III. Descriptions

5. Pions and Nucleons: Chiral Effective Field Theory

Or: What I Do for a Living

References: [(Goldstone: CL 5; Ryd 8.1-3); Scherer/Schindler: Primer $\chi$EFT;
CL 5; Ryd 8.1-2; Ber 2, 3; Ericson/Weise: Pions and Nuclei Chap. 9;
lectures 1-6 of Fleming’s EFT online course at MIT; and much more – see me!]
(a) Matching Expectations

What Holds the Nucleus Together?

1953

In the past quarter century physicists have devoted a huge amount of experimentation and mental labor to this problem – probably more man-hours than have been given to any other scientific question in the history of mankind. […]

The glue that holds the nucleus together must be a kind of force utterly different from any we yet know.

[Hans A. Bethe: “What holds the nucleus together?”, Scientific American 189 (1953), no. 2, p. 58]

2007

Effective Field Theory

Effective field theories provide a powerful framework for solving physical problems that are characterized by a natural separation of distance scales. They are particularly important tools in QCD, where the relevant degrees of freedom are quarks and gluons at short distances and hadrons and nuclei at longer distances. Indeed, at energies below the proton mass, the most notable features of QCD are the confinement of quarks and the spontaneous breaking of QCD’s chiral symmetry. Chiral perturbation theory is an effective field theory that incorporates both; when applied to mesons it is a mature theory. Perhaps the most striking advances in chiral effective field theory have come in its application to few-nucleon systems. This has yielded precise results for nucleon-nucleon forces and also produced consistent three-nucleon forces. This opens the way for precision analyses of

We look for an approach which • connects to QCD; • is efficient;
• provides falsifiable predictions with reliable theoretical uncertainties.
The Bridge From QCD To Nuclear Physics

\[ \mathcal{L}_{QCD} = \sum_q \bar{\Psi}_q [i \gamma^\mu D^\mu_q - m_q] \Psi_q - \frac{1}{2} \text{tr}[F^{\mu\nu} F_{\mu\nu}] \]

with few parameters: \( \alpha_s(Q_0^2) + 6 \) masses.

Nucleon & Few-N System: gateway to quantitative understanding of nuclear structure from QCD.

Rich low-energy Structure & patterns all emerge from “simple” QCD.

Explain Life: abundances of \(^{12}\text{C}\), \(^{16}\text{O}\), . . .

Explain Interactions/Scattering/Production/ . . .

We know \( \sim 3000 \) nuclei, \( \sim 300 \) stable, \( \gg 10^5 \) excitations – and many are unknown.

Different regions and energy scales need different, efficient descriptions.

(Example: Lattice-QCD will not explain \(^{235}\text{U}\).)
\( B = ZM_p + NM_n - M_A \)

Bound state: \( B > 0 \)

\( \geq 4p \) not bound

\( ^3Li(ppp) 6.8 \text{MeV} \)

\( pp \) not bound

\( ^3He (ppn) I = \frac{1}{2}, 7.7 \text{MeV} (ppnn) I = 0 \)

\( ^4He \) decay rapidly

\( ^4He: \text{No excitations below threshold; very stable!} \)

\( ^3H \) stable

\( \beta^+ \beta^- \alpha \)

Tally of States: 44 isospins allowed for \( A \leq 6 \)

4(5) stable: \( d, ^3H, ^3He, ^6Li \)

11 unstable: \( ^3Li \) & \( ^5(H,He, Li, Be) \) & \( ^6(H, B) \): 1 level

\( ^4(H, Li), ^6(He, Be): \geq 4 \) levels; 50 excitations, only 2 bound

Notable No-Shows: few-\( n \), \( pn(I = 1) \), \( pp \), \( 4p \), \( A = 5 \) unstable,

\textbf{Whence the Patterns?}
Few-Nucleon Spectra “Should” follow from QCD

Whence the Patterns? How much is special to QCD?
What Is “Low Energy”? QCD Spectrum Above Nucleon Hass Gap

- **Low-energy excitations** at scales $p_{\text{typ}} \lesssim 300\text{MeV}$:
  - Lightest mesons produced: pion $m_\pi \approx 140\text{MeV}$
  - Lowest resonance: $\Delta(1232)$, $M_\Delta - M_N \approx 300\text{MeV}$
  - Bound states of nucleons, e.g. $E_B[d(np)] \approx 2.2\text{MeV}$: $p_{\text{bind}} \approx \sqrt{M E_B} \approx 45\text{MeV}$

- **High-energy excitations** at scales $p_{\text{typ}} \gtrsim 1000\text{MeV}$:
  - Next-lightest meson $m_\rho, \omega \approx 770\text{MeV}$
  - Next-lowest excitation $M_{N^*} - M_N \approx 600\text{MeV}$
  - Strange excitations only as $s$-quark pair: $2M_K \approx 1000\text{MeV}$
  - String constant $\sigma \approx 1\text{GeV/fm}$, $\alpha_s(1\text{GeV}^2) \to 1, \ldots$

QCD at low resolution/energy: Confinement/infrared slavery of quarks & gluons.

⇒ **Rearrange into effective low-energy degrees of freedom**: $N = (p_n^p), \Delta(1232), \pi^a$.

$L[N, \Delta, \pi^a]$: any interaction imaginable. ⇒** Small, dimension-less expansion parameter?!?**
To probes with wavelength $\lambda$, object of size $R$ appears
point-like for $\lambda \gg R$, blurry for $\lambda \gtrsim R$, composed for $\lambda \lesssim R$.

• Example Expansion in Radiation Multipoles: $P_{El} \xrightarrow{\lambda \gg R} \sum_{\text{ang. mom. } l} a_l \left( \frac{R}{\lambda} \right)^{2l}$

Series converges if $Q = \frac{\text{target size } R}{\text{resolution } \lambda} < 1 \implies \text{error-estimate, space for improvement}$

• Example Gravity: Strings/Branes/Whathevers $\implies$ very heavy objects: General Relativity

$\implies v \ll c$: attraction between two bodies $V_{grav} = -\frac{G_N M m}{r}$

$\implies m \ll M$, small drop $h \ll r_E = 6,400\text{km}$: $V_{grav} = -\frac{G_N M m}{r_E + h} \approx V_0 + m \frac{G_N M}{r_E^2} h$

$\implies$ Find parameter $g = 9.81\text{ms}^{-2}$ from underlying theory, or fit by Leaning-Tower experiment.

New form more efficient for pendulum $h \sim 1\text{m} \ll r \sim 6,400\text{km}$: Save time & effort.
The Onion We Call Nature: The World Is Effective

All Physics Theories applicable only in a limited energy range (except in-effective TOE...).
Take Uncertainty Relation Seriously: $\Delta x \Delta p \geq \hbar$

**The EFT Tenet** Weinberg 1979

Short-distance physics does not have to be right for a good calculation, because a low-energy process cannot probe details of the high-energy structure.

---

**Effective Field Theories**: method for multiple, separate scales:

Identify those degrees of freedom and symmetries which are **appropriate** to resolve the **relevant** Physics at the scale of interest.

Turn into **systematic approximation** of real world, allowing for **estimate of theoretical uncertainties involved**.
Ingredients:

**Separation of scales** by breakdown-scale $\bar{\Lambda}_{\text{EFT}}$:

- **high momenta** $q_{\text{high}} \gtrsim \bar{\Lambda}_{\text{EFT}}$ → simplify complicated/unknown UV into **Low-Energy Coefficients (LECs)**: contact interactions.
- **low momenta** $q_{\text{low}} \ll \bar{\Lambda}_{\text{EFT}}$

**Effective (relevant) degrees of freedom** → correct IR-Physics

**Symmetries at low scales** constrain interactions. Lorentz, gauge, ...
Ingredients:

Separation of scales by breakdown-scale $\bar{\Lambda}_{\text{EFT}}$:

- **High momenta** $q_{\text{high}} \gtrsim \bar{\Lambda}_{\text{EFT}} \rightarrow$ simplify complicated/unknown UV into **Low-Energy Coefficients (LECs)**: contact interactions.

- **Low momenta** $q_{\text{low}} \ll \bar{\Lambda}_{\text{EFT}}$

Effective (relevant) degrees of freedom $\rightarrow$ correct IR-Physics

Symmetries at low scales constrain interactions. Lorentz, gauge, . . .

Recipe:

Write down most general Lagrangean permitted by ingredients. $\rightarrow$ infinitely many terms

Order in small expansion parameter

\[
Q = \frac{\text{typ. low momenta } q_{\text{low}}}{\text{breakdown scale } \bar{\Lambda}_{\text{EFT}}} = \frac{1/(\text{resolution } \lambda)}{1/(\text{target size } R)} \ll 1:
\]

$\rightarrow$ low-$q$ expansion of $\mathcal{L}$: Estimate importance of LECs & graphs before explicit calculation by *Naïve Dimensional Analysis & Naturalness Assumption.*

Determine LECs at desired accuracy from underlying theory or (simple) low-mom. observables.
Ingredients: 

Separation of scales by breakdown-scale $\bar{\Lambda}_{\text{EFT}}$:

- high momenta $q_{\text{high}} \gtrsim \bar{\Lambda}_{\text{EFT}}$ → simplify complicated/unknown UV into Low-Energy Coefficients (LECs): contact interactions.
- low momenta $q_{\text{low}} \ll \bar{\Lambda}_{\text{EFT}}$

Effective (relevant) degrees of freedom → correct IR-Physics

Symmetries at low scales constrain interactions. Lorentz, gauge, ...

Recipe:

Write down most general Lagrangean permitted by ingredients. → infinitely many terms

Order in small expansion parameter

$$Q = \frac{\text{typ. low momenta } q_{\text{low}}}{\text{breakdown scale } \bar{\Lambda}_{\text{EFT}}} = \frac{1/(\text{resolution } \lambda)}{1/(\text{target size } R)} \ll 1:$$

→ low-$q$ expansion of $\mathcal{L}$: Estimate importance of LECs & graphs before explicit calculation by Naïve Dimensional Analysis & Naturalness Assumption.

Determine LECs at desired accuracy from underlying theory or (simple) low-mom. observables.

Result:

Model-independent, universal, systematic, unique: Predictions with estimate of uncertainties.

Finite accuracy with minimal number of parameters at each order. “Space from Improvement”
[...], you’re not really making any assumption that could be wrong, unless of course Lorentz invariance or quantum mechanics or cluster decomposition is wrong, [...]

[...] As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you’re simply writing down the most general theory you could possibly write down.

[Steven Weinberg: *What is quantum field theory, and what did we think it is?* [hep-th/9702027]]

“I know of no proof, but I am sure it’s true. That’s why it’s called a »folk theorem«.” [Weinberg, Chiral Dynamics 2009 (Bern)]

“EFT = Symmetries + Parameterisation of Ignorance”???

WHAT CAN POSSIBLY GO WRONG???
Know Your Limitations!

Serve With Caution:

Check assumptions:
- $p_{\text{typ.}} \sim \Lambda_{\text{EFT}} \implies Q \ll 1$?
  - “EFTs carry seed of their own destruction.”
    [D. R. Phillips]
- No separation/jungle of scales? e.g. $N^*$ at 2 GeV
- Wrong constituents/degrees of freedom?
  - new d.o.f. e.g. QED at 100 GeV without $W, Z$
  - change of d.o.f. over phase transition
    - e.g. $N, \pi \rightarrow$ quarks, gluons
- Nature does not have assumed symmetry?
  - e.g. impose Parity in weak interactions

Check Quantitatively Predicted Convergence Pattern:
- Order by order smaller corrections.
- Order by order less cut-off/RScheme dependence.

Falsifiability: Convergence to Nature tests assumptions. – After theory uncertainties determined.
Separation of Scales: $\chi$EFT from QCD

\[ \mathcal{L}_{\text{QCD}} = \bar{q} \left[ i \partial + g A - m \right] q - \frac{1}{2} \text{tr}[F_{\mu\nu} F^{\mu\nu}] \]

Effective low-energy degrees of freedom: $N = \binom{p}{n}$, $\pi^a$, $\Delta(1232)$

\[ \mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2 \pi^a \pi^a + \ldots \]

\[ + N^\dagger \left[ i D_0 + \frac{\bar{D}^2}{2M_N} + \frac{g_A}{2f_\pi} \bar{\sigma} \cdot \bar{D} \pi + \ldots \right] N + C_0 \left( N^\dagger N \right)^2 + \ldots \]

**Parameters:** lattice QCD or low-energy data.

**Correct long-range Physics + all interactions.**

**Short-range:** minimal parameter-set at given order.

**Systematic ordering** in $Q = \frac{\text{typ. momentum}}{\text{breakdown scale}} \sim \frac{m_\pi}{1 \text{ GeV}} \approx \frac{1}{5 \ldots 7}$

$\implies$ **Controlled approximation:** model-independent, error-estimate.

