PHYS 6610: Graduate Nuclear and Particle Physics I

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II. Phenomena

1. Shapes and Masses of Nuclei

Or: Nuclear Phenomeology

References: [PRSZR 5.4, 2.3, 3.1/3; HG 6.3/4, (14.5), 16.1; cursorily PRSZR 18, 19]
(a) Getting Experimental Information

heavy, spinless, composite target \( \Rightarrow M \gg E' \approx E \gg m_e \to 0 \)

\[
\frac{d\sigma}{d\Omega} \bigg|_{\text{lab}} = \left( \frac{Z\alpha}{2E \sin^2 \frac{\theta}{2}} \right)^2 \cos^2 \frac{\theta}{2} \frac{E'}{E} \left| F(q^2) \right|^2
\]

Mott: spin-\( \frac{1}{2} \) on spin-0, \( m_e = 0, M_A \neq 0 \)

when nuclear recoil negligible: \( M_A \gg E \approx E' \)

\( q^2 = (k - k')^2 \to -q^2 = -2E^2 (1 - \cos \theta) \)

translate \( q^2 \leftrightarrow \theta \leftrightarrow E \)

max. mom. transfer: \( \theta \to 180^\circ: q^2 \to -(2E)^2 \)

min. mom. transfer: \( \theta \to 0^\circ: q^2 \to 0 \)

For \( E = 800 \text{ MeV} \) (MAMI-B), \( ^{12}\text{C} \) (\( A = 12 \)):

\[
\begin{array}{ccc}
\theta & \sqrt{-q^2} = |\vec{q}| & \Delta x \approx \frac{1}{|\vec{q}|} \\
180^\circ & 1500 \text{ MeV} & 0.15 \text{ fm} \\
90^\circ & 1000 \text{ MeV} & 0.2 \text{ fm} \\
30^\circ & 400 \text{ MeV} & 0.5 \text{ fm} \\
10^\circ & 130 \text{ MeV} & 1.5 \text{ fm} \\
\end{array}
\]

includes \( E/M \) (recoil) effects
“Typical” Example: $^{40,48}$Ca measured over 12 orders

**Fig. 5.7.** Differential cross-sections for electron scattering off the calcium isotopes $^{40}$Ca and $^{48}$Ca [Be67]. For clarity, the cross-sections of $^{40}$Ca and $^{48}$Ca have been multiplied by factors of 10 and $10^{-1}$, respectively. The solid lines are the charge distributions obtained from a fit to the data. The location of the minima shows that the radius of $^{48}$Ca is larger than that of $^{40}$Ca.

**$\theta$-dependence, multiplied by 10, 0.1!!**

$^{40}$Ca has less slope $\implies$ smaller size

$|\vec{q}|$-dependence fm$^{-1}$

[PRSZR] [HG 6.3]
Characterising (Spherically Symmetric) Charge Densities

\[ F(\vec{q}^2) := \frac{4\pi}{Ze} \int_0^\infty dr \frac{r}{q} \sin(qr) \rho(r) , \quad \text{normalisation } F(\vec{q}^2 = 0) = 1 \]

In principle: Fourier transformation \( \Rightarrow \) need \( F(\vec{q}^2 \to \infty) \): impossible

\[ \Rightarrow \text{Ways out:} \]

(i) Characterise just object size:

\[ F(\vec{q}^2 \to 0) = \frac{4\pi}{Ze} \int_0^\infty dr \frac{r}{q} \left[ qr - \frac{(qr)^3}{3!} + O((qr)^5) \right] \rho(r) \]

\[ = \frac{4\pi}{Ze} \int_0^\infty dr \, r^2 \, \rho(r) - \frac{q^2}{3!} \frac{4\pi}{Ze} \int_0^\infty dr \, r^2 \times r^2 \rho(r) + O((qr)^5) \]

\[ \Rightarrow \langle r^2 \rangle = -\frac{3!}{3} \left. \frac{dF(\vec{q}^2)}{d\vec{q}^2} \right|_{\vec{q}^2=0} \] (square of) root-mean-square (rms) radius
Ways Out: (ii) Assume Charge Density, Calculate its FF, Fit

\[ \rho(r) \] point-like \hspace{1cm} \rho(r) \text{ exponential} \hspace{1cm} \rho(r) \text{ Gaussian} \hspace{1cm} \rho(r) \text{ uniform sphere} \hspace{1cm} \rho(r) \text{ Fermi function} \]

