II. Phenomena

5. \( e^+ e^- \rightarrow \) Leptons and Quarks

Or: Why We Believe in Things We Don’t See

References: [PRSZR 9.1/3; PRSZR 15/16 (cursorily); HG 10.9, 15.1-7; HM 11.1-3; Tho 9.6]
(a) Recap $e^+ e^- \to \mu^+ \mu^-$: Massless Point-Fermions

For massless point-fermions $X$ of charge $Z_X$: $R \equiv \frac{\sigma(e^+ e^- \to X \bar{X})}{4\pi\alpha^2/(3s)} \to \sum Z_X^2$ with $\sqrt{s} \geq 2M_X$

\[ \sigma_{cm} = \frac{4\pi(Z\alpha)^2}{3s} = Z^2 \frac{21.7\text{nb}}{E_{cm}^e[\text{GeV}]} \]

\[ \frac{d\sigma}{d\Omega}_{cm} = \frac{(Z\alpha)^2}{4s} (1 + \cos^2 \theta) \]

Ang. distrib. characteristic of spin-$\frac{1}{2}$.

$\Theta \frac{1}{137}$, simple to interpret, $e^+ e^-$ collider cheap

$\Theta$ Directly probes only charges, not strong int.
(b) Leptoproduction and Lepton Universality

Threshold $\sqrt{s_{\text{min}}} = 2M_l$: muon $M_\mu = 0.106$ GeV (1936), tau lepton $M_\tau = 1.777$ GeV (1975)

Lifetime $\tau(\tau^\pm \rightarrow e^- \bar{\nu}_e \nu_\tau$ or $\mu^- \bar{\nu}_\mu \nu_\tau) = 3 \times 10^{-13}$ s $\gg \tau_{\text{elmag,strong}} \implies$ weak decay

Even at $\gamma = \frac{E}{M_\tau} \approx 50$, $\tau$ lepton travels $c\gamma\tau_\tau = 10^{-2}$ m before decay $\implies$ not in detector

Experiments at $E \gg M_\tau, M_\mu$:

$R(\mu^+\mu^-$ or $\tau^+\tau^-) = 1$

$\implies Z_\mu = Z_\tau = 1 = Z_e$

Fig. 9.3. Cross-sections of the reactions $e^+e^- \rightarrow \mu^+\mu^-$ and $e^+e^- \rightarrow \tau^+\tau^-$ as functions of the centre of mass energy $\sqrt{s}$ (from [Ba85] and [Be87]). The solid line shows the cross-section (9.6) predicted by quantum electrodynamics. [PRSZR]

$\implies$ Lepton Universality Hypothesis: Leptons couple with same form & strengths, and differ only by mass & charge (thresholds etc. different, but not fundamental couplings).

BaBar at SLAC 2012: branching ratio $B \rightarrow \mu$ or $\tau$ [Phys. Rev. Lett. 109 (2012) 101802]

LHCb at CERN 2015: branching ratio $D, D^* \rightarrow \mu$ or $\tau$ [Phys. Rev. Lett. 115 (2015) 111803]

Lepton Universality may be broken:
Small ($10^{-2}$) but significant ($3.9\sigma$?) and important (Baryogenesis).

$\Rightarrow$ Beyond-Standard-Model Physics?
(c) $e^+ e^- \rightarrow \text{Hadrons}$: Overview

**non-resonant:**
well-reproduced by $\sigma \propto \frac{1}{s}$

**resonances:**
have quantum numbers of $\gamma^*$:
- $I = 0$ or 1, $J^{PC} = 1^{--}$
- *wide* at low $s$, *narrow* at high $s$

$$R = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-)}$$

increases after each resonance region

$\Longrightarrow$ Hadron contains point-like, charged fermions with “small” masses.

[PDG 2012 46.6]
(d) Nonresonant Hadron Production (High Energies)

Produce point spin-$\frac{1}{2}$ particle: 
\[
\left. \frac{d\sigma}{d\Omega} \right|_{\text{cm}} = \frac{(Z\alpha)^2}{4s} \left( 1 + \cos^2 \theta \right)
\]

Angular distribution of 2-jet event consistent with $1 + \cos^2 \theta \implies$ Evidence for point-fermions.

\[\sqrt{s} = 34 \text{ GeV}\]
Counts Quark Charges AND Colours

\[ R(s) := \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \sum_{\text{quarks } i \text{ with } 2M_i \leq \sqrt{s}} Z_i^2 \]

\[ R = N_c \sum_q Z_q^2 = 3 \sum_q Z_q^2 \text{ universal hidden property} \]

\[ \begin{align*}
\rho & = \frac{2}{3} \text{up} + \frac{1}{3} \text{down} + \frac{1}{3} \text{strange} = \frac{6}{9} \\
\phi & = \frac{10}{9} \\
\omega & = \frac{10}{9} + \left(\frac{2}{3}\right)^2 \text{charm} \\
\psi(2S) & = \frac{10}{9} + \left(\frac{1}{3}\right)^2 \text{bottom} = \frac{11}{9} \\
J/\psi & = 6 \\
\gamma & = 10 \\
Z & = 11 \\
\end{align*} \]
No free quarks seen. Each quark of initial $q\bar{q}$ pair carries energy $E \gg m_q$; fly in opposite directions.