$\implies$ **Chiral Effective Field Theory $\chi$EFT $\equiv$ low-energy QCD**

**Symmetries constrain form of interactions:** gauge, Lorentz, iso-spin, …

**Most important for QCD:** *Chiral Symmetry & its Breaking.* $\implies$ Now!
(d) Chiral Symmetry: Nambu-Goldstone Theorem

The Pion Is Special

$\pi^\pm, \pi^0$ form the $I(J^P) = 1(0^-)$ pseudo-scalar iso-triplet —

$$m^\pm_\pi = 139.57\text{MeV}, \ m^0_\pi = 134.97\text{MeV} \ll 1\ \text{GeV}: \text{typ. QCD scale}$$

Next non-strange hadron $m_{\omega\rho} \approx 5\frac{1}{2} m_\pi \implies \text{Pion is lightest hadron, by far.}$

$\implies \text{Mediates interaction with longest range } R \sim \frac{1}{m_\pi} \sim 1.4\text{fm}; \text{decays only weakly.} \implies \text{HW}$

Why is the pion mass so much smaller than any typical QCD scale $\sim 1\text{GeV}$?
Review: Chiral Symmetry in QCD

QCD in chiral basis: 

\[
\begin{pmatrix}
q_R^\dagger \\
q_L^\dagger
\end{pmatrix}
\begin{pmatrix}
E - gA_0 + \vec{\sigma} \cdot (\vec{p} + g\vec{A}) \\
m
E + gA_0 - \vec{\sigma} \cdot (\vec{p} - g\vec{A})
\end{pmatrix}
\begin{pmatrix}
q_R \\
q_L
\end{pmatrix},
q_R/L = \begin{pmatrix}
u_R/L \\
d_R/L
\end{pmatrix}
\]

\[\Rightarrow\] For \(m = 0\) invariant under separate transformations \(R \in SU_R(2), L \in SU_L(2)\) in flavour space:

\[q_R \rightarrow Rq_R = e^{-i\theta_R^a \frac{\tau^a}{2}} q_R, \quad q_L \rightarrow Lq_L = e^{-i\theta_L^a \frac{\tau^a}{2}} q_L\]

\[\Rightarrow SU_R(2) \times SU_L(2)\] chiral symmetry

\[\Rightarrow\text{Noether Theorem: } \begin{pmatrix} 2 \times 3 \end{pmatrix}\text{ conserved currents & charges:} \]

\[j_R^{\mu a} = \bar{q}_R \gamma^\mu \frac{\tau^a}{2} q_R : \partial_\mu j_R^{\mu a} = 0\]

and

\[j_L^{\mu a} = \bar{q}_L \gamma^\mu \frac{\tau^a}{2} q_L : \partial_\mu j_L^{\mu a} = 0\]

More convenient are these linear combinations:

\[V^a_\mu : j^R_\mu + j^L_\mu \rightarrow j_\mu^a \]

\[A^a_\mu : j^R_\mu - j^L_\mu \rightarrow j_\mu^a \]

Symmetry realised in Nature?

\[H (Q^a_5 |\text{state} \rangle) = Q^a_5 (H |\text{state} \rangle) = E (Q^a_5 |\text{state} \rangle) \]

\[P (Q^a_5 |\text{state} \rangle) = -Q^a_5 (P |\text{state} \rangle) \]

\[\Rightarrow Q^a_5 |\text{state} \rangle \text{ should have same mass, opposite parity.}\]
QCD in chiral basis:

\[
(E - gA_0 + \vec{\sigma} \cdot (\vec{p} + gA_0)) m \begin{pmatrix} q_R \\ q_L \end{pmatrix} = \begin{pmatrix} E + gA_0 - \vec{\sigma} \cdot (\vec{p} - gA_0) \end{pmatrix} \begin{pmatrix} m E \\ gA_0 - \vec{\sigma} \cdot (\vec{p} - gA_0) \end{pmatrix} \begin{pmatrix} q_R \\ q_L \end{pmatrix}, \quad q_{R/L} = \begin{pmatrix} u_{R/L} \\ d_{R/L} \end{pmatrix}
\]

For \( m = 0 \) invariant under separate transformations \( R \in SU_R(2), \ L \in SU_L(2) \) in flavour space:

\[
q_R \rightarrow R q_R = e^{-i\theta_R^a \frac{\tau^a}{2}} q_R \quad q_L \rightarrow L q_L = e^{-i\theta_L^a \frac{\tau^a}{2}} q_L
\]

\[\implies SU_R(2) \times SU_L(2) \text{ chiral symmetry}\]

**Noether Theorem:** \( 2 \times 3 \) conserved currents & charges:

\[
\begin{align*}
 j^\mu_{Ra} &= \bar{q}_R \gamma^\mu \frac{\tau^a}{2} q_R : \partial_\mu j^\mu_{Ra} = 0 \\
 j^\mu_{La} &= \bar{q}_L \gamma^\mu \frac{\tau^a}{2} q_L : \partial_\mu j^\mu_{La} = 0
\end{align*}
\]

More convenient are these linear combinations:

**Vector Current:** \( V^a_\mu := j^\mu_{Ra} + j^\mu_{La} = \bar{q} \gamma^\mu \frac{\tau^a}{2} q \)

\[\implies \text{Vector Charges: } Q^a = \int d^3r \ q^\dagger \frac{\tau^a}{2} q\]

Symmetry in Nature? **YES:** \( SU_V(2) \text{ isospin} \), e.g. \( m_{\pi^+} = m_{\pi^-} \approx m_{\pi^0} \implies I, I_3 = Q^3 \) label states ✓
Review: Chiral Symmetry in QCD

QCD in chiral basis: \((q_R^\dagger, q_L^\dagger)\left(\begin{array}{c} E - gA_0 + \vec{\sigma} \cdot (\vec{p} + g\vec{A}) \\ m \\
- E + gA_0 - \vec{\sigma} \cdot (\vec{p} - g\vec{A}) \end{array}\right)\left(\begin{array}{c} q_R \\ q_L \end{array}\right), \quad q_R/L = \left(\begin{array}{c} u_R/L \\ d_R/L \end{array}\right)\)

\(\Rightarrow\) For \(m = 0\) invariant under separate transformations \(R \in SU_R(2), \ L \in SU_L(2)\) in flavour space:

\[ q_R \rightarrow R q_R = e^{-i \theta_R^a \tau_a^R} q_R \quad q_L \rightarrow L q_L = e^{-i \theta_L^a \tau_a^L} q_L \]

\(\Rightarrow\) Noether Theorem: \(2 \times 3\) conserved currents & charges:

\[ j_{\mu a}^R = \bar{q}_R \gamma_\mu \tau_a^R q_R : \partial_\mu j_{\mu a}^R = 0 \quad \text{and} \quad j_{\mu a}^L = \bar{q}_L \gamma_\mu \tau_a^L q_L : \partial_\mu j_{\mu a}^L = 0 \]

More convenient are these linear combinations:

Vector Current: \(V_\mu^a := j_{\mu a}^R + j_{\mu a}^L = \bar{q} \gamma_\mu \tau_a^R q \quad \Rightarrow\) Vector Charges: \(Q^a = \int d^3r \ \bar{q}^\dagger \gamma_\mu \frac{\tau_a^R}{2} q\)

Symmetry in Nature? YES: \(SU_V(2)\) isospin, e.g. \(m_{\pi^+} = m_{\pi^-} \approx m_{\pi^0} \Rightarrow I, I_3 = Q^3\) label states \(\checkmark\)

Axial Current: \(A_\mu^a := j_{\mu a}^R - j_{\mu a}^L = \bar{q} \gamma_\mu \gamma_5 \frac{\tau_a^R}{2} q \quad \Rightarrow\) Axial Charges: \(Q^a = \int d^3r \ \bar{q}^\dagger \gamma_\mu \gamma_5 \frac{\tau_a^R}{2} q\)

Symmetry realised in Nature?

\[ H (Q^a_5 |\text{state}\rangle) = Q^a_5 (H |\text{state}\rangle) = E (Q^a_5 |\text{state}\rangle) \]

\[ P (Q^a_5 |\text{state}\rangle) = -Q^a_5 (P |\text{state}\rangle) \]

\(\Rightarrow Q^a_5 |\text{state}\rangle\) should have same mass, opposite parity.
Chiral Symmetry Is Broken!

\[ Q_5^{a} |\text{state}\rangle \text{ should have same mass, opposite parity.} \]

but we see

no parity doublets in Nature:

\[ m[I(J^+)] \neq m[I(J^-)] \]

neither for baryons...

\[ \Rightarrow \text{Axial } SU_A(2) \text{ symmetry generated by } Q_5^{a} \text{ is broken although } \mathcal{L} \text{ is invariant: Noether??} \]
Nambu-Goldstone Symmetry Breaking

Example: Global $U(1)$ Symmetry Breaking in the Nonlinear $\sigma$ Model

**Landau-Ginzburg model:** Complex scalar field $\mathcal{L}_{LG} = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - \lambda \left[ \Phi^\dagger \Phi - a^2 \right]^2$

Like Higgs field, but now no gauge symmetry.

$\lambda < 0$

$\lambda > 0, a \neq 0$
Example: Global $U(1)$ Symmetry Breaking in the Nonlinear $\sigma$ Model

And here as a movie with sound (click picture):
Nambu-Goldstone Symmetry Breaking

Example: Global $U(1)$ Symmetry Breaking in the Nonlinear $\sigma$ Model
We used classical reasoning so far:

Ball is at exactly one point.
Quantum Mechanics:

Probability amplitude of ball spread out.

\[ \Rightarrow \text{Ground-state wave function has same symmetry.} \]

(QM version of Noether)
Quantum Mechanics:

Probability amplitude of ball spread out.

\[ \text{Ground-state wave function has same symmetry.} \]

(QM version of Noether)

Measurement/interaction:

collapse of wave function.

Localised wave function is superposition of eigenstates.

With time, wave function dissipates back into symmetric form (with energy loss).

Transition amplitude

\[ \propto \sum_{\text{states } k} a_k(t) e^{-iE^{(k)}t} \]

\[ \Rightarrow \text{Finite dissipation timescale } \sim \frac{1}{E^{(1)} - E^{(0)}} \]
And here as a movie with sound (click picture):

QM: Goldstone's Spontaneously Broken Symmetry

localised wave function disperses into ground state

Spontaneous and Explicit Symmetry Breaking

The Nambu–Goldstone Mode in Quantum Mechanics
Quantum Mechanics and QFT: (Sloppy) Semiclassical Reasoning

**QM → Quantum Field Theory:** Collapse at each point to *different* angle.

Localised wave function is still superposition of eigenstates, but neighbours couple via \( \vec{\nabla} \Phi \)-terms: **free/kinetic energy, interactions**.

**Transition amplitude** (symbolic) with energy densities \( \varepsilon_i^{(k)} \) of state \( k \) at site \( i \), \( \varepsilon_{ij}^{(k)} \) of \( k \) on link \( (ij) \) in spat. volume \( V \):

\[
\propto \sum_{\text{states } k} a_{ij}^{(k)}(t) \exp \left[ -i t \int \left( \sum_{\text{site } i} \varepsilon_i^{(k)} + \sum_{\text{link (ij)}} \varepsilon_{ij}^{(k)} \right) \right]
\]

Random phases cancel in mean, unless all \( \varepsilon_i^{(k)} \approx \varepsilon_{ij}^{(k)} \approx 0 \) aligned close to ground state (WKB).

– But then \( V \to \infty \implies \text{Still zero transition probability} \).

**Corollary Elitzur’s Theorem:** Only **global** continuous symmetries can be broken in QFT, and only in the **infinite volume**. Local (gauge) symmetries **cannot** be broken. [Phys. Rev. D. 12 (1975) 3978]

**Intuitive Proof:** Local symmetry acts only on finite/countable number of degrees of freedom.

\[
\text{In practise, metastable when dissipation timescale } \gg \text{ relevant timescales (life of universe).}
\]
Quantum Mechanics: Noether’s Theorem: Symmetries of $\mathcal{L}$ must be symmetries of the ground state.
Quantum Field Theory: $\infty$ many degrees of freedom $\Rightarrow$ not necessarily true (loophole in Noether).

Take a **global, continuous symmetry** of $\mathcal{L}$ which is generated by $N$ conserved charges.