Very simple form for heavy nuclei: Fermi Distribution

\[ \rho_N(r) = \frac{\rho_0}{1 + \exp \frac{r-c}{a}} = \frac{A}{Z} \rho_{\text{charge}} \]

**experiments:** radius at half-density \( c \approx A^{1/3} \times 1.07 \text{ fm} \):

translate hard-sphere: \( R = \sqrt{\frac{5}{3} \langle r^2 \rangle} \approx A^{1/3} \times 1.21 \text{ fm} \)

**experiments:** \( a \approx 0.5 \text{ fm} \): largely independent of \( A \!; \)

related to surface/skin thickness \( t = a \times 4 \ln 3 \approx 2.40 \text{ fm} \) (drop from 90% to 10%)
Better parameterise as sum of a few Gauß’ians:

$$\rho_{\text{charge}}(r) = \sum_i b_i \exp\left(-\frac{(r - R_i)^2}{\delta^2}\right)$$

⇒ “line thickness” = parametrisation uncertainty

\[\rho_N \approx \frac{0.17 \text{ nucleons}}{\text{fm}^3} \approx \frac{1}{(1.4 \times (2 \times 0.7\text{fm}))^3} \approx 3 \times 10^{11} \frac{\text{kg}}{\text{litre}} = 300 \frac{\text{tonnes}}{\text{mm}^3} = 3 \text{ space shuttles} \text{ mm}^3\]
Putting $\rho_N$ In Perspective

$\rho_N \approx \frac{0.17 \text{ nucleons}}{\text{fm}^3} \approx \frac{1}{(1.4 \times (2 \times 0.7\text{fm}))^3} \approx 3 \times 10^{11} \frac{\text{kg}}{\text{litre}} = 300 \frac{\text{tonnes}}{\text{mm}^3} = 3 \text{ space shuttles \ mm}^3$

Density

- Water
- Solids
- Neutron stars
- White dwarfs
- Nuclear Matter
- Black holes

$10^{-5}$ $10^0$ $10^5$ $10^{10}$ $10^{15}$ g/cm$^3$

[HG fig 1.3]
So far: no spin, no deformation, but spherically-symmetric $F(q^2)$ is the exception, not the rule.

**Deformation Example Deuteron** $d(np)$ with $J^{PC} = 1^{+-}$: $L = \vec{J} - \vec{S}$, $S = 1$

\[
L = 1_J \otimes 1_s = 0 \oplus 1 \oplus 2 \implies L = 0 \text{ (s-wave) or } L = 2 \text{ d-wave} - \text{Parity forbids } L = 1.
\]

Angular momentum $\implies$ magnetic moment $\implies$ spin-spin interaction/spin transfer ($\theta \to 180^\circ$, helicity)

**Charge FF:**
\[
G_C(q^2) = \frac{e}{3} \sum_{m_J = -1}^{1} \langle m_J | J^0 | m_J \rangle
\]

avg. of hadron density, $G_C(0) = 1$

**Magnetic FF:**
\[
G_M(q^2) = \frac{M_d}{M_N} \kappa_d, \text{ mag. moment } \kappa_d = 0.857
\]

**Quadrupole FF:**
\[
G_Q(q^2)
\]

Quadrupole moment $Q_d := G_Q(0) = Ze \int d^3r \left( 3z^2 - r^2 \right) \rho_{\text{charge}}(\vec{r}) = 0.286 \text{ fm}^2$