$\implies$ Generate light $q\bar{q}$ pairs out of vacuum.

Rearrange to dress bare quarks into baryons & mesons:

timescale $\gg$ pair-production

$\implies$ Production & hadronisation: 2-step process:

incoherent sum of $q\bar{q}$-pair productions

$$\frac{d\sigma}{d\Omega}(ee \to hX) = \sum_q \frac{d\sigma}{d\Omega}(ee \to q\bar{q}) \left[D^h_q(z) + D^h_{\bar{q}}(z)\right]$$

which are weighted by

Quark-fragmentation functions

$D^h_q, D^h_{\bar{q}}(z = \frac{E_h}{E_q})$ related to PDFs $q(x)$

by crossing & time-reversal symmetries.
Resonant Hadron Production at Low Energies

Broad $J^{PC} = 1^{--}$ resonances, $\tau \sim 10^{-[22...24]}s \implies$ strong process.

$\sqrt{s} [\text{GeV}]$ [PDG 2012 46.7]

$\omega^0(782 \text{ MeV})$: decay $\rightarrow \pi^+ \pi^0 \pi^-$; no isospin partners $\implies I = 0$

$\rho^0(770 \text{ MeV})$: decay $\rightarrow \pi^+ \pi^-$; isospin partners $\rho^{\pm,0} \implies I = 1$

Spin-isospin-quark content e.g. $|\rho^+\rangle = -|u^\uparrow \bar{d}^\uparrow\rangle$

Resonances in close proximity $\implies$ strong interference!

Vector Meson Dominance (VMD) Model [Sakurai 1960/69]:

Elmag. dominated by these mesons, e.g. in $\gamma N \sim \rho, \omega, \phi$
Something interesting should indeed happen around $800\text{MeV}$:

Lowest hadron production threshold: $\sqrt{s} = 2m_\pi$ from $e^+e^- \rightarrow \pi\pi$.

$\pi\gamma$ coupling from pion form factor for space-like $q^2 < 0$ (see II.2.g):

$$J_\pi^\mu = -ie (p'^\mu - k'^\mu) \overline{\text{pion FF}} \frac{a^2}{a^2 - q^2} \text{ with } a^2 = \frac{6}{\langle r^2_\pi \rangle} \approx (740\text{MeV})^2 \text{(exp)}$$

Apply **crossing symmetry/analytic continuation** into time-like region $q^2 = s > 0$:

$$\Rightarrow \text{ Expect pole/very large amplitude/resonance in } J^{PC} = 1^{--} \text{ processes around}$$

$$q^2 = s = a^2 \approx (740\text{MeV})^2$$

Agrees with $\omega/\rho$-meson quantum numbers and $m_\omega \approx m_\rho \approx 775\text{MeV}$.

But be careful: Exp. only gives rough form factor with uncertainties.

$$\Rightarrow \text{ Analytic continuation needs “reasonable” assumptions.}$$
**φ Resonance: The Strange Quark**

Narrow resonance: \( \Gamma_\phi = 4.4 \text{ MeV} \) since \( 2m_K = 990 \text{ MeV} \) \( \Rightarrow \) only 30 MeV of phase space!

\( \phi \rightarrow \pi\pi\pi \) decay very small, although \( m_\phi - 3m_\pi = 600 \text{ MeV} \) much bigger.

\( \Rightarrow \) Attribute to new quark flavour: **Strange Quark**; strangeness \( S \) conserved in strong int.

\( K^+K^- , K^0\bar{K}^0 \) isospin doublets \( \Rightarrow \)

New charge formula: \[ Q = \frac{\text{Baryon}}{2} + I_3 + \frac{S}{2} \]

Gell-Mann–Nishijima relation

\[ Q_s = -\frac{1}{3}, B_s = \frac{1}{3} \]

But strangeness of strange is \( S(s) = -1 \): That's strange! (but a definition...)
Interpret Strangeness (or Strong Hypercharge $Y = S + B$) as quantum number, orthogonal to Isospin.