Here: $SU_R(2) \times SU_L(2)$: $3$ charges $Q^a_R = \& 3$ charges $Q^a_L$

Let the vacuum state show only $n < N$ symmetries (i.e. VEV has $n$ of these symmetries).

Here: isospin $SU_V(2)$: $3$ charges $Q^a = Q^a_R + Q^a_L$

Then the vacuum is only annihilated by $n$ Noether charges

and one finds

- **$n$ massive fields**, $m \propto $VEV
- $Q^a_A |\text{vac}\rangle \neq 0$
- and $(N-n)$ **massless fields**

  carry the quantum numbers of the $(N-n)$ broken symmetries
  and do not interact for momenta $p \rightarrow 0$.

---

Symmetry Scenarios of Vacuum in QFT with Noether Charges $Q^a_N$

**Either Wigner-Weyl mode:** unbroken/invariant:

$e^{i Q^a_N \theta_a} |\text{vac}\rangle = |\text{vac}\rangle \Leftrightarrow Q^a_N |\text{vac}\rangle = 0$, symmetry in spectrum

**Or Nambu-Goldstone mode:** spontaneous breaking:

$e^{i Q^a_N \theta_a} |\text{vac}\rangle \neq |\text{vac}\rangle \Leftrightarrow Q^a_N |\text{vac}\rangle = |\text{Goldstones}\rangle \neq 0$

generates massless fields $\pi^a(x)$, symmetry not seen in spectrum.
SSB (Spontaneous Chiral Symmetry Breaking) Order Parameter

(e) Chiral Perturbation Theory: Mesons in $\chi EFT$ [Weinberg 1979, Gasser/Leutwyler, ...]

$\chi EFT$ relates quark parameters ($m_q, \ldots$) and pion/low-energy parameters ($m_\pi, f_\pi, \ldots$).

Prior: “Current Algebra”: hard, unknown corrections. [Gell-Mann, ... 1964-]

Now: Chiral Low Energy Theorems: simpler, systematic corrections.

[Weinberg, Gasser/Leutwyler/... 1979-]

Pion Decay: Mass, Fixing Parameter $f_\pi = [92.21 \pm 0.15]{\text{MeV}}$ [PDG 2014]

Some use $f_\pi = \sqrt{2}[92.21 \pm 0.15]{\text{MeV}} \approx 130{\text{MeV}}$. 
Explicit Chiral Symmetry Breaking: Tilting the Hat

Landau-Ginzburg analogy:

\[ V_{\text{LG}} = \lambda (\Phi^\dagger \Phi - a^2)^2 \]
\[ \rightarrow 2a^2 \lambda \sigma^2 + 2\sqrt{2}\lambda a \sigma^3 + \lambda \sigma^4 \]
\[ = m_\sigma^2 / 2 \]

Leave \( \sigma \) alone, only affect \( \pi \), but keep broken-symmetry along angle:

\[ V_{\text{LG}} - \varepsilon a^4 \left[ e^{i\pi/\sqrt{2}a} + e^{-i\pi/\sqrt{2}a} \right] \]
\[ = V_{\text{LG}} - 2\varepsilon a^4 \cos \left[ \frac{\pi}{\sqrt{2}a} \right] \]
\[ \rightarrow V_{\text{LG}} - 2\varepsilon a^4 + \frac{1}{2} \varepsilon^2 \sigma^2 - \frac{\varepsilon}{48} \pi^2 + O(\pi^6) \]
\[ = m_\pi^2 \]

\( m_\pi = 0 \)

\( m_\sigma \gg m_\pi \neq 0 \) for \( \varepsilon \ll \lambda \)
$m_\pi$ from quark condensate ($\chi$SB order param.): $m_\pi^2 = -\frac{m_q}{f_\pi^2} \langle \bar{u}u \rangle + \mathcal{O}(m_q^2)$, 

where we assumed isospin symmetry: $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \frac{1}{2} \langle \bar{q}q \rangle$ of iso-vector $q = (u, d)$

phenomenology: $\langle \bar{u}u \rangle \approx -(250 \text{ MeV})^3 \approx -2 \text{fm}^{-3} \gg \rho_{\text{nucl. matter}} = 0.17 \text{ fm}^{-3} = (100 \text{ MeV})^3$

lattice QCD: $\langle \bar{u}u \rangle = -(251 \pm 7 \pm 11) \text{ MeV}^3$ [JLQCD: Phys. Rev. Lett. 98 (2007) 172001]

Predicted $m_\pi$-dependence consistent with lattice QCD. $\implies$ Confirms $\chi$ EFT, chiral symmetry.

$\chi$ EFT ok up to $m_\pi \lesssim 600 \text{ MeV}$, in line with breakdown scale $\Lambda_{\chi \text{EFT}} \sim m_\rho \sim 1 \text{ GeV}$.
LO Lagrangean and $\pi\pi$ S-Wave Scattering Lengths

$$\mathcal{L}_{\chi EFT}^{\text{LO}} = \frac{f_\pi^2}{4} \text{tr}[(\partial_\mu U)\dagger(\partial^\mu U)] + \frac{m_\pi^2 f_\pi^2}{4} \text{tr}[U + U\dagger]$$

Parameter-free prediction for LO $\pi\pi$ scattering lengths:

$$a_{I=0} = \frac{7\pi}{2} \left( \frac{m_\pi}{4\pi f_\pi} \right)^2 = +0.16\text{fm}$$

$$a_{I=2} = -\pi \left( \frac{m_\pi}{4\pi f_\pi} \right)^2 = -0.05\text{fm}$$

Goldstone boson decouples.

$\Rightarrow$ Confirms chiral symmetry.
Goldberger-Treiman Relation

Relate $\pi N$ coupling and pion parameters as $g_{\pi NN} = g_A \frac{M_N}{f_\pi} + \text{corrections.}$

Data:

$g_{\pi NN} = 13.21^{+0.11}_{-0.05}$ pion-photoproduction $\gamma N \rightarrow \pi N$ [SS 4.3]

$g_A = 1.2695(29)$ axial coupling in neutron decay $n \rightarrow p e^- \bar{\nu}_e$ [SS 4.3]

$f_\pi = 92.21(15)$ pion decay $\pi^+ \rightarrow \mu^- \nu_\mu$ [PDG 2014]

$\Rightarrow$ Goldberger-Treiman Discrepancy $\Delta_{GT} = 1 - \frac{g_A M_N}{g_{\pi NN} f_\pi} = [2.15^{+0.89}_{-0.51}]\%$ indeed tiny.

Another consequence of chiral symmetry and its breaking!
Chirally Symmetric Interactions in the $\pi N$ System

Goldberger-Treiman: $\pi N$ matrix element is $\langle p|\pi^-|n\rangle = g_{\pi NN} M_N f_\pi \sqrt{2} \bar{u}_p(p')\gamma_5 u_n(p)$.

Re-establish $\gamma^\mu$ as before: $M_N u_n(p) = p u_n(p)$, $\bar{u}_p(p')M_N = \bar{u}_p(p') p'$ on-shell.

Recall $\sqrt{2} \pi^- = \pi^1 + i\pi^2$. 
Chirally Symmetric Interactions in the $\pi N$ System

Goldberger-Treiman: $\pi N$ matrix element is $\langle p | \pi^- | n \rangle = g_A \frac{M_N}{f_\pi} \sqrt{2} \bar{u}_p(p') \gamma_5 u_n(p)$.

Re-establish $\gamma^\mu$ as before: $M_N u_n(p) = \not{p} u_n(p), \bar{u}_p(p') M_N = \bar{u}_p(p') \not{p}'$ on-shell.

Recall $\sqrt{2} \pi^- = \pi^1 + i \pi^2$.

$\implies$ Chiral symmetry derives prefactor of isospin symmetric $\pi N$ interaction stated in [ResReg]:

$$\chi_{\text{symmetry}}: \quad U = e^{i \frac{\pi^a \tau_a}{f_\pi}} \text{ curvature of SSB potential prescribes more interactions with odd # of $\pi^a$.}$$

$$\sim \frac{g_A}{2 f_\pi} \not{q} \gamma_5$$

$$\sim \frac{g_A}{2 f_\pi} \not{q}_{123} \gamma_5$$

$$\sim \frac{g_A}{2 f_\pi} \not{q}_{12345} \gamma_5$$
Chirally Symmetric Interactions in the $\pi N$ System

Goldberger-Treiman: $\pi N$ matrix element is $\langle p | \pi^- | n \rangle = g_A M_N \frac{\sqrt{2}}{f_\pi} \bar{u}_p(p') \gamma_5 u_n(p)$.

Re-establish $\gamma^\mu$ as before: $M_N u_n(p) = \not{p} u_n(p), \bar{u}_p(p') M_N = \bar{u}_p(p') \not{p'}$ on-shell.

Recall $\sqrt{2} \pi^- = \pi^1 + i \pi^2$.

$\implies$ Chiral symmetry derives prefactor of isospin symmetric $\pi N$ interaction stated in [ResReg]:

\[
\begin{align*}
\tau^a & : -g_A 2f_\pi \not{q} \gamma_5 \tau^a \rightarrow -g_A 2f_\pi \not{q} \cdot \vec{\sigma} \\
\end{align*}
\]

for particle in Dirac basis of $\gamma^\mu$. Needed in HW!

$\chi$ symmetry: $U = e^{i \pi^a \tau^a}$ curvature of SSB potential prescribes more interactions with odd # of $\pi^a$. 

\[
\begin{align*}
\sim g_A 2f_\pi \not{q}_{123} \gamma_5 \\
\sim g_A 2f_\pi \not{q}_{12345} \gamma_5 \\
\end{align*}
\]

Gauge $q_\mu \rightarrow q_\mu + e Z_{\pi} A_\mu \implies \varepsilon^\mu : -i Z_{\pi} e \frac{g_A}{2f_\pi} \varepsilon^\mu \gamma_\mu \gamma_5$

nonrel. : $\vec{e} \cdot \vec{\sigma}$

$\implies \chi$ sym. prescribes factor for charged-pion photoproduction. Used to determine $g_{\pi N}$.
Chirally Symmetric Interactions in the $\pi N$ System

**Goldberger-Treiman:** $\pi N$ matrix element is $\langle p |\pi^- | n \rangle = g_A \frac{M_N}{f_\pi} \sqrt{2} \bar{u}_p(p')\gamma_5 u_n(p)$.

Re-establish $\gamma^\mu$ as before: $M_Nu_n(p) = \not{p} u_n(p), \bar{u}_p(p')M_N = \bar{u}_p(p')\not{p}'$ on-shell.

Recall $\sqrt{2} \pi^- = \pi^1 + i\pi^2$.

$\implies$ Chiral symmetry derives prefactor of isospin symmetric $\pi N$ interaction stated in [ResReg]:

\[ \frac{q}{2f_\pi} \not{\tau}^a \rightarrow \frac{g_A}{2f_\pi} \not{q} \cdot \not{\sigma} \] for particle in Dirac basis of $\gamma^\mu$. Needed in HW!

$\chi$ symmetry: $U = e^{i\frac{\pi^a q_\mu}{f_\pi}}$ curvature of SSB potential prescribes more interactions with odd # of $\pi^a$.

\[ \sim \frac{g_A}{2f_\pi} q_{123} \gamma_5 \] \[ \sim \frac{g_A}{2f_\pi} q_{12345} \gamma_5 \]  

**Gauge** $q_\mu \rightarrow q_\mu + eZ_\pi A_\mu \implies$  

$\varepsilon^\mu_{\gamma^a \gamma^b} \rightarrow -iZ_\pi e \frac{g_A}{2f_\pi} \varepsilon^\mu_{\gamma^\mu} \gamma_5$  

"Kroll-Rudermann term"

$\implies \chi$ sym. prescribes factor for charged-pion photoproduction. Used to determine $g_{\pi NN}$.

**First $N\pi\pi$ interaction** comes from curvature of SSB potential, like in $\chi$:

\[ \frac{1}{4f_\pi^2} (\not{q} + \not{q}') \varepsilon_{\gamma^a \gamma^b \gamma^c} \tau_c \] charge-transfer, no $g_A$!  

"Weinberg-Tomozawa term"
Sketch of the HW Problem: $\pi N$ Scattering Length

**LO $\chi$EFT amplitudes:**

- **Isospinology** in [ResReg: II.3.f]: $N \otimes \pi^a : \frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2} \implies 2$ amplitudes $M_{2I+1=4}, M_2$.
  - Diagrams $\equiv i \times \text{amplitude} = iT^{ab} = i[\delta^{ab} T^+ - i\epsilon^{abc} \tau_c T^-]$.
  - Most interested in prediction involving SSB curvature: **Weinberg-Tomozawa** $\frac{1}{4f_\pi^2} (\phi + \phi') \epsilon^{abc} \tau_c$.