\[
\left| \frac{d\sigma}{d\Omega} \right|_{\text{lab}} = \left( \frac{Z\alpha}{2E \sin^2 \frac{\theta}{2}} \right)^2 \cos^2 \theta \frac{E'}{2E} \times \left[ \left( G_C^2 + \frac{2\tau}{3} G_M^2 + \frac{8\tau^2 M_d^4}{9} G_Q^2 \right) + \frac{4\tau(1 + \tau)}{3} G_M^2 \tan^2 \frac{\theta}{2} \right]
\]

with $\tau = -\frac{q^2}{4M_d^2}$ as before

$\implies$ Dis-entangle by $\theta$ & $\tau$ dependence.
Deuteron Form Factors

[Garçon/van Orden: [nucl-th/0102049]] does not include most recent data (JLab), but pedagogical plot. High-accuracy data up to $Q^2 \gtrsim (1.4 \text{ GeV})^2$

**Theory quite well understood:**

Embed nucleon FFs into deuteron, add:

- Deuteron bound by short-distance + tensor force: one-pion exchange $N^\dagger \vec{\sigma} \cdot \vec{q}\pi(q)N$

  ➔ Photon couples to charged meson-exchanges and many more!!

Open issues: zero of $G_Q$, form for $Q \gtrsim 3 \text{ fm}^{-1}$, …
(c) Nuclear Binding Energies (per Nucleon)

\[ \frac{B}{A} \] rapidly increases from deuteron \((A = 2)\): \(1.1 \text{ MeV}/A\)

to about \(^{12}\text{C}\): \(7.5 \text{ MeV}/A\)

for \(A \gtrsim 16\) (oxygen) remains around \(7.5 \ldots 8.5 \text{ MeV}/A\)

maximal for \(^{56}\text{Fe}–^{60}\text{Co}–^{62}\text{Ni}\): \(8.5 \text{ MeV}/A\)

small decrease to \(A \approx 250\) (U): \(7.5 \text{ MeV}/A\)

\[ \Rightarrow \] “typically”, fusion gains (much) energy up to Fe; fission gains (some) energy after Fe.

\[ \Rightarrow \] Fe has relatively large abundance: product of both exothermal fusion and fission.
Bethe-Weizsäcker Mass Formula & Interpretation: Liquid Drop Model

Know \( < 3000 \) nuclei \( \implies \) roughly parametrise ground-state binding energies, not only for stable nuclei

**Total binding energy: SEMF**

\[
B = a_V A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(N - Z)^2}{4A} - \delta \quad (1935/36)
\]

- \( a_V = 15.67 \text{ MeV} \) volume term
- \( a_s = 17.23 \text{ MeV} \) surface tension
- \( a_C = 0.714 \text{ MeV} \) Coulomb repulsion
- \( a_a = 93.15 \text{ MeV} \) (a)symmetry/Pauli term
- \( \delta = \begin{cases} 
-11.2 \text{ MeV} & \text{Z & N even} \\
0 & \text{Z or N odd} \\
+11.2 \text{ MeV} & \text{Z & N odd} 
\end{cases} \) pairing term

\( \implies \) Well-separated, quasi-free nucleons, next-neighbour interactions like in liquid.