**One Can Show:** symmetry group in Nature extends from $SU_I(2) \rightarrow SU_{flavour}(3)$ acting on $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$:

$Ladder Operators I_\pm, U_\pm, V_\pm$ raise/lower along triangle sides.
Constructing Multiplets from The Fundamental Representations

Combine **Weight Diagrams** like in $SU(2)$:

- $j = \frac{1}{2}$
  - $-\frac{1}{2}$
  - $\frac{1}{2}$

- $j = 1$
  - $-1$
  - $0$
  - $+1$

- $j = \frac{3}{2}$
  - $-\frac{3}{2}$
  - $-\frac{1}{2}$
  - $\frac{1}{2}$
  - $\frac{3}{2}$

**Example:** $q\bar{q}$ combinations give Meson Octet & Singlet.

\[ \begin{array}{c}
\text{Example: } q\bar{q} \text{ combinations give Meson Octet & Singlet.}
\end{array} \]
Lowest-Mass Meson Octets: Natural Isospin Doublets $K^+K^0$ & $K^-\bar{K}^0$

**Constituent picture $\implies$ Gell-Mann–Okubo mass formula:**

$$m_{\text{meson}} = M_{\text{bind}} + \sum_i m_{q_i}?$$

$m_s \approx 360\text{MeV}$?

**$SU_f(3)$-breaking in Ground-State Octet:**

$$\frac{\text{diff. } \pm 350\text{MeV}}{\text{avg. } 320\text{MeV}} \approx \frac{1!}{10}$$

in Excited Octet: $\frac{\pm 80\text{MeV}}{850\text{MeV}} \approx \frac{1}{10}$

**Example Octet-Breaking in $\phi$:**

$$m_\phi - 3m_\pi \approx 600\text{MeV}$$

$$\gg m_\phi - 2m_K \approx 30\text{MeV}$$

but $\pi$ decay tiny, while decays to 85% into Kaons

$$\implies |\phi\rangle \approx |s\bar{s}\rangle$$

and not $|\phi\rangle = \frac{1}{\sqrt{3}}[|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle]$

$$m_s = m_K - m_\rho \approx 120\text{MeV}?$$

or $(m_\phi - m_\rho)/2 \approx 120\text{MeV}?$
\[ \pi N / KN \rightarrow X : \text{Lowest-Mass Baryon Multiplets: Octet & Decouplet} \]

**SU_f(3) Breaking in Baryon Octet:**

\[
\begin{align*}
\Delta^- & \rightarrow \Delta^0 & \Delta^+ & \rightarrow \Delta^{++} \\
(3/2)^- & \rightarrow 0 & (1/2)^+ & \rightarrow 3/2 \\
\Sigma^{*-} & \rightarrow \Sigma^0 & \Sigma^{*+} & \rightarrow \Sigma^{*0} \\
(1/2)^- & \rightarrow 1 & (3/2)^+ & \rightarrow 2 \\
\Xi^- & \rightarrow \Xi^0 & \Xi^{*-} & \rightarrow \Xi^{*0} \\
(1)^- & \rightarrow 2 & (3)^+ & \rightarrow 3 \\
S^+ & \rightarrow 1 & S^0 & \rightarrow 0 \\
S^- & \rightarrow 3 & S^{*-} & \rightarrow 2 \\
S & \rightarrow 0 & S^* & \rightarrow 1 \\
\end{align*}
\]

\[ M_{\text{baryon}} = M_{\text{bind}} + \sum_i m_{q_i} \]

\[ \downarrow \text{linear increase} \]

\[ m_s \approx 210\text{MeV} ? \]

**GMO:**

\[ 1100\text{MeV} \approx \frac{1}{6} \]

**in Baryon Decouplet:**

\[ 1400\text{MeV} \approx \frac{1}{10} \]

\[ \downarrow \text{linear increase} \]

\[ m_s \approx 140\text{MeV} ? \]

Gell-Mann 1962: predict quantum numbers & mass of \( \Omega^- \). Dedicated experiment found it.
What and How Good Is The Constituent Quark Model?

Assumptions: (cf. ”dressed” electron in solid)
- “Naked” QCD quarks dressed into constituent quarks inside hadrons.
- Constituent quarks determine bulk of quantum numbers;
- still point-like/”elementary”, but with anomalous magnetic momenta.

Add a spin-spin term $H_{\text{spin}} \propto \frac{\vec{\sigma}_{q1} \cdot \vec{\sigma}_{q2}}{m_{q1} m_{q2}}$ with “universal” prefactor to match hadron masses & magnetic moments.

But conflicting answers:
- Constituent masses for mesons & baryons different.
- Point-like but not fundamental particles: constituent quarks $\neq$ QCD (current) quarks of DIS.
- Couplings $> 1$. $\implies$ Perturbative treatment of non-perturbative interaction inconsistent!
- Confinement problem unsolved: If perturbative, then no confinement.

The Constituent Quark Model is not QCD– It is a QCD-inspired Model, at best!
Misconceptions lead to self-inflicted puzzles/crisis (spin-puzzle, missing resonances, EMC effect, . . .).