  $\implies$ Consider **charge-transfer or charged pion scattering**; track WT term (the one without $g_A$).
  - $\implies$ Go for $T^-$: coefficient of operator $\epsilon^{abc} \tau_c$.
Sketch of the HW Problem: $\pi N$ Scattering Length

LO $\chi$EFT amplitudes:

Isospinology in [ResReg: II.3.f]: $N \otimes \pi^a : \frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2} \implies 2$ amplitudes $M_{2I+1=4}, M_2$.

- Diagrams $= i \times$ amplitude $= iT^{ab} = i[\delta^{ab} T^+ + i\varepsilon^{abc} \tau_c T^-]$.

- Most interested in prediction involving SSB curvature: \textbf{Weinberg-Tomozawa} $\frac{1}{4f^2} (q + q') \varepsilon^{abc} \tau_c$.

\implies \text{Consider charge-transfer or charged pion scattering; track WT term (the one without $g_A$!).}

\implies \text{Go for $T^-$: coefficient of operator $\varepsilon^{abc} \tau_c$.}

- Use crossing symmetry $k \longleftrightarrow -k'$ for “crossed” diagram.

- For cross sections, sum all 3 amplitudes, square total: QM interference matters! (not in HW)
Sketch of the HW Problem: $\pi N$ Scattering Length

**Isospinology** in [ResReg: II.3.f]: $N \otimes \pi^a: \frac{1}{2} \otimes 1 = \frac{1}{2} \oplus \frac{3}{2} \implies 2$ amplitudes $\mathcal{M}_{2I+1=4}, \mathcal{M}_2$.

- **Diagrams** $= i \times \text{amplitude} = i T^{ab} = i[\delta^{ab} T^+ - i\epsilon^{abc} \tau_c T^-].$
  - symmetric
  - antisymmetric

- Most interested in prediction involving SSB curvature: **Weinberg-Tomozawa** $\frac{1}{4f^2 \pi} (\phi + \phi') \epsilon^{abc} \tau_c$.

  $\implies$ Consider **charge-transfer or charged pion scattering; track WT term** (the one without $g_A$!).

  $\implies$ Go for $T^-$: coefficient of operator $\epsilon^{abc} \tau_c$.

- Use crossing symmetry $k \longleftrightarrow -k'$ for “crossed” diagram.

- For cross sections, sum all 3 amplitudes, square total: QM interference matters! (not in HW)

- **Scattering length is easier** (HW): Zero-momentum scattering $k = k' = (m_\pi, 0), p = p' = (M_N, 0)$.

  - Match cross section to definition of scattering length $a$:
    $$\sigma(\vec{p} = \vec{k} = 0) = 4\pi a^2 = \int d\Omega \frac{|\mathcal{M}|^2}{64\pi^2 s} \implies a^\pm = \frac{1}{8\pi \sqrt{s}} T^\pm$$

- **Chiral limit**, compare to experiment, error-assessment.
**πN Scattering: \( \chi \) EFT and Data**

<table>
<thead>
<tr>
<th>( \chi ) EFT without WT (i.e. not really “LO”)</th>
<th>( a^+ \text{[10}^{-4}\text{MeV}^{-1}] )</th>
<th>( a^- \text{[10}^{-4}\text{MeV}^{-1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi ) EFT without WT (i.e. not really “LO”)</td>
<td>-0.680</td>
<td>+5.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LO ( \chi ) EFT (really, with WT) parameter-free</th>
<th>( a^+ \text{[10}^{-4}\text{MeV}^{-1}] )</th>
<th>( a^- \text{[10}^{-4}\text{MeV}^{-1}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PWA-I [Koch 1986]</td>
<td>-0.7 ± 0.1</td>
<td>+6.6 ± 0.1</td>
</tr>
<tr>
<td>PWA-II [Matsinos 1997]</td>
<td>+0.20 ± 0.12</td>
<td>+5.8 ± 0.1</td>
</tr>
<tr>
<td>pionic hydrogen [Schröder 2001]</td>
<td>-0.27 ± 0.36</td>
<td>+6.59 ± 0.30</td>
</tr>
<tr>
<td>( N^2 )LO ( \chi ) EFT (data extraction)[Siemens/...1602.02640]</td>
<td>+0.2 ± 0.1</td>
<td>+5.91 ± 0.04</td>
</tr>
</tbody>
</table>

\( \chi \) EFT with Weinberg-Tomozawa favoured. \( \Rightarrow \) Curvature of \( \chi \) SSB potential!

Partial wave analysis tricky: \( \Delta(1232) \)!

\( \Rightarrow \) Substantial uncertainties.

Still, higher-order \( \chi \) EFT agreement ok.
When $\chi$EFT Does Not Work: “Ruler Plots”

$\chi$EFT: $M_N(m_\pi) - M_N(m_\pi = 0) \propto m_q \propto m_\pi^2$.

Lattice: $M_N = 800.0\text{MeV} + 1.0m_\pi!$ WHY??
Example: induced electric dipole radiation from harmonically bound charge, damping $\Gamma$ [Lorentz/Drude 1900/1905]

$$\vec{d}_{\text{ind}}(\omega) = \frac{q^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\Gamma \omega} \vec{E}_{\text{in}}(\omega)$$

$$=: 4\pi \alpha_{E1}(\omega) \text{"displaced volume"} \ [10^{-4} \text{ fm}^3]$$

electric scalar dipole polarisability

Dis-entangle interaction scales, symmetries & mechanisms with & among constituents.

$\implies$ Clean, perturbative probe of $\Delta(1232)$ properties, nucleon spin-constituents, $\chi$iral symmetry of pion-cloud & its breaking.

Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2 \rangle$...[PDG]
The Low-Energy Method: Chiral Effective Field Theory

Degrees of freedom $\pi, N, \Delta(1232) + \text{all interactions allowed by symmetries}$: Chiral SSB, gauge, iso-spin,…

$\Longrightarrow$ **Chiral Effective Field Theory** $\chi$ EFT $\equiv$ low-energy QCD

$$\mathcal{L}_{\chi\text{EFT}} = (D_\mu \pi^a)(D^\mu \pi^a) - m_\pi^2 \pi^a \pi^a + \cdots + N^\dagger \left[ i D_0 + \frac{\bar{D}^2}{2M} + \frac{g_A}{2f_\pi} \bar{\sigma} \cdot \bar{D} \pi + \cdots \right] N + C_0 \left( N^\dagger N \right)^2 + \cdots$$

Controlled approximation $\Longrightarrow$ **Model-independent, error-estimate**.

Expand in $\delta = \frac{M_\Delta - M_N}{\Lambda_\chi} \approx 1\text{ GeV} \approx \sqrt{\frac{m_\pi}{\Lambda_\chi}} = \frac{p_{\text{typ}}}{\Lambda_\chi} \ll 1$ (numerical fact) [Pascalutsa/Phillips 2002].
Unified Amplitude: gauge & RG invariant set of all contributions which are
in low régime $\omega \lesssim m_\pi$ at least $N^4$LO ($e^2 \delta^4$): accuracy $\delta^5 \lesssim 2\%$;
or in high régime $\omega \sim M_\Delta - M_N$ at least NLO ($e^2 \delta^0$): accuracy $\delta^2 \lesssim 20\%$.

Unknowns: short-distance $\delta \alpha, \delta \beta \iff$ Fit static $\alpha_{E1}, \beta_{M1}$ (offset). $\implies$ Predict $\omega$-dependence.
Nucleon Polarisabilities from Consistent Database

**Polarisabilities: Energy-dependent Multipoles** of real Compton scattering at fixed energy.

\[
T = \left[ \text{Powell: point spin-} \frac{1}{2} \text{ with anomalous mag. moment} \right] + 2\pi \omega^2 \left[ \alpha_E(\omega) \vec{E}^2 + \beta_M(\omega) \vec{B}^2 + \gamma_{E1E1}(\omega) \vec{\sigma} \cdot (\vec{E} \times \hat{E}) + \ldots \right]
\]

\[\theta_{\text{lab}} = 45^\circ, \quad \theta_{\text{lab}} = 60^\circ, \quad \theta_{\text{lab}} = 110^\circ, \quad \theta_{\text{lab}} = 133^\circ, \quad \theta_{\text{lab}} = 155^\circ\]

\(\omega \ll m_\pi\): more than “static + slope”! \(\implies\) Understand **dynamics** to extrapolate from data to \(\omega = 0\).

\(\implies\) Compress rich dynamics into few numbers.
The purpose of this Editorial is to discuss the importance of including uncertainty estimates in papers involving theoretical calculations of physical quantities.

It is not unusual for manuscripts on theoretical work to be submitted without uncertainty estimates for numerical results. In contrast, papers presenting the results of laboratory measurements would usually not be considered acceptable for publication. The question is to what extent can the same high standards be applied to papers reporting the results of theoretical calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers are aimed at understanding physical processes, and it is important to have a quantitative idea of the uncertainties involved in the calculations. It is all too often the case that the numerical results are presented without uncertainty estimates. Authors sometimes say that it is difficult to arrive at error estimates. Should this be considered an adequate reason for omitting them? In order to answer this question, we need to consider the goals and objectives of the theoretical (or computational) work being done. Theoretical papers are aimed at understanding physical processes, and it is important to have a quantitative idea of the uncertainties involved in the calculations.

For example, in scattering processes involving complex systems, the comparison with experiment itself provides a test of our theoretical understanding. However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made. Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and whenever possible, to present the data with error bars.

**Theoretical uncertainty: Truncation of Physics**

\[ \alpha^p_{E1} = 10.65 \pm 0.35_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}} \]

**Non-Theory Errors:** Numerical ➞ better computers. Statistical/parameter ➞ better data.

**Scientific Method:** Quantitative results with corridor of theoretical uncertainties for falsifiable predictions. Need procedure which is established, economical, reproducible: room to argue about “error on the error”.

“Double-Blind” Theory Errors: Assess with pretense of no/very limited data.
physical effects not included in the calculation from the beginning, such as electron correlation and relativistic corrections. It is of course never possible to state precisely what the error is without in fact doing a larger calculation and obtaining the higher accuracy. However, the same is true for the uncertainties in experimental data. The aim is to estimate the uncertainty, not to state the exact amount of the error or provide a rigorous bound.

There are many cases where it is indeed not practical to give a meaningful error estimate for a theoretical calculation; for example, in scattering processes involving complex systems. The comparison with experiment itself provides a test of our theoretical understanding. However, there is a broad class of papers where estimates of theoretical uncertainties can and should be made. Papers presenting the results of theoretical calculations are expected to include uncertainty estimates for the calculations whenever practicable, and especially under the following circumstances:

1. If the authors claim high accuracy, or improvements on the accuracy of previous work.
2. If the primary motivation for the paper is to make comparisons with present or future high precision experimental measurements.
3. If the primary motivation is to provide interpolations or extrapolations of known experimental measurements.

These guidelines have been used on a case-by-case basis for the past two years. Authors have adapted well to this, resulting in papers of greater interest and significance for our readers.

\[ \alpha_{E1}^{p} = 10.65 \pm 0.35_{\text{stat}} \pm 0.2_{\Sigma} \pm 0.3_{\text{theory}} \]

Non-Theory Errors: Numerical \(\rightarrow\) better computers. Statistical/parameter \(\rightarrow\) better data.

Theoretical uncertainty: Truncation of Physics

\[ EFT \text{ claim: systematic in } Q = \frac{p_{\text{typ}}}{\Lambda_{EFT}} \]

Scientific Method: Quantitative results with corridor of theoretical uncertainties for falsifiable predictions.

Need procedure which is established, economical, reproducible: room to argue about “error on the error”.

“Double-Blind” Theory Errors: Assess with pretense of no/very limited data.
What Does “Conservative” Theory Uncertainty Mean?

χEFT proton $[10^{-4} \text{fm}^3]$: $\alpha_{E1}^p = 10.6 \pm 0.4_{\text{stat}} \pm ???_{\text{th}} = 12.5_{\text{LO}} - 2.3_{\text{NLO}} + 0.4_{\text{N^2LO}} \pm ???_{\text{th}}$

Observable as series:

$\mathcal{O} = c_0 + c_1 \delta^1 + c_2 \delta^2 + \text{unknown} \times \delta^3$

$\implies$ Setting $\delta \approx 0.4$, estimate next term “most conservatively” as $|\text{unknown}| c_3 \lesssim \max\{|c_0|; |c_1|; |c_2|\}$.