- \( \rho_N \approx 0.17 \text{ fm}^{-3} \implies \) saturation
- \( \text{fewer neighbours on surface} \implies \text{less } B \) of protons \( \implies \) tilt to \( N > Z \)
- Pauli principle \( \implies \) tilt to \( N \sim Z \)
- Opposite spins have net attraction, \( \text{wf overlap decreases with } A \implies A\)-dependence
Valley of Stability around $N \gtrsim Z$

$$B = a_V A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(A - 2Z)^2}{4A} - \frac{\delta}{A^{1/2}}$$

$\Rightarrow$ Parabola in $Z$ at fixed $A$ with $Z_{\text{min}} = \frac{a_a A}{2(a_a + a_c A^{2/3})} \lesssim \frac{A}{2}$
Valley of Stability

Probe nuclear interactions by pushing to "drip-lines":

Facility for Rare Isotope Beams FRIB at MSU

The valley of stability

nuclei with excess nucleons move down the valley towards stability
the neutron side of the valley is poorly understood - scientists aren't sure where the dripline lies

stable elements

A = 56

A = 150
(d) Application: Nuclear Fission

For $A > 60$, fission can release energy, but must overcome fission barrier.

Let's assume fission into 2 equal fragments.

**Estimate:** infinitesimal deformation into ellipsoid (egg) with excentricity $\epsilon$ at constant volume

\[ E(\text{sphere}) - E(\text{ellipsoid}) \approx \frac{\epsilon^2}{5} \left( 2a_s A^{2/3} - a_C Z^2 A^{-1/3} \right) \]

Fission barrier classically overcome when $\leq 0$:

\[ \frac{Z^2}{A} \sim \frac{2a_s}{a_C} \approx 48 \quad \text{e.g. } Z > 114, A > 270 \]

below: QM tunnel prob. $\propto \exp \left( -2 \int \sqrt{2M(E-V)} \right)$ between points with $r(E=V)$

**Induced Fission:** importance of pairing energy

$n + \frac{238}{92} U \rightarrow \frac{239}{92} U$: even-even $\rightarrow$ even-odd

\[ \Longrightarrow \text{invest pairing } E_\delta = \frac{\delta=11.2\text{MeV}}{\sqrt{239}} = 0.7\text{MeV} \]

$n + \frac{235}{92} U \rightarrow \frac{236}{92} U$: even-odd $\rightarrow$ even-even

\[ \Longrightarrow \text{gain pairing energy } \frac{\delta}{\sqrt{236}} = -0.7\text{MeV} \Longrightarrow \text{can use thermal neutrons (higher } \sigma!) \]
Nuclear Interactions Saturate: \( \rho_N \approx \frac{0.17 \text{ nucleons}}{\text{fm}^3} \to \text{const. in heavy nuclei} \)

Nucleons “separate but close”: Distance between nucleons in nucleus \( \approx 1.4 \times \text{nucleon rms diameter} \).

Fermi distribution at temperature \( T = 0 \) for \( N = Z \):
occupy all levels, 2 spins, proton & neutron, \( N = Z \)
\[
\rho_N = \rho_p + \rho_n = 2 \int \frac{d^3k}{(2\pi)^3} \left[ n_p(\vec{k}) + n_n(\vec{k}) \right] = \frac{2}{3\pi^2} k_F^3
\]

\( \Rightarrow \) Fermi momentum (max. nucleon momentum) \( k_F = \frac{3}{2} \sqrt{\frac{3\pi^2 \rho_N}{2}} \approx 1.3 \text{fm}^{-1} \approx 260 \text{MeV} \approx 2m_{\pi} \).

Liquid-gas transition for temperature \( T \uparrow \)
Liquid-Drop Model: heat \( \Rightarrow \) evaporation \( T \) via \( E \)-distrib. of collision fragments:
\[
N(E) \propto \sqrt{E} \exp[-E/T] \text{ (Maxwell)}
\]

but need many fragments, angle-indep.
exp: \( T \approx 5 \text{MeV} \) in finite symmetric nuclei.

