I. Tools

7. Scattering and Decay of Particles

Or: How Long to Count

References: [HH; HG 10.1-2, 5.7/12; PRSZR 4; HM 4.3, 2.10, 4.4; PDG 48, 48.5, 49]
Garbage-In – Garbage-Out
What An Experiment Really Is (Ideally)

**Beam Cleanup:** remove charged undesireds by $\vec{B}$

**Collimators:** make sure *all* beam hits target
eliminate “beam halo” (cotravelling undesireds)
#1: define, #2: remove scatters, #3: make sure

**Charged-Beam Dump:** use Cu; after bend to reduce backscatter; measure charged-beam flux by Faraday cup; often most radioactive piece during run

**Target:** typ. $1\text{ mol} \approx 6 \times 10^{23}$ particles to avoid *multiple scattering*;

gas @STP: $6 \times 10^{23} \text{ dm}^{-3} \leftrightarrow 1000$ liquid/solid: $6 \times 10^{23} \text{ cm}^{-3}$;

often cooled to K or mK (liquid H, $^4\text{He}, \ldots$) & polarised

**If you are a beam, everything looks like a target:**

Nature cannot separate between *signal (good)* and *noise (bad)*:

**contaminations:** scatter from wrong reaction, atomic $e^-$, container, impurities/stabilising compounds (e.g. NaPO$_3$ for P), collimators, beam dump; environment: concrete, cosmics,\ldots

**Detector:**
collimator often defines angle

**Data Acquisition:**
hardware/software filters, event recording,\ldots

\[\equiv \text{ student} \equiv \text{ paper}\]
target has length $d$

typical target density for liquid/solid:

$$\frac{1 \text{ particle}}{\text{Ångstrom}} \approx 1 \times 10^{30} \text{m}^{-3}$$

for gas:

$$\frac{6 \times 10^{23} \text{ particles}}{22 \text{ litres} \approx 1 \text{ mol}} \times \frac{\text{pressure}}{1 \text{ bar}} \approx \frac{1}{4} \times 10^{26} \text{m}^{-3} \times \frac{\text{pressure}}{[\text{bar}]}$$
Summary Electron Scattering Cross Sections

Lowest-order Feynman graph

\[
\frac{\text{d} \sigma}{\text{d} q^2} = \frac{Z^2}{2E^2m^2} F^2(q^2) \cos \theta E^2 \cos \frac{\theta}{2} \times |F(q^2)|^2
\]

\( e^\dagger \) Coulomb scattering on infinitely heavy, composite spin-0 target:

- elastic, unpolarised, cm-frame
- lepton tensor: scatter off virtual \( \gamma \)

\( e^\dagger \) full electromagnetic scattering on massive composite spin-0 target:

- elastic, unpolarised, lab-frame
- lepton current (electromagnetic)

\( e^\dagger \mu^\dagger \) scattering on massive spin-1 target without structure:

- elastic, unpolarised, lab-frame
- hadron tensor: scatter off virtual meson

\( e^\dagger \) on composite, massive spin-1 target: form factors \( F_1(q^2) \): Dirac, \( F_2(q^2) \): Pauli

- elastic, unpolarised, lab-frame
- lepton current (electromagnetic)

\( e^\dagger \) inelastic, inclusive scattering: \( E^\prime \) independent variable, \( p' \) not detected

- elastic, unpolarised, lab-frame
- lepton current (electromagnetic)

Structure functions \( F_1, F_2 \) are not the Dirac, Pauli FFs of Eq. (1.7.5)!
**Classical Mechanics:** resonance frequencies reveal properties of materials.

**Electrodynamics:** Lorentz-Drude model, resonance fluorescence

\[ \vec{E}_{\text{in}}(\omega) \]

**Quantum Mechanics:** interference \( \rightarrow \) resonance even when no bound states.

Figure 5.30: Scattering of a particle with energy \( E \) from a one-dimensional potential well. Classically, all incident particles will be transmitted. Quantum mechanically, at small energies, the transmission coefficient \( T \) is unity only at certain energies. The appearance of *transmission resonances* in the behavior of the transmission as a function of particle energy \( E \) is shown at the right. [HG]
Describe Resonance as Creation & Decay of Unstable Particle

\( \sigma(1 + 2 \rightarrow BC \ldots) \propto |\mathcal{M}(1 + 2 \rightarrow A^* \rightarrow BC \ldots)|^2 \)

**IF[!]! Modelled as Nonrelativistic Breit-Wigner:**

Collision with total cm-energy \( E_{\text{cm}} \), relative momentum \( \vec{k}_{\text{cm}} \), spins \( S_1, S_2 \).

\[ \implies \text{Produces resonance at } E_0, \text{ total decay width } \Gamma_{\text{total}}, \text{ spin } J. \]

\[ \implies A^* \rightarrow BC \ldots \text{ (final state fully specified)}. \]

\[
\sigma(1 + 2 \rightarrow A^* \rightarrow BC \ldots) = \frac{2J + 1}{(2S_1 + 1)(2S_2 + 1)} \frac{4\pi}{|\vec{k}_{\text{cm}}|^2} \Gamma_{\text{total}}^2 / 4 \frac{B_{\text{in}}^{1+2 \rightarrow A^*} B_{\text{out}}^{BC \ldots} \Gamma_{\text{total}}^2 / 4}{(E_{\text{cm}} - E_0)^2 + \Gamma_{\text{total}}^2 / 4}
\]

\( \Gamma_{\text{total}} \): decay width into any final state: “Full Width at Half-Maximum” FWHM

\( \Gamma_{A^* \rightarrow BC \ldots} = B_{\text{out}}^{BC \ldots} \times \Gamma_{\text{total}} \): partial decay width into specific final state \( BC \ldots \)

\[ \Gamma_{\text{total}} = \sum_{\text{all finals}} \Gamma_{BC \ldots}, \sum_{\text{all finals}} B_{\text{out}}^{BC \ldots} = 1 \]

**Branching Ratios:** \( B_{\text{out}}^{BC \ldots} \): percentage of resonances decaying into specific final state \( BC \ldots \)

\( B_{\text{in}} = B^{1+2} \) by detailed balance: “probability” to produce \( A^* \) by colliding \( 1 + 2 \).
Be Wary of Breit-Wigner Parametrisations in Hadron Physics!

Must account for energy constraints (thresholds) in decay! \(\implies\) energy-dependent width \(\Gamma_{BW}(s)\)

**Relativistic Breit-Wigner parametrisation:** proposed by PDG, often used but not unique

\[
M_{res} = \frac{\sqrt{s} \Gamma_{BW}^{\text{elastic}}(s)}{s - M_{BW}^2 + i \sqrt{s} \Gamma_{BW}^{\text{total}}(s)}
\]

**BUT** Breit-Wigner parametrisations work only for narrow, well-separated resonances!

Problems:

- \(\mathcal{M} = M_{res} + M_{\text{background}}\): split is arbitrary!
- Where does background start/end?
- Resonances overlap \(\implies\) interference!

\(\implies\) Only positions \(s_R\) and residues \(\Gamma_{\text{residue}}(s_R)\) of poles in scattering amplitude \(\mathcal{M}\) are unique!

\[
\sqrt{s_R} \neq M_{BW} - \frac{i \Gamma_{BW}}{2}:
\]

Breit-Wigner mass is not pole position!

More in PHYS 6710: Nuclear & Particle Physics II
Next: 8. Electron Scattering

Familiarise yourself with: [HM 4, 6.1/3-6/9/11/13, 8]