No infinite sampling pool; data fixed; more data changes confidence.

$\implies$ For probabilistic interpretation, call upon the Reverend Bayes!

see e.g. BUQYEYE collaboration [Furnstahl/Phillips/... 1506.01343]

New information increases level of confidence.

$\implies$ Smaller corrections, more reliable uncertainties.

Bayes makes you specify your premises/assumptions about series.

Priors: leading-omitted term dominates ($\delta \ll 1$); putative distributions of all $c_k$’s and of largest value $\bar{c}$ in series.

“Least informed/informative”: All values $c_k$ equally likely, given upper bound $\bar{c}$ of series.

“Any upper bound”: In-uniform prior sets no bias on scale of $\bar{c}$.

\[
\text{pr}(c|\bar{c}) \propto \frac{1}{c}, \epsilon \to 0
\]
Quantifying One’s Beliefs in $\mathcal{O} = Q^n (c_0 + c_1 Q + c_2 Q^2 + \ldots)$

**Information:** Convergence LO $\rightarrow$ NLO $\rightarrow$ $N^2$LO gives probable “largest number” $R = \delta^k \max\{|c_0| \ldots |c_{k-1}|\}$.

**Result:** Posterior $\equiv$ **Degree of Belief (DoB)** that next term $c_k \delta^k$ differs from order-$k$ central value by $\delta$.

$$\text{pr}(\Delta|\text{max. } R, \text{order } k) \propto \int_0^\infty d\bar{c} \text{ pr}(\bar{c}) \text{ pr}(c_k = \frac{\Delta}{\delta^k} | \bar{c}) \prod_n \text{ pr}(c_n | \bar{c}) \rightarrow \frac{k}{k+1} \frac{1}{2R} \left\{ 1 \left( \frac{R}{|\Delta|} \right)^{k+1} \right\} \begin{cases} |\Delta| \leq R \\ |\Delta| > R \end{cases}$$

<table>
<thead>
<tr>
<th>order</th>
<th>DOB in $\pm R$</th>
<th>$\sigma$: 68%</th>
<th>$\Delta$ (95%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>50%</td>
<td>1.6 $R$</td>
<td>$11R = 7\sigma$</td>
</tr>
<tr>
<td>Gauß</td>
<td>68.27%</td>
<td>1.0 $R$</td>
<td>$2.0\sigma$</td>
</tr>
</tbody>
</table>

**Degree of Belief (DoB) interval** $[\bar{c}_0, \bar{c}_1, \ldots]$. 

**Result diagram:** 
- $\text{pr}(c_k | \text{max. } c_0 \ldots c_{k-1})$ after $k$ tests
- $\Delta/R = c_k/\max\{c_0 \ldots c_{k-1}\}$
- $\text{DoB}$
Quantifying One’s Beliefs in $\mathcal{O} = Q^n(c_0 + c_1 Q^1 + c_2 Q^2 + \ldots)$

**Information:** Convergence LO→NLO→N^2LO gives probable “largest number” $R = \delta^k \max\{|c_0| \ldots |c_{k-1}|\}$.

**Result:** Posterior $\equiv$ Degree of Belief (DoB) that next term $c_k \delta^k$ differs from order-$k$ central value by $\delta$.

$$\Pr(\Delta|\text{max. } R, \text{order } k) \propto \int_0^\infty \mathrm{d} \bar{c} \Pr(\bar{c}) \Pr(c_k = \frac{\Delta}{\delta^k} | \bar{c}) \prod_n \Pr(c_n | \bar{c}) \rightarrow k \frac{1}{k+1} \frac{1}{2R} \left\{ \begin{array}{ll} 1 & |\Delta| \leq R \\ \left( \frac{R}{|\Delta|} \right)^{k+1} & |\Delta| > R \end{array} \right. $$

**pdf of $c_k/\max\{c_0..c_{k-1}\}$ after $k$ tests**

<table>
<thead>
<tr>
<th>Order</th>
<th>DOB in $\pm R$</th>
<th>$\sigma$: 68%</th>
<th>$\Delta(95%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>50%</td>
<td>1.6 $R$</td>
<td>11$R = 7\sigma$</td>
</tr>
<tr>
<td>NLO</td>
<td>66.7%</td>
<td>1.0 $R$</td>
<td>2.7$R = 2.6\sigma$</td>
</tr>
<tr>
<td>Gauß</td>
<td>68.27%</td>
<td>1.0 $R$</td>
<td>2.0$\sigma$</td>
</tr>
</tbody>
</table>

$\Delta/R = c_k/\max\{c_0..c_{k-1}\}$
Quantifying One’s Beliefs in \( \mathcal{O} = Q^n(c_0 + c_1 Q^1 + c_2 Q^2 + \ldots) \)

**Information:** Convergence LO → NLO → N^2LO gives

\[ R = \delta^k \text{max}\{|c_0|, \ldots |c_{k-1}|\}. \]

**Result:** Posterior \( \equiv \text{Degree of Belief (DoB)} \) that next term \( c_k \delta^k \) differs from order-\( k \) central value by \( \delta \).

\[ \text{pr}(\Delta|\text{max. } R, \text{order } k) \propto \int_0^\infty d\bar{c} \, \text{pr}(\bar{c}) \, \text{pr}(c_k = \frac{\Delta}{\delta^k}|\bar{c}) \prod_{n} \text{pr}(c_n|\bar{c}) \rightarrow k \frac{1}{k+1} \frac{1}{2R} \left\{ \begin{array}{ll} 1 & |\Delta| \leq R \\ \left( \frac{R}{|\Delta|} \right)^{k+1} & |\Delta| > R \end{array} \right. \]

**pdf of \( c_k/\text{max}\{c_0\ldots c_{k-1}\} \) after \( k \) tests**

<table>
<thead>
<tr>
<th>order</th>
<th>DOB in ( \pm R )</th>
<th>( \sigma ): 68%</th>
<th>( \Delta(95%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>50%</td>
<td>1.6( R )</td>
<td>11( R ) = 7( \sigma )</td>
</tr>
<tr>
<td>NLO</td>
<td>66.7%</td>
<td>1.0( R )</td>
<td>2.7( R ) = 2.6( \sigma )</td>
</tr>
<tr>
<td>N^2LO</td>
<td>75%</td>
<td>0.9( R )</td>
<td>1.8( R ) = 1.9( \sigma )</td>
</tr>
<tr>
<td>( k )</td>
<td>( \frac{k}{k+1} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauß</td>
<td>68.27%</td>
<td>1.0( R )</td>
<td>2.0( \sigma )</td>
</tr>
</tbody>
</table>

For “high enough” order, largest number \( R \) limits \( \gtrsim 68\% \) degree-of-belief interval.

**Varying priors:** When \( k \geq 2 \) orders known, DoBs with different assumptions about \( \bar{c}, c_n \) vary by \( \lesssim \pm 20\% \).

**Posterior pdf not Gauß’ian:** Plateau & power-law tail.— Do not add in quadrature in convolution!

\[ \implies \text{Interpretation of all theory uncertainties, with these priors; } “A \pm \sigma”: 68\% \text{ DoB interval } [A - \sigma; A + \sigma]. \]
Prior Choice of “Natural Size”? (SCOTUS: I Know It When I see It.)

Observable/Series \( \mathcal{O} = c_0 + c_1 \delta^1 + c_2 \delta^2 + \text{unknown} \times \delta^3 \) with “naturally-sized coefficients” \( c_i \).

\[ p(c_k|\bar{c}) \]

“Least informative/informed”: characterised by 1 number: \( \bar{c} \).

More informed choices: more complicated structures, more thought, more parameters: \( \bar{c} \), typ. size, spread,\

\[ p(c_k|\bar{c}) \quad \text{Gaußian} \]

\[ p(c_k|\bar{c}) \quad \text{Goldilocks} \]

[BUQEYE (Wesolowski/Klco/...):] When \( k \geq 2 \) orders known, DoBs with different assumptions about \( \bar{c} \), \( c_n \) vary by \( \lesssim \pm 20\% \) for some “reasonable priors”.

PHYS 6610: Graduate Nuclear and Particle Physics I, Spring 2018

H. W. Grießhammer, INS, George Washington University
Bayes provides well-defined procedure!

Example: $\chi$ EFT predicts nucleon polarisabilities

\[ \gamma_{M1M1} = [2.2 \pm 0.5_{\text{stat/indirect}} \pm 0.6_{\text{th}}] \times 10^{-4} \text{fm}^4 \]

MAMI [2015]: \[3.2 \pm 0.9_{\text{stat}}\] \times 10^{-4} \text{fm}^4

Bayes in EFTs also used to estimate:

- Momentum-dependent expansion parameter : $Q(k) = \frac{\text{low momentum}}{\text{breakdown}} \Lambda_{\text{EFT}}$
- Breakdown scale $\Lambda_{\text{EFT}}$
- Momentum-dependent data-weighting for LEC fitting/extraction
- Build LEC hierarchy into fit
- “Model quality” $\equiv$ correctness of EFT assumptions, …

[BUQYE collaboration Furnstahl/Phillips/… 1506.01343, 1511.03618,…]

Finally quantitative theoretical uncertainties which make the EFT falsifiable.
Physical Models vs. Physical Theories – A Sliding Scale

Model: Parametrise data, Capture some aspects with lots of data – no “fail” but “tuning”. Cargo Cult mode.

The Trouble With Nuclear Physics

In fact the trouble in the recent past has been a surfeit of different models [of the nucleus], each of them successful in explaining the behavior of nuclei in some situations, and each in apparent contradiction with other successful models or with our ideas about nuclear forces.


Theory: Predictive, comprehensive, prescriptive, may fail. Explain-All-To-Some-Degree mode.

Gelman’s Totalitarian Principle/Swiss Basic Law/
Weinberg’s “Folk Theorem”: Throw In the Kitchen Sink

As long as you let it be the most general possible Lagrangian consistent with the symmetries of the theory, you’re simply writing down the most general theory you could possibly write down.


Quality Check: EXISTENCE: Are there Theory-uncertainties/errors?

EXISTENCE: Clear discussion how they are assessed?
NN System is gateway to understanding microscopic structure of nuclear structure from QCD.
Few-Nucleon Systems: Complexity, Patterns, Bridge

\[ B = ZM_p + N M_n - M_A \]

**Binding energy [MeV]**

- Stable: \( B > 0 \)
- Unstable: \( B \leq 0 \)

**black box: stable**

Decays: \( \beta^+ \) \( \beta^- \) \( \alpha \)

**Tally of States:** 44 isospins allowed for \( A \leq 6 \)

- 4(5) stable: \( d, ^3H, ^3He, ^6Li \)
- 11 unstable: \( ^3Li \) & \( ^5(H,He,Li,Be) \) & \( ^6(H,B): 1 \) level
- \( ^4(H,Li), ^6(He,Be): \geq 4 \) levels; 50 excitations, only 2 bound

**Notable No-Shows:** few-\( n \), \( pn(I = 1) \), \( pp \), \( 4p \), \( A = 5 \) unstable, ...

**Whence the Patterns?**
Few-Nucleon Spectra “Should” follow from QCD

Whence the Patterns? How much is special to QCD?
Non-Relativistic Reduction of an EFT for $Q = \frac{p_\text{typ.}}{M} \ll 1$

Find kinetic energy $T = p_0 - M \ll M$ of free nonrelativistic boson (including spin is “trivial”):

$$L = \Phi^\dagger \left[ (T + M)^2 - \vec{p}^2 - M^2 \right] \Phi = \left( \sqrt{2M} \Phi^\dagger \right) \left[ T - \frac{\vec{p}^2}{2M} - \frac{\vec{p}^4}{8M^3} + \ldots \right] \left( \sqrt{2M} \Phi \right)$$

relative $O(Q^2)$

 playable field $\phi$

$\implies$ Treat higher orders in $\beta \approx \frac{|\vec{p}|}{T + M}$ as perturbation:

$\implies$ Propagator $\frac{i}{T - \frac{\vec{p}^2}{2M} + i\epsilon}$ has only one pole: $T = \frac{\vec{p}^2}{2M} - i\epsilon > 0 \implies$ No anti-particles. ✓

$\implies T \sim \frac{\vec{p}^2}{2M} \ll |\vec{p}| \ll M$ as expected – and Pauli spinors $N = \left( \begin{array}{c} p \\ n \end{array} \right)$, isospin $\otimes \left( \begin{array}{c} \uparrow \\ \downarrow \end{array} \right)$, spin.$\otimes$
Non-Relativistic Reduction of an EFT for \( Q = \frac{p_{\text{typ.}}}{M} \ll 1 \)