Neutron-\( E \)-distrib. in \( ^{235}\text{U} \) fission: \( T = 1.29 \text{MeV} \)

compare to water
Nuclei Are Not “Nuclear Matter”

SEMFT of finite nuclei:

$$\frac{B}{A} = a_V - \frac{a_s}{A^{1/3}} - a_C \frac{Z^2}{A^{4/3}} - a_a \frac{(N-Z)^2}{4A^2} - \frac{\delta}{A^{3/2}} \approx 8.5\text{MeV}$$

Infinite nuclear matter: no surface, Coulomb negligible, no pairing

$$\Rightarrow \frac{B}{A} \approx a_V = 15.6\text{MeV}.$$

grand canonical ensemble:

$$Z = \text{tr} \exp \left( -\frac{1}{T} \left[ H - \mu_p N_p - \mu_n N_n \right] \right)$$

$$-T \ln Z = -PV = E - TS - \mu_p N_p - \mu_n N_n$$

$$\Rightarrow \text{pressure} -P = \mathcal{E} - Ts - \mu_p \rho_p - \mu_n \rho_n$$ with $\mathcal{E}$: energy density, $s$: entropy density

Need to extrapolate or solve nuclear many-body problem: specify interactions!

Descriptions agree at $\rho_0 \approx 0.16\text{fm}^{-3}$ — here $\chi$ EFT ($\pi$, $N$, $\Delta(1232)$) [Fiorilla/...[arXiv:1111.3688 [nucl-th]]]

Empirical first-order liquid-gas phase transition
for infinite, symmetric ($N = Z$) nuclear matter at critical temperature $T_c = [16\ldots18]\text{MeV}$.

Chemical potential at temperature $T = 0$:

$$\mu_N(T = 0) = M_N - \frac{B}{A} = [939 - 16]\text{MeV} \approx 923\text{MeV}$$
A First Phase Diagram of Nuclear Matter: $N = Z$

$T_c \ll m_\pi, M_N \implies$ symmetric nuclear matter close to liquid-gas transition: just inside liquid phase.

Density $\rho(T, \mu_N)$ of stable nuclear matter depends on $T, \mu$. 

$T_c \approx 15\text{MeV}$
Again relatively good agreement between descriptions – here $\chi$EFT [Fiorilla/... [arXiv:1111.3688 [nucl-th]]].

\[ Z/A = 0 \]

\[ Z/A = 0.5 \]

$\rightarrow$ Nuclear-matter density $\rho(N - Z)$ decreases as $Z/A$ decreases.

– Nuclear Matter becomes unbound for $Z/A \lesssim 0.1$.

– Why is pure-neutron matter unbound (a gas)?? (Pauli-principle??)

– In early 1900’s, neutron “invented” to mitigate Coulomb repulsion between protons.

\[ \text{So why no binding when I take all protons away?} \]

– Lattice QCD: Neutron matter might actually be bound for larger $m_\pi > 600$ MeV (controversial).
So Why Are There Neutron Stars??

**Gravity** compacts interior $\implies$ saturation point shifts: $\rho_0 \approx 0.16 \text{fm}^{-3} \rightarrow 3\rho_0$, holds neutrons together.

How to extrapolate to there – and how to extrapolate from $Z \approx 0.4A$ to $Z \lesssim 0.1A$ (“neutron” star!)?

**Taylor in** $N - Z \over A$:

$\mathcal{E}(\rho, \ N - Z \over A) = \mathcal{E}_0(\rho_0, \ N - Z \over A = 0) + \left[ a_a(\rho_0) \over 4 \ + \ {d(a_a/4) \over d\rho} \bigg|_{\rho_0} \right] \left(N - Z \over A \right)^2 + \ldots$

Nuclei (SEMF): “(a)symmetry energy” $a_a(\rho_0)/4 \approx 22 \text{ MeV}$; nucl. matter: [29…33] MeV

slope $L = 3 {d(a_a/4) \over d \ln \rho} \bigg|_{\rho_0} = [40 \ldots 62] \text{ MeV}$. Method: compare different $Z/A$ nuclei & extrapolate.

