

Problem Sheet 1

Due date: 05 September 2018 **16:00**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.

Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.

I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.

News and .pdf-files of Problems also at <http://home.gwu.edu/~hgrie/lectures/math-methods18/math-methods18.html>.

1. ANIMAL PHYSICS (3P):

- a) (2P) A Canada goose (mass about 8 kg) has a cruising speed of about 20 m s^{-1} . An ostrich is the fastest running bird, with speeds up to 75 km h^{-1} . Determine the cruise speed which an ostrich would have (mass up to 150 kg) in flight – assuming it would have a comparable wing-form. Explain why ostriches are not very likely to fly.
- b) (1P) Mammals in the arctic are pretty big. Explain this by comparing their heat production (proportional to the number of cells the animal has) to their heat loss through their surface.

2. DIMENSIONAL ANALYSIS (6P):

By matching dimensions, we can discover some very fundamental relations of modern physics – without understanding the underlying theory.

Warning: You get the points for a readable and concise *derivation*, not for stating the result! I know you could look up the numbers anywhere.

- a) (2P) LORD RAYLEIGH'S PROBLEM We know nothing of Hydrodynamics, but we can nonetheless estimate wave velocities in some liquids. Consider a water wave advancing in “infinitely” deep water under the action of gravity on Earth (standard gravitational acceleration g). Estimate the wave-length of the wave, depending on its velocity. Does the result depend on the density of the liquid?
- b) (2P) Estimate the size at which an object with mass M will collapse due to gravity in classical mechanics. You will notice that you need a constant with dimensions of a velocity, in addition to Newton's constant. What could that be? The exact relation is found as the SCHWARZSCHILD RADIUS of General Relativity.
- c) (2P) It does not always work: We are looking for a “electrodynamic theory of gravitation”: Relate the gravitational mass of the electron to the electromagnetic charge, the gravitational constant, and appropriate constants of Nature – without resort to other electron properties. Is something like this result found in any theory of fundamental forces?

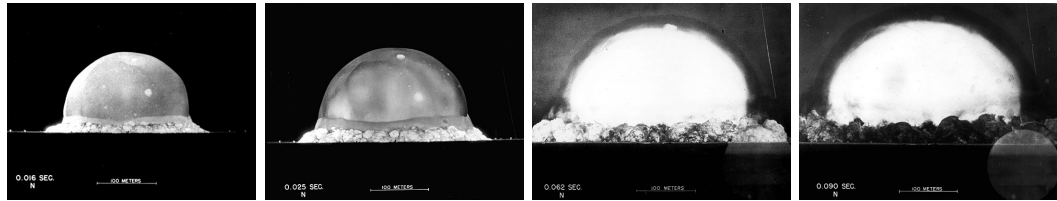
3. MECHANICAL SIMILARITY (4P):

We investigate a bit more the potential which is a homogeneous function of degree ν , i.e. $U(\alpha\vec{r}) = \alpha^\nu U(\vec{r})$. We showed in the lecture that if a curve $\Gamma = \{\vec{r}\}$ is a solution to the equation of motion, then the re-scaled curve $\alpha\Gamma = \{\alpha\vec{r}\}$ solves the rescaled equation when one re-scales the time variable appropriately.

- a) (2P) We now consider more carefully the initial condition: Γ is as solution to the initial condition $\vec{r}(t=0) = \vec{r}_0, \dot{\vec{r}}(t=0) = v_0$. What are the initial conditions on $\alpha\Gamma$ for a scaled solution?
- b) (2P) Consider now re-scaling not in \vec{r} but in time t , keeping \vec{r} un-scaled. Find the ratio of the times on a path when you re-scale the particle masses but keep their potential energy the same.

Please turn over.

- GUESSTIMATE (3P): What is the typical volume of a human cell? Estimate the number of cells in your body. Write an “essay” which explains how you come up with the result, step-by-step. Do not cheat – using google does not count. Pretend you do it using Mental Math.
- NUCLEAR EXPLOSION (5P): Well into the fifties, the energy released from a nuclear explosion was classified information, but films showing the expansion of fireball or of the famous “mushroom cloud” were released. Based on a film declassified in 1947, British physicist Sir Geoffrey Taylor used dimensional estimates for the ‘fireball to deduce the energy released by the first nuclear bomb test on 16 July 1945, “Trinity” [Proc. R. Soc. A201 (1950), 159 and 175]. The censors had overlooked that the still shots contain both time marks and length scales. Here are a few pictures and data of the radius r of the fireball as function of elapsed time t :



t [ms]	0.52	0.66	0.80	1.08	1.36	1.65	1.93	3.53	4.07	4.61	25.0	53.0	62.0
r [m]	28.8	31.9	34.2	38.9	42.8	46.0	48.7	61.1	64.3	67.3	130.0	175.0	185.0

Convince yourself that the energy released can reasonably depend only on the time elapsed, the cloud size at that time and a typical density (of what?). Verify that the data given obeys the scaling law of the dimensional estimate and estimate the strength of the explosion (4×10^{12} Joule= 1 kiloton TNT).

Historical Note: Since the Americans officially still treated the Trinity test yield as classified, Taylor got into a lot of trouble, but hopefully helped people realise the craziness of a nuclear arms race.

- TAYLOR EXPANSION: (4P) Using “Mental Math” tricks to avoid taking derivatives, find the first and second non-trivial order (i.e. the second term which contains a power of the parameter x):

$$\cos[\pi\sqrt{1-x^2}] \text{ for } x \rightarrow 0 \qquad \exp[-x/(1+x^3)^{\frac{1}{4}}] - \exp[-x] \text{ for } x \rightarrow 0 .$$

- MENTAL MATH: (5P) Given the following binomial equations:

$$x^2 + x - 85,000 = 0 \qquad 85,000x^2 + x - 1 = 0$$

- (3P) Find approximate solutions to these equations, without resort to “exact” formulae or electronic means, and without using a Taylor expansion of the binomic formula. Write an “essay” which explains how you come up with the result, step-by-step.
- (2P) Estimate (with giving reasons!) the accuracy of each of your results. Do *not* use the exact result or the binomial formula. Pretend that you do not know the exact result!