Find kinetic energy \( T = p_0 - M \ll M \) of free nonrelativistic boson (including spin is “trivial”):

\[
\mathcal{L} = \Phi^\dagger \left[ (T + M)^2 - \vec{p}^2 - M^2 \right] \Phi = \left( \sqrt{2M} \Phi^\dagger \right) \left[ T - \frac{\vec{p}^2}{2M} - \frac{\vec{p}^4}{8M^3} + \ldots \right] \left( \sqrt{2M} \Phi \right)
\]

\[
\text{relative } \mathcal{O}(Q^2) \text{ non-rel. field}
\]

\( \Rightarrow \) Treat higher orders in \( \beta = \frac{\left| \vec{p} \right|}{T + M} \approx \frac{\left| \vec{p} \right|}{M} \) as perturbation:

\[
\Rightarrow \text{Propagator } \frac{i}{T - \frac{\vec{p}^2}{2M} + i\epsilon}
\]

\( \Rightarrow \text{Treat } T - \frac{\vec{p}^2}{2M} + i\epsilon + \frac{\vec{p}^4}{8M^3} + \ldots \text{ as } T \approx \frac{\vec{p}^2}{2M} \ll |\vec{p}| \ll M \) as expected – and Pauli spinors \( N = \begin{pmatrix} p \end{pmatrix}_{n} \otimes \begin{pmatrix} |\uparrow\rangle \end{pmatrix}_{\text{isospin}} \otimes \begin{pmatrix} |\uparrow\rangle \end{pmatrix}_{\text{spin}} \)

\( \Rightarrow \text{Kinetic-energy term for Nucleons: } \mathcal{L}_N = N^\dagger \left[ T - \frac{\vec{p}^2}{2M} - \frac{\vec{p}^4}{8M^3} + \ldots \right] N \)

Pion-exchange in \( t \)-channel between nonrelativistic nucleons becomes instantaneous:

\[
\frac{(q_0 = T_1 - T_2)^2}{(q = \vec{p}_1 - \vec{p}_2)^2} \ll 1 \quad \text{and} \quad \frac{i}{q_0^2 - q^2 - m^2_{\pi}} \rightarrow \frac{-i}{q^2 + m^2_{\pi}} + \ldots
\]

\( \Rightarrow \text{Use non-relativistic QM for few-}N \text{ bound states from potentials: } \text{Schrödinger eq.,. . .} \)
(h) Understanding Two-Nucleon Systems

Quantum Numbers of the $NN$ System

Couple 2 nucleons with spin $S = \frac{1}{2}$, isospin $I = \frac{1}{2}$:

$$\vec{S} = \frac{\vec{s}_1}{2} + \frac{\vec{s}_2}{2}$$

$$\vec{I} = \frac{\vec{\tau}_1}{2} + \frac{\vec{\tau}_2}{2}$$

| $S$ | $m_s$ | $\frac{1}{\sqrt{2}} [| \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle]$ | $I = 0$ | $I_3 = 0$ | $\frac{1}{\sqrt{2}} [|pn\rangle - |np\rangle]$ | anti-symmetric
|---|---|---|---|---|---|
| $0$ | $0$ | $\frac{1}{\sqrt{2}} [| \uparrow \downarrow \rangle - | \downarrow \uparrow \rangle]$ | $I = 0$ | $I_3 = 0$ | $\frac{1}{\sqrt{2}} [|pn\rangle - |np\rangle]$ | anti-symmetric
| $1$ | $+1$ | $| \uparrow \uparrow \rangle$ | $I = 1$ | $I_3 = +1$ | $|pp\rangle$ | symmetric
| $1$ | $0$ | $\frac{1}{\sqrt{2}} [| \uparrow \downarrow \rangle + | \downarrow \uparrow \rangle]$ | $I = 1$ | $I_3 = 0$ | $\frac{1}{\sqrt{2}} [|pn\rangle + |np\rangle]$ | symmetric
| $1$ | $-1$ | $| \downarrow \downarrow \rangle$ | $I = 1$ | $I_3 = -1$ | $|nn\rangle$ | symmetric

$$\Psi_{NN \text{total}}^{\text{spin}} = |\text{spin}\rangle \otimes |\text{isospin}\rangle \otimes |\text{orb. ang. mom. } Y_{lm}(\theta, \phi)\rangle \otimes |\text{radial } (r)\rangle$$

Pauli Principle: Total wave function anti-symmetric under exchange of identical fermions:

$$(-)^1 = (-)^{S+1} (-)^{I+1} (-)^L \implies S + I + L \text{ must be odd!}$$

Angular momentum coupling: Eigenvalues to $\vec{J}^2 = (\vec{L} + \vec{S})^2$ are $J = 0, 1, 2, \ldots$

Lowest Partial Waves in Spectroscopic Notation $^{2S+1}L_J$:

$I = 1$ in $pp, np, nn$: $^1S_0; ^3P_{0,1,2}; ^1D_2; \ldots$

$I = 0$ only in $np$: $^3S_1; ^1P_1; ^3D_{1,2,3}; \ldots$

Waves with same $J^P$ mix, most importantly: $^3S_1 - ^3D_1$ (see in a minute...
In the $^{1}S_0$- and $^3S_1-^3D_1$ waves: Unnatural Scales are Natural

Zero-momentum cross section $\sigma(k = 0) = 4\pi a^2$.

If $a > 0 \implies$ bound state with energy $B \approx \frac{1}{2\mu a^2}$

Geometrical meaning of scattering length $a$: “target size”.

Hard-sphere collisions: $a = 2R_N$

implies “Natural size”: $a \sim R_N \sim \frac{1}{m_\pi} \approx 1.5$ fm Yukawa range.

$\pi N$ System: $a^+ = \frac{0.008}{m_\pi}$, $a^- = \frac{0.078}{m_\pi}$ anomalously small – understood: chiral symmetry.
\( \text{NN in the } ^1S_0 \text{- and } ^3S_1 - ^3D_1 \text{ Waves: Unnatural Scales are Natural} \)

Zero-momentum cross section \( \sigma(k = 0) = 4\pi a^2 \).

If \( a > 0 \) \( \implies \) bound state with energy \( B \approx \frac{1}{2\mu a^2} \)

Geometrical meaning of scattering length \( a \): “target size”.

Hard-sphere collisions: \( a = 2R_N \) \( \implies \) “Natural size”: \( a \sim R_N \sim \frac{1}{m_\pi} \approx 1.5 \text{ fm Yukawa range} \).

\[ \pi N \text{ System: } \quad a^+ = \frac{0.008}{m_\pi}, \quad a^- = \frac{0.078}{m_\pi} \] anomalously small – understood: chiral symmetry.

\( \pi N \text{ System: } \quad a(1S_0) = [-23.71 \pm 0.03] \text{ fm} \approx \frac{-16.9}{m_\pi} \) nearly bound

\( \approx \) range \( \sim \frac{1}{m_\pi} = 1.4 \text{ fm} \)

\( \rho N \text{ System: } \quad a(3S_1) = [+5.432 \pm 0.005] \text{ fm} \approx \frac{3.9}{m_\pi} \) bound

The Deuteron \( I(J^{PC}) = 0(1^-) \) \( [2S+1L_J = (^3S_1-^3D_1)] \) is the only NN bound state:

binding energy \( B_d = 2.2244573 \text{ MeV} \) \( \ll \) natural size \( \frac{m_\pi^2}{M_N} \approx 20 \text{ MeV} \) if Yukawa alone (dim. an.)

Unnaturally shallow bound state & large scattering lengths:
Yukawa should be attractive, but not too much: compensate by repulsive core.
Necessary fine-tuning not yet fully understood in QCD!
**Unnatural Scales Obscure Chiral Power-Counting**

cf. hg 1511.00490 [nucl-th]

**Phenomenology:** Non-relativistic system with shallow (real/virtual) bound-state \(\implies\) LO non-perturbative.

\[
T_{NN}(E \sim \frac{p^2 k^2}{M}) \sim Q^{-1}
\]

Power-Counting:

\[
\frac{T_{LO}}{Q^m} = \frac{V_{LO}}{Q^m} + \frac{V_{LO} G_{NN}^{\text{nonrel.}}}{Q^{2m+3-2}} \implies m = -1
\]

**Examples:** NRQCD/NRQED  \(\chi\)EFT  EFT(\(\not\!p\))

Coulomb

\[
A_0, \bar{p} \cdot \vec{A} \sim \nu^{-1}
\]

\[
C_0 \sim Q^{-1}
\]

\[
- \frac{g_A^2}{4f^2_\pi} \tau_1 \cdot \tau_2 \frac{\sigma_1 \cdot q \sigma_2 \cdot \bar{q}}{\bar{q}^2 + m^2_\pi} \sim Q^{-1}
\]

**\(T_{NN}\) non-perturbative only in bound-state dynamics:** \(E \sim \frac{Q^2}{M}\)
Generic Structure of the $NN$ Potential at Low Energies

Pions $\pi^a$ & non-relativistic nucleons: $N = \left( \begin{array}{c} p \\ n \end{array} \right)_{\text{isospin}} \otimes \left( \begin{array}{c} |\uparrow\rangle \\ |\downarrow\rangle \end{array} \right)_{\text{spin}}$

**Most general form** which depends only on relative distance $r$ of nucleons (“local”) and is isospin, rotation, parity symmetric:

$$V_{NN}(\vec{r}, \vec{\sigma}_i, \tau^a_i, \vec{L}) = \delta_{I0} V^{(I=0)} + 4\delta_{I1} V^{(I=1)}$$

with $V^{(I)} = V_C^{(I)} + \vec{\sigma}_1 \cdot \vec{\sigma}_2 V^S_{S} + \vec{L} \cdot \vec{S} V^{(I)}_{LS} + S_{12}(\vec{e}_r) V^{(I)}_T$

**Tensor operator** $S_{12}(\vec{e}_r) = 3(\vec{\sigma}_1 \cdot \vec{e}_r)(\vec{\sigma}_2 \cdot \vec{e}_r) - \vec{\sigma}_1 \cdot \vec{\sigma}_2 = 6(\vec{S} \cdot \vec{e}_r)^2 - 4\delta_{S1}$:

- Analogous to elmag. dipole-dipole $V_{dd} = -\frac{3(\vec{\mu}_1 \cdot \vec{e}_r)(\vec{\mu}_2 \cdot \vec{e}_r) - \vec{\mu}_1 \cdot \vec{\mu}_2}{r^3}$.

- Mixes partial waves with same $J$ and parity, most importantly: $^3S_1-^3D_1$.

**Solve Schrödinger**

$$\left[ E - V + \frac{\vec{\nabla}^2}{M_N} \right] \Psi_{NN} = 0 \text{ or Lippmann-Schwinger } T = V + TGV$$

in Partial-Wave basis: Decouple into 1-dimensional problems for $S = 0$; but $(2 \times 2)$ for $S = 1$. 
EFT at Leading Order (LO): One Pion Exchange

Pion lightest meson $\rightarrow$ dominates $V_{NN}$ at large distance

$\pi N$ symmetries: chiral, isospin, parity, rotation, plus simplest form: fewest derivatives

\[ \vec{q} \rightarrow \pi^a \pi \rightarrow \frac{g_A}{2f_\pi} \tau^a \vec{\sigma} \cdot \vec{q} \]

No interaction for $\vec{q} \rightarrow 0$: $\pi$ decouples by chiral symmetry.

\[ \rightarrow \text{One Pion Exchange Potential (OPE)} \]
[\[ V_{OPE} = -\frac{g_A^2}{4f_\pi^2} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{\vec{q}^2 + m_\pi^2} \tau^a_1 \tau^a_2 \]

$\vec{\sigma} \cdot \vec{q}$ Spin-dependent: strongest attraction for $\vec{q}$ along opposite $N$ spin.

\[ \rightarrow \pi N \text{ is a } P\text{-wave interaction, like magnetic dipole in external field } \vec{\sigma} \cdot \vec{B}. \]

\[ \rightarrow NN \text{ interacts like dipole-dipole: tensor force, angle-dependent.} \]

$\tau^a_1 \tau^a_2 = 2I(I+1) - 3$: Isospin-dependent “iso-tensor” interaction.

\[ \rightarrow \text{Study partial-wave decomposition in isospin, spin and angular momentum!} \]
The Deuteron $I(J^{PC}) = 0(1^{-+})[^3S_1-^3D_1]$

\[ \Psi_d(r) \frac{1}{\sqrt{4\pi r}} \left[ u(r) S_{\text{wave}} + S_{12}(\vec{e}_r) \frac{w(r)}{\sqrt{8}} \right] \chi_{1M} \leftarrow \text{spin-triplet wf} \]

with \( \int_0^\infty dr [u(r)^2 + w(r)^2] = 1 \)