**Taylor in** $(\rho - \rho_0)$:

$\mathcal{E}(\rho, \ N - Z \over A) = \mathcal{E}_0(\rho_0, \ N - Z \over A = 0) + {d^2 \mathcal{E} \over d\rho^2} \bigg|_{\rho_0} (\rho - \rho_0)^2 + \ldots$

$\rho = \rho_0 + K(\rho_0)(\rho - \rho_0)^2 + \ldots$ justified for $\rho$ (neutron star) $= 3\rho_0$??

**Compressibility of nuclear matter** $K(\rho) = 9\rho {d^2 \mathcal{E} \over d\rho^2} > 0$ for stable nuclear matter at density $\rho$.

Test dependence on $(\rho, N - Z)$ in neutron skin of heavy nuclei, collective excitations & **extrapolate**!

At $\rho_0$, $N = Z$: compressibility $K = k_F^2(\rho_0) {d^2 \mathcal{E} \over d\rho^2} \bigg|_{\rho_0} = [210 \pm 10] \text{MeV}$. Wide agreement.

At $\rho_0$, pure neutron matter: $K \approx 600 \text{MeV}$, error $\pm 100 \text{MeV}$ or more.

**People disagree!** Number here from [Vretenar/... PRC68 (2003) 024310]
**Problem:** Neutrons have no charge $\rightarrow$ higher-order effect & weak interactions.

**Inference Depends on Theory:** skin only skin-deep, not “nuclear matter”,…

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Experimental values of the $^{208}_{\text{Pb}}$ neutron skin thickness ($\Delta r_{np}$), which is related to the neutron matter pressure at $\rho = 2/3 \rho_0$, agree better with calculations that include 3-nucleon forces.
When you compress nucleons, additional energy can be converted into new particles: baryons (Λ(1440),...) mesons (kaon,...), resonances/excitations (Δ(1232),...), exotics...

⇒ Influence on neutron-star radius,...! ⇒ Later.
Need third axis with chemical potential $\mu_I = \mu_p - \mu_n$ for $Z - N$ to place neutron stars.

**Box 3**
**Features of the QCD Phase Diagram at Low Temperature and High Density**

The 3-dimensional QCD phase diagram at high baryonic $\mu_B$ and moderate isospin $\mu_I$ densities has a rich and yet largely unexplored structure: a critical endpoint separates a smooth cross-over from a first order as well as a chiral phase transition at high baryon densities. New and exotic phases like quarkyonic matter or color superconducting phases might appear at baryonic high densities. At very high $\mu_B$ a superfluid color-flavor-locked phase is speculated on. Supernovae are formed at initial proton fractions $\approx 0.4$ which reduce to $\approx 0.1$ for cold neutron stars. Heavy-ion collisions at FAIR or NICA energies are expected to probe this region as well as the conjectured phase boundaries to quarkyonic or fully deconfined matter.

[NuPECC Long-Range Plan 2017 p. 89]
**Fig. 5.9.** Spectrum of electron scattering off $^{12}$C. The sharp peaks correspond to elastic scattering and to the excitation of discrete energy levels in the $^{12}$C nucleus by inelastic scattering. The excitation energy of the nucleus is given for each peak. The 495 MeV electrons were accelerated with the linear accelerator MAMI-B in Mainz and were detected using a high-resolution magnetic spectrometer (cf. Fig. 5.4) at a scattering angle of 65.4°. *(Courtesy of Th. Walcher and G. Rosner, Mainz)*

**SEM6 does not explain nuclear level spectrum.**
Beyond the SEMF/Liquid Drop

cursory look at [PRSZR 18, 19]

Difference Semi-Empirical Mass Formula SEMF – Experiment


**Magic numbers** 2, 8, 20, 28, 50, 82, 126 for Z or N more stable than SEMF ➞ Shell-like structure?

binding energy difference
Mass Formula - experiment
Example of Single-Particle Models: 3 Minutes on the Shell Model

**Single-Particle Models:** Individual nucleon moves in average potential created by all other nucleons.

⇒ Neglect feedback of motion onto potential. Saturation, short-range forces ⇒ $V(r) \propto \rho(r)$