Predictions for low-mass baryons & mesons misleading (e.g. inconsistent $m_s$).
Predictions for high-mass baryons & mesons adequate $\rightarrow$ quarkonia: QCD perturbative.
**High-\(E\) Resonant Hadron Production: Quarkonia**

**Huge cross section, tiny width.**

\(J/\psi(3097\text{ MeV})\) (November Revolution 1974)

width \(\Gamma = 0.093\text{ MeV}\)

indicates electromagnetic decay.

\[\Delta R = \frac{4}{3} = N_c Z_c^2 \implies Z_c = \frac{2}{3}\]

charm quark: \(J/\psi = c\bar{c}\) charmonium

\(\Upsilon(9470\text{ MeV})\) (1977)

width \(\Gamma = 0.052\text{ MeV}\)

\[\Delta R = \frac{1}{3} = N_c Z_b^2 \implies Z_b = -\frac{1}{3}\]

bottom quark: \(\Upsilon = b\bar{b}\) bottomium

\[\implies Q = \frac{B}{2} + I_3 + \frac{S + C + B + T}{2}\]

generalised Gell-Mann–Nishijima

“Toponium” (Fermilab 1995): resonance at

\[\sqrt{s} = 2M_t \approx 340\text{GeV}\] in \(p\bar{p}\) collisions.
Example of Excited States: $\psi \rightarrow \gamma X$ Photon Decay Spectrum

Very narrow states, decaying electromagnetically.

Fig. 13.5. The photon spectrum in the decay of $\psi (2^3S_1)$, as measured in a crystal ball, and a sketch of the so extracted charmonium energy levels. The strong peaks in the photon spectrum represent the so numbered transitions in the sketch. The continuous lines in the sketch represent parity changing electric dipole transitions and the dashed lines denote magnetic dipole transitions which do not change parity [K886].

[PRSZR]
Comparing Quarkonium Spectra to Positronium in QED

\( J/\psi \) and \( \Upsilon \) are \( J^P = 1^- \) resonances. \( \Rightarrow \) 1\(^3\)\(S_1\) excited states, not \( J = 0 \) ground states 1\(^1\)\(S_0\).

\[ M(\text{quarkonium}) < 2M(\text{heavy-light meson}) \Rightarrow \text{strong decay forbidden} \Rightarrow \text{elmag., small rates} \]

D mesons: (\( \bar{c}u \)) etc  
B mesons: (\( \bar{b}u \)) etc  
K mesons: (\( \bar{s}u \)) etc.

Does not apply to \( \phi \): \( 2m_K < m_\phi \Rightarrow \text{strong decay but small phase space} \).

Nomenclature \( N^{2S+1L_J} \)

Spectra adjusted to same \( 1^{1S_0} - 2^{1S_0} \) gap. [PRSZR]

\( \Rightarrow \text{Quarkonia (heavy-heavy): Coulombic potential for low states – different for high states.} \)
6. Summary: The Path to QCD

- Hadrons contain near-massless charged spin-$\frac{1}{2}$ point-particles. \[ \text{partons} \rightarrow \text{quarks} \]
- Parton masses do not set masses of light mesons, nucleons. \[ \text{hadron masses from strong int.} \]
- 6 quark flavours: u, d, s, c, b, t – only charges $\pm \frac{2}{3}, \pm \frac{1}{3}$, \[ Q = \frac{\text{Baryon}}{2} + I_3 + \frac{S + C + B + T}{2}. \]
- Approx. hadron mass multiplets: SU$_f$(2) \[ \begin{pmatrix} u \\ d \end{pmatrix} \]; less well for SU$_f$(3) \[ \begin{pmatrix} u \\ d \\ s \end{pmatrix} \]. \[ \text{flavour symmetry} \]
- Quarks come in 3 colours ($\Delta^{++}(u^\uparrow u^\uparrow u^\uparrow)$, R in $e^+ e^- \rightarrow \text{hadr}$). \[ \text{colour degree of freedom} \]
- Quarks only differ by mass & charge (and related effects). \[ \text{flavour & colour universality} \]
- Neutral, strongly int. hadron constituents carry large fractions of its momentum & spin. \[ \text{gluons} \]
- Strong int. QED-like & perturbative as $E, m_q \uparrow \infty$ (quarkonia, 3-jet events). \[ \text{Asymptotic Freedom} \]

Identify gluons with colour carriers?

- No free quarks seen. \[ \text{Quark Confinement Hypothesis plausible, unproven} \]
- No free gluons seen. \[ \text{Gluon Confinement Hypothesis plausible, unproven} \]
- No states with net nonzero colour seen. \[ \text{Colour Neutrality Hypothesis plausible, unproven} \]

Find a theory which explains all this, and Nuclear Physics – quantitatively!
Next: III. Descriptions

Non-Abelian Gauge Theories

Familiarise yourself with: [HM 14.1-4, 2.15; HG 12.3; CL 8.1]