Asymptotic wave function decays with “binding momentum” \( \gamma = \sqrt{MB_d} = 45.70 \ldots \text{MeV} \):

\[ \lim_{r \to \infty} \left( \frac{u(r)}{w(r)} \right) \propto \left( \frac{1}{\eta} \right) \exp[-\gamma r] \text{ with asymptotic } D\text{-to-}S \text{ wave ratio } \eta_{\text{exp}} = 0.02544 \]

$D$ wave \( \implies \) deformation \( \implies \) electric quadrupole moment $Q_d = [0.2859 \pm 0.0003]$ fm$^3$ \([II.1.b]\)

Tests Kroll-Rudermann in coupling to $\pi^\pm$ exchange:
Deuteron and $^3S_1-^3D_1$: The Partial-Wave Projected LO OPE

Project into partial waves & Fourier transform:

\[
V_{OPE} = -\frac{g_A^2}{4f_\pi^2} \frac{(\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})}{q^2 + m_\pi^2} \tau_1^a \tau_2^a
\]

Central Potential is Yukawa: \[V_C(r) = -\frac{g_A^2 m_\pi^2}{16\pi f_\pi^2} \frac{e^{-m_\pi r}}{r} < 0 \quad \text{chiral limit} \quad \rightarrow 0\]

Tensor Potential: \[V_T(r) = -\frac{g_A^2 m_\pi^2}{16\pi f_\pi^2} \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right) \frac{e^{-m_\pi r}}{r} < 0 \quad \text{chiral limit} \quad \rightarrow -\frac{3g_A^2}{16\pi f_\pi^2} \frac{1}{r^3}\]

Strength: \[
\frac{g_A^2 m_\pi^2}{16\pi f_\pi^2} \quad \text{Goldberger-Treiman}
\frac{g_{\pi NN}^2}{4\pi} \quad \frac{m_\pi^2}{4M_N^2} = \alpha_{NN} \quad \frac{m_\pi^2}{4M_N^2} = \text{“nuclear”} \quad \alpha_{NN} \approx 13.9 \quad \rightarrow \text{Nonperturbative!}
\]

\[V_{OPE}[S = 0] = V_C(r) \times \begin{cases} 
-3 & : \text{repulsive for } I = 0, \text{ i.e. } L \text{ odd} \\
+1 & : \text{attractive for } I = 1, \text{ i.e. } L \text{ even} (^1S_0!)
\end{cases}\]

\[V_{OPE}[S = 1] = \frac{1}{3} \left[ V_C(r) + S_{12}(\vec{e}_r) V_T(r) \right] \times \begin{cases} 
+3 & : \text{attractive for } I = 0, \text{ i.e. } L \text{ even (deuteron!)} \\
-1 & : \text{repulsive for } I = 1, \text{ i.e. } L \text{ odd}
\end{cases}\]

Pion tensor force couples $S$ and $D$ waves in deuteron:

\[
\frac{1}{M} \frac{\partial^2}{\partial r^2} \begin{pmatrix} u(r) \\ w(r) \end{pmatrix} = \begin{pmatrix} -E + V_C(r) \\ \sqrt{8} V_T(r) \end{pmatrix} - E + \frac{6}{Mr^2} + [V_C(r) - 2V_T(r)] \begin{pmatrix} u(r) \\ w(r) \end{pmatrix}
\]

centrifugal
The Problem: Wave Functions Collapse at Short Range

For \((m_\pi r) \to 0\) (short distance/chiral limit): \[-\frac{3g_A^2}{16\pi f_\pi^2} \frac{1}{r^3} \begin{pmatrix} 0 & \sqrt{8} \\ \sqrt{8} & -2 \end{pmatrix}\] with EVals \(\begin{pmatrix} 4 \\ -2 \end{pmatrix}\)

A little project: Sensitivity of phase-shift on short-distance with shooting method. [HH: QM-I/II]

\[
V_C(r) = -0.9 \text{ fm}^2 e^{-0.7 \frac{r}{\text{fm}}} \quad \text{up to } R = 0.1 \text{ fm}^{-1} = 20 \text{ MeV}
\]

\[
V_C(r) = -0.073 e^{-0.7 \frac{r}{\text{fm}}} \quad \text{up to } R = 3 \text{ fm}
\]

Use “realistic” parameters for \(V_C(1S_0)\) & \(V_T(3S - D_1)\).

\[\implies V_C \text{ stronger than } V_T \text{ for } r \gtrsim 3 \text{ fm.}\]

Only at short distances does \(V_T\) win.

\(k_{cm} = 20 \text{ MeV} \ll m_\pi \ll \frac{2\pi}{R}: \quad \text{EFT Folk Theorem}\)

Expect no sensitivity on short-distance, i.e. on \(R\) or form.

✓ for \(V_C\) (Coulombic)
The Solution to Collapsing Wave Functions: EFT

The EFT Tenet

Weinberg 1979

Short-distance physics does not have to be right for a good calculation, because a low-energy process cannot probe details of the high-energy structure.

$\chi$: EFT: long-range/low-energy correct.

$\Rightarrow$ Add short-range repulsive core to stabilise system against collapse!

Simplest: Point-interaction $- ic$

without structure/derivative/form factor

renders cutoff-independence at all(!) $k$.

$RGE$: Adjust CT strength $c(R = \frac{1}{\Lambda})$ with $R = \frac{1}{\Lambda \lesssim \Lambda_{\chi}}$ so that observables cutoff-independent.

Initial condition set by one datum: scatt. length, $B_d, \ldots$; $O(k)$ predicted, only residual $\Lambda$-dep.

In line with unnaturally shallow bound state & large scattering lengths in $^3S_1$ and $^2S_0$:

OPE should be attractive, but not too much: compensate by repulsive core.
Check: Observables dependent on cut-off $\Lambda = 1/R$ at LO with $c(\Lambda)$ fixed by $B_d$?

Phase-shift $\delta(\text{cut-off } \Lambda)$:

- $E_{\text{lab}} = 10\text{MeV}$
- $50\text{MeV}$
- $100\text{MeV}$
- $190\text{MeV}$

Similar for all repulsive waves & attractive singlets ($\pm 1r$).
Check: Observables dependent on cut-off $\Lambda = 1/R$ at LO with $c(\Lambda)$ fixed by $B_d$?

Other channels: Need 4 more, new, momentum-dependent LECs for low attractive triplets: $^3P_{0,2}, ^3D_{2,3}$.

Low attractive P/D-wave triplets:

Centrifugal barrier suppresses tunneling into “dangerous region” $r \rightarrow 0$.

$\Rightarrow$ Collapse suppressed for high $l$. 
## EFT Power Counting Comparison

All but WPP: RGE as construction principle, but different approximations at short-range lead to variant interpretations.

**Proposed order** $Q^n$ at which counter-term enters **differs.** $\implies$ Predict different accuracy, # of parameters.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^{-1}$</td>
<td>LO of $^1S_0$, $^3S_1$, OPE plus $^3D_1$, $^3SD_1$</td>
<td>plus $^3P_{0,2}$, $^3D_2$</td>
<td>plus $^3P_{0,2}$</td>
<td></td>
</tr>
<tr>
<td>$Q^{-2}$</td>
<td>none</td>
<td>LO of $^3P_{0,1,2}$, $^3PF_2$, $^3F_2$, $^3D_2$</td>
<td>LO of $^3SD_1$, $^3D_1$, $^3F_2$</td>
<td>none</td>
</tr>
<tr>
<td>$Q^0$</td>
<td>none</td>
<td>NLO of $^1S_0$</td>
<td>none</td>
<td>none</td>
</tr>
<tr>
<td>$Q^{1}$</td>
<td>LO of $^3SD_1$, $^1P_1$, $^3P_{0,1,2}$; NLO of $^1S_0$, $^3S_1$</td>
<td>none</td>
<td>none</td>
<td>LO of $^3SD_1$, $^1P_1$, $^3P_1$, $^3PF_2$; NLO of $^3S_1$, $^3P_0$, $^3P_2$; $N^2$LO of $^1S_0$</td>
</tr>
</tbody>
</table>

| # at $Q^{-1}$ | 2 | 4 | 5 | 4 |
| # at $Q^0$ | +0 | +7 | +5 | +1 |
| # at $Q^1$ | +7 | +3 | +0 | +8 |

**total at $Q^1$** | 9 | 14 | 10 | 13 |

With same $\chi^2$/d.o.f., proposal with least parameters **wins:** minimum information bias.
Partial Wave Analysis and (More) Phenomenological Potentials

**Nijmegen Partial Wave Analysis** 1993-present: > 6000 \(pp\) and \(np\) scattering data for \(p \lesssim 300\) MeV

**Sketch of Phenomenological approaches:** long-range: OPE – short-range: reasonable guesses...

**One Boson Exchange Potentials (Bonn BC, Paris,...)**: \(V_{\text{core}} = \sum_{\omega, \rho, \sigma, ...} g_i^2 \times \text{(spin-structure)} \frac{e^{-m_i r}}{r} \)

**Short-Distance Core (Nijmegen 93, AV18, Reid,...)**: e.g. \(V_{\text{core}}(r) \sim \sum_{LSJ} A^{[2S+1LJ]} \frac{1 + \exp \left( -\frac{r - r_0}{a} \right)}{r} \)

Suitably flexible, \(\sim 40\) parameters, same fit quality: Systematic? Resolved for \(p < 300\) MeV?

Deuteron wave functions agree at large distances, disagree at short ones – but **no Physics there!**
Few-Nucleon Interactions in $\chi$EFT

<table>
<thead>
<tr>
<th>Type</th>
<th>LO</th>
<th>NLO</th>
<th>$N^2$ LO</th>
<th>$N^3$ LO</th>
</tr>
</thead>
<tbody>
<tr>
<td>2N ints</td>
<td>$\propto p^2$</td>
<td>$\propto p^4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 parameter</td>
<td>+7 parameter</td>
<td>+0 parameter</td>
<td>+15 = 24 param.</td>
</tr>
<tr>
<td>$\chi^2$ d.o.f in np</td>
<td>36.2</td>
<td>10.1</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>3N ints</td>
<td></td>
<td></td>
<td>$D$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>parameter-free, in progress</td>
<td></td>
</tr>
<tr>
<td>4N ints</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Typical momentum breakdown scale: $\ll 1$

**Long-Range:** correct symmetries and IR degrees of freedom: **Chiral Dynamics**

**Short-Range:** symmetries constrain contact-ints to simplify UV: **Minimal parameter-set**

Hierarchy: 2NF-effects $\gg$ 3NF-effects $\gg$ 4NF-effects

$\chi^2$ d.o.f in np:
- 2N ints: 36.2
- 3N ints: 10.1
- 4N ints: 1.10 (AV 18: 1.04, 40 param.)

Parameter-free, in progress
(i) Selected (Biased) Accomplishments

\textit{np} Scattering Phase Shifts: Bands Estimate Higher-Order Effects

[Epelbaum/...1412.0142]

Less free parameters than traditional.

Converges order-by-order – and even to Nature.