Light Nuclei: Gaußian profile; Heavy nuclei: Fermi/Woods-Saxon potential

$$V(r) = \frac{-V_0}{1 + \exp \frac{r-c}{a}}$$

**Full QM: Solve Schrödinger Equation**

- Fermi Gas/Liquid Model: 3-dim. potential square-well with depth $V_0$.
- 3-dim Harm. Oscillator $E_{h.o.} = (N_x + N_y + N_z + \frac{3}{2}) \hbar \omega$; ang. mom. $l = N - 2(\# \text{ wf nodes} - 1)$

**Refinement Coulomb:**

proton sees charges, neutron not.

⇒ $V_0^p > V_0^n$.

*Refinement Spin-Orbit Coupling* $V_{ls}(r) \vec{l} \cdot \vec{s}$: like (??) fine structure in H atom, where it is tiny $\mathcal{O}(\alpha^2)$.

Nucleon $\vec{s} \otimes \vec{l} = \vec{j} \implies l \in \{j - \frac{1}{2}; j + \frac{1}{2}\} \implies \vec{l} \cdot \vec{s} = \frac{1}{2}[(\vec{l} + \vec{s})^2 - \vec{r}^2 - \vec{s}^2] = \frac{1}{2}[j(j+1) - l(l+1) - \frac{3}{4}]$

⇒ $\Delta E_{ls} = (l + \frac{1}{2}) \langle V_{ls} \rangle$  

**Experiment:** $\langle V_{ls} \rangle \approx -20 \text{MeV} < 0 $ huge

(heavy & close constituents), opposite sign to H atom.

And, of course, many more refinements...
Example of Single-Particle Models: 3 Minutes on the Shell Model

Each state with 2 protons & 2 neutrons (spin!); pairing $\implies$ closed shells do not contribute.

$\implies$ **Gaps** at *magic numbers* $2, 8, 20, 28, 50, 82, 126$.

$\implies$ Spin-orbit responsible for **gaps** at *magic numbers* $28, 50, 82, 126$.

[Maria Goeppert Mayer/Wigner/Jensen 1949 + developments: “Periodic table” of nuclei]

Very good *close to shell closure* (“valence nucleons”; incl. magnetic moments!), bad-ish off-closure.
Liquid Drop Is Example of A Collective Model

Collective Model: Nucleons lose individuality, form continuous fluid/gas.

Example Collective Vibrations/Shape Oscillations: shape of nucleus deformed.

Example Compressibility of Nuclear Matter: “monopole mode” $J^{PC} = 0^{+-}$: radial oscillations.

Experiment: excitation energy $\approx 80A^{-1/3} \text{MeV} \gg$ any other mode

$\Rightarrow$ Nuclear matter pretty incompressible (except for interior of Neutron stars!).

Example Giant Electromagnetic Dipole Resonance: p & n oscillate against each other.

$\Rightarrow$ Coherent elmag. excitation $\propto Z^2$; huge resonance.
Example Collective Rotations

Non-spherical nucleus rotates around non-symmetry axis, inertia $I$:

$$E_{\text{rot}} = \frac{j^2}{2I} = \frac{J(J+1)}{2I}$$

“rotation bands”

$\Rightarrow$ characteristic spacing

$$\Delta E \propto (2J + 1).$$

**Experiment:** Inertia $I <$ rigid ellipsoid, but $I >$ irrotational flow (superfluid)

$\Rightarrow$ Nucleus like raw egg.

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**Fig. 18.14.** Energy levels of $^{152}$Dy [Sh90]. Although the low energy levels do not display typical rotation bands, these are seen in the higher excitations, which implies that the nucleus is then highly deformed.
Next: 2. Hadron Form Factors & Radii

Familiarise yourself with: [HM 8.2 (th); HG 6.5/6; Tho 7.5;