text{total}}(s)}$$

$\nabla$ BUT Breit-Wigner parametrisations work only for narrow, well-separated resonances!

Problems:

$\implies$ HW

$- \mathcal{M} = \mathcal{M}_{res} + \mathcal{M}_{background}$: split is arbitrary!

Where does background start/end?

$- \text{Resonances overlap } \implies \text{interference!}$

$\implies$ Only positions $s_R$ and residues $\Gamma_{\text{residue}}(s_R)$ of poles in scattering amplitude are unique!

$\sqrt{s_R} \neq M_{BW} - i \frac{\Gamma_{BW}}{2}$:

Breit-Wigner mass is not pole position!

More in PHYS 6710: Nuclear & Particle Physics II
Next: 7. Electron Scattering

Familiarise yourself with: [HM 4, 6.1/3-6/9/11/13, 8]