<table>
<thead>
<tr>
<th></th>
<th>LO</th>
<th>NLO</th>
<th>N^2LO</th>
<th>N^3LO</th>
<th>AV 18</th>
</tr>
</thead>
<tbody>
<tr>
<td># of parameters</td>
<td>2</td>
<td>+7</td>
<td>+0</td>
<td>+15</td>
<td>24</td>
</tr>
<tr>
<td>(\chi^2/d.o.f) in np</td>
<td>36.2</td>
<td>10.1</td>
<td>1.06</td>
<td>1.04</td>
<td></td>
</tr>
</tbody>
</table>
Scattering Observables at $E_{\text{cm}} = 50 \& 200 \text{MeV}$
Polarisation transfer coefficients at $E_{cm} = 22.7$ MeV

Bands estimate theoretical uncertainties by higher-order effects.
Few-Nucleon Interactions in $\chi$EFT

**Long-Range:** correct symmetries and IR degrees of freedom: **Chiral Dynamics**

**Short-Range:** symmetries constrain contact-ints to simplify UV: **Minimal parameter-set**

<table>
<thead>
<tr>
<th>N$^2$LO</th>
<th>N$^3$LO</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Diagram](2N ints)</td>
<td>![Diagram](4N ints)</td>
</tr>
<tr>
<td>![Diagram](3N ints)</td>
<td>![Diagram](2 parameter)</td>
</tr>
<tr>
<td>![Diagram](4 parameter)</td>
<td>![Diagram](3 parameter)</td>
</tr>
</tbody>
</table>

Typ. momentum breakdown scale $\ll 1$

**Hierarchy:**
- 2NF-effects $\gg$ 3NF-effects $\gg$ 4NF-effects

<table>
<thead>
<tr>
<th>$\chi^2$ d.o.f. in np</th>
<th>LO</th>
<th>NLO</th>
<th>$N^2$LO</th>
<th>$N^3$LO</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="36.2" alt="Diagram" /></td>
<td><img src="10.1" alt="Diagram" /></td>
<td><img src="1.10" alt="Diagram" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>+2 parameter</td>
</tr>
<tr>
<td>+7 parameter</td>
</tr>
<tr>
<td>+0 parameter</td>
</tr>
<tr>
<td>+15 parameter</td>
</tr>
</tbody>
</table>

| ![Diagram](3 parameter-free, in progress) | ![Diagram](2 parameter-free) | ![Diagram](Parameter-free) |

$\chi^2$ d.o.f. in np

- 2N ints: 2 parameter
- 3N ints: 0 parameter
- 4N ints: parameter-free, in progress

**PHYS 6610: Graduate Nuclear and Particle Physics I, Spring 2018**

H. W. Grießhammer, INS, George Washington University
FIG. 9: (Color online) Calculated (red dots) ground state energies in MeV using chiral LO, NLO, and $N^2LO$ NN interactions at $R = 1.0 \text{ fm}$ (without SRG evolution) based on the NN forces only in comparison with experimental values (blue levels). Red error bars indicate NCCI extrapolation uncertainty and shaded bars indicate the estimated truncation error at each chiral order as defined in the Introduction.
FIG. 15: (Color online) Results for ground state energies per nucleon of closed (sub)shell nuclei, showing chiral uncertainties as presented in the Introduction (gray bars) compared with the two alternative uncertainty estimates (green and pale red bars), discussed in the text. No numerical many-body uncertainties are shown. All results correspond to $R = 1.0$ fm and SRG $\alpha = 0.08$ fm$^4$ except for $^{48}$Ca at N$^3$LO, where the results for $\alpha = 0.04$ fm$^4$ were taken due to the unavailability of the ones for $\alpha = 0.08$ fm$^4$. For comparison, the experimental values are also shown as the blue bars.
FIG. 10: (Color online) Calculated (red dots) ground state magnetic moments of light nuclei up to $A = 10$ at LO, NLO, and N$^2$LO with $R = 1.0$ fm in comparison with experimental values (blue horizontal lines). Red error bars indicate NCCI extrapolation uncertainty and shaded bars indicate the estimated truncation error at each chiral order as defined in the Introduction.
Starting on Spectra of Less-Light Nuclei (with 3NI)


TABLE I: NLEFT results and experimental (Exp) values for the lowest even-parity states of $^{16}$O (in MeV). The errors are one-standard-deviation estimates which include both statistical Monte Carlo errors and uncertainties due to the extrapolation $N_t \to \infty$. The notation is identical to that of Ref. [20].

<table>
<thead>
<tr>
<th>$J^P_{11}$</th>
<th>LO (2N)</th>
<th>NNLO (2N)</th>
<th>+3N</th>
<th>+4N_{eff}</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^+_1</td>
<td>-147.3(5)</td>
<td>-121.4(5)</td>
<td>-138.8(5)</td>
<td>-131.3(5)</td>
<td>-127.62</td>
</tr>
<tr>
<td>0^+_2</td>
<td>-145(2)</td>
<td>-116(2)</td>
<td>-136(2)</td>
<td>-123(2)</td>
<td>-121.57</td>
</tr>
<tr>
<td>2^+_1</td>
<td>-145(2)</td>
<td>-116(2)</td>
<td>-136(2)</td>
<td>-123(2)</td>
<td>-120.70</td>
</tr>
</tbody>
</table>

Chiral Equation of State for $Z = N$ Nuclear Matter: 3NI Important

$V_{\text{low, } k}$ NN from $N^3LO$ (500 MeV)

$NN + 3N$

$E/A$ [MeV]

$A/A$ [MeV]
Figure 4 | Properties of the nuclear equation of state and neutron star radii based on chiral interactions. a, The symmetry energy $S_v$ and b, the slope $L$ of the symmetry energy at predicted saturation densities versus the point-proton radius in $^{48}$Ca. c, Pressure-radius relationship for a neutron star of mass $M=1.4M_\odot$ (red band) from the phenomenological expression of refs. 30,31. The predicted pressure (horizontal orange band) constrains the neutron star radius (vertical yellow band).

Fig. 2 Energy per baryon in pure neutron matter for different supernova EoS, compared to results of $\chi$EFT (grey band [228]), from Ref. [229].

[Blaschke/...1803.01836]  [Hagen/...1509.07169]

Error bars in $\chi$EFT vs. no error bars in models. – More work needed!
**EFT and Lattice QCD: Exploring Alternative Worlds**

**χEFT**: long-range well understood, short-range QCD encoded in minimal parameter-set.

\[ \chi \text{EFT: Chiral symmetry dictates extrapolation in } m_q \propto m^2 \pi, \ldots, \text{volume, lattice-spacing.} \]

**Future**: Fully dynamical simulations utilising \( \chi \)EFT for long-range part.

- \( ^1S_0 \) with bound state for \( m_\pi > 160 \text{ MeV} \)? \( \implies \) \( nn, pp \) bound!
- What is the deuteron binding energy for \( m_\pi \neq 140 \text{ MeV} \)?
- Explain fine-tuning of \( NN \)-scattering lengths, origin of few-\( N \) interactions.
- Fix parameters hard to determine experimentally: weak int.'s test SM; \( \pi NN \) & \( YN \)-couplings...
Merger of EFT and lattice has started exploring how few-nucleon systems emerge from QCD.

Surprisingly little change in few-nucleon systems – but $nn$ becomes bound when $m_\pi$ increased!
Another aspect of few-nucleon Physics: measure neutron properties on stable target.

- **Nucleon Structure**: average of neutron & proton polarisabilities: \( \chi \) EFT, Disp. Rel.: p-n difference is small [hg/Pasquini/... 2005]

- **Parameter-free One-Nucleon Contributions**:

- **Parameter-free charged Meson-Exchange Currents** dictated in \( \chi \) EFT by gauge & chiral symmetry:

Model-independently subtract binding \( \Rightarrow \chi \) EFT: quantify reliable uncertainties.

Test charged-pion component of \( NN \) force.
Neutron Polarisabilities & Nuclear Binding

**Experiment:** More charge & MECs $\Rightarrow$ more counts $\Rightarrow$ heavier nuclei

**Theory:** Reliable only if nuclear binding & levels accurate $\Rightarrow$ lighter nuclei

Find *sweet-spot* between competing forces: deuteron, $^3$He, $^4$He.

Use Complementing Targets of Opportunity.

**Deuteron, $^4$He:** sensitive to average p+n polarisabilities $\Rightarrow$ neutron pols

$^3$He: sensitive to $2\alpha_{E1}^p + \alpha_{E1}^n$ & $2\beta_{M1}^p + \beta_{M1}^n$ $\Rightarrow$ neutron pols.

Model-independently subtract binding effects.

$\Rightarrow$ $\chi$EFT: quantify reliable uncertainties.

Chirally consistent 1N & few-N: potentials, wave functions, currents, $\pi$-exchange.

Test charged-pion component of $NN$ force.
Example: induced electric dipole radiation from harmonically bound charge, damping $\Gamma$ [Lorentz/Drude 1900/1905]

$$\vec{d}_{\text{ind}}(\omega) = \frac{q^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\Gamma\omega} \vec{E}_{\text{in}}(\omega)$$

$$= 4\pi \alpha_{E1}(\omega) \ "\text{displaced volume}" \ [10^{-4} \text{ fm}^3]$$

Electric scalar dipole polarisability

Dis-entangle interaction scales, symmetries & mechanisms with & among constituents.

$\implies$ Clean, perturbative probe of $\Delta(1232)$ properties, nucleon spin-constituents, chiral symmetry of pion-cloud & its breaking.

 Fundamental hadron properties, like charge, mass, mag. moment, $\langle r_N^2 \rangle$...[PDG]
Hadron Polarisabilities: GW Leads Connecting Data & QCD

GW focus

Needs to be phrased as energy-difference: \[ \Delta E = -2\pi \alpha_{E1}^{(N)} \bar{E}^2. \]

Neither Approach Uses The Other To Fit!

[lattice: Lujan/Alexandru/Freeman/Lee arXiv:1411.0047 [hep-lat];
chiral extrapolation: hgrie/McGovern/Phillips arXiv:1511.01952 [nucl-th];
Downie/Feldman take data at HI\gamma S, MAMI,...]
Neutron \approx \text{proton polarisabilities}; \text{exp. error dominates.}

Downie, Feldman,\ldots\text{spokespersons of Compton efforts at HI} \gamma S, \text{MAMI,}\ldots
The special status of pions and kaons in QCD and their marked impact on the long-distance structure of hadrons can be systematically encoded in an effective theory, applicable to processes at low energy. This effective theory, as well as emerging LQCD calculations, can provide benchmark predictions for so-called polarizabilities that parameterize the deformation of hadrons due to electromagnetic fields, spin fields, or even internal color fields. Great progress has been made in determining the electric and magnetic polarizabilities. Within the next few years, data are expected from the High Intensity Gamma-ray Source (HIGS) facility that will allow accurate extraction of proton-neutron differences and spin polarizabilities. JLab also explores aspects of hadron structure.

**HIGS (DOE):** a central goal; > 3000 hrs committed at 60 – 100 MeV
- proton doubly & beam pol. (E-06-09/10)
- deuteron beam pol. (E-18-09, running)
- $^3$He unpol & doubly pol. (E-07-10, E-08-16)
- $^4$He unpol
- $^6$Li unpol. (E-15-11, first!)

**A2 @ MAMI (DFG: 5-year SFB):** running, data cooking and planned
- proton 100 – 400 MeV: beam & target pol. deuteron, $^3$He, $^4$He unpol., beam & target pol.

**MAXlab:** data cooking
- deuteron 100 – 160 MeV: unpol.
Observable \( \mathcal{O} = c_0(m_\pi) + c_1(m_\pi) \delta^1 + c_2(m_\pi) \delta^2 + \text{unknown} \times \delta^3. \)

χEFT provides coefficients \( c_i \) with explicit \( m_\pi \)-dependence, parameters fixed at \( m_\pi^{\text{phys}} \).

**Propagating Uncertainties:** Bayesian order-by-order as before, now at each \( m_\pi \).

\[ \rightarrow \text{Conservatively expand in } \delta(m_\pi) = \frac{m_\pi}{\Lambda_\chi} \approx 0.4 \times \frac{m_\pi}{m_\pi^{\text{phys}}} \text{ and fade as } \delta \to 1, \text{ i.e. } m_\pi \to \frac{m_\pi^{\text{phys}}}{0.4}. \]

**Cottingham Σ Rule:** \( \beta_{M1}^v \) is one of several inputs into the proton-neutron self-energy difference:

\[ M_{p-n} = M_{p-n}^{\text{strong}} + M_{p-n}^{\text{em,elastic}} - A \beta_{M1}^v \]

Only physical point has no substantial isospin splitting.

\[ \rightarrow \text{SPECULATION: Neutron lifetime may be shortened for larger } m_q, \]

with impact on Big Bang Nucleosynthesis, Anthropic Principle…?
\( \chi \text{EFT is low-energy QCD} \)

Unified, systematic description, rooted in QCD. Universally parameterise short-range int’s. Bridge from (lattice) QCD to Nuclear Structure.

**Correlations**, neutron properties, iso-spin & P-violation, Unique signals of chirality, 3NF, 4NF, \ldots \quad \text{QCD}

Reliable predictions for processes hard-to-access:

**Astro- & Neutrino-Physics**: supernovae, Big Bang, \ldots

**Beyond the Standard Model**: low-energy precision

**Alternative Worlds**: vary \( m_q, \alpha_s, N_c, \ldots \)

fundamental questions

conceptual advances \quad \text{concrete examples}

Many cliffs still to take, but the view is already wonderful!

[chart adapted from G. Henning]
6. The Standard Model – and Beyond

Lots of answers, but each raises more questions! ➡️ Your Turn!
Much more to come in PHYS 6710:

Nuclear and Particle Physics II: The Return of The Theorist

Spring 2019 – watch this space!