PHYS 6110: Mathematical Methods of Theoretical Physics

Syllabus

PHYS 6110, section 10, in conjunction with PHYS 6130, section 10.

Teacher: Dr. Harald W. Griesshammer, Corcoran Hall 306, 202-994-3849, hgrie@gwu.edu .

Lectures: *Tuesday/Thursday* 14:00 to 15:40 in Corcoran Hall 309 (100 min each, for 4 credits). "Snow Days" (possible slots for rescheduled lectures): Fri 08:20 to 10:00 in Corcoran Hall 309 (alt. Mon 08:20 to 10:00 in Corcroan Hall 309). – All to be confirmed! No lectures: Details to be announced. – Watch the website!

Homework Due: Wednesdays at 16:00 sharp

Surgery hours: *Fridays at 8:20 in Corcoran Hall 309* (or immediately after Friday lecture) to discuss the problem sheets and for questions, discussions and suggestions; duration: till all questions are answered. More office hours by appointment weekdays after 15:00 in my office. Best email what and when to discuss to make sure I have time and am prepared.

Mid-Term Exam: Friday, 26 Oct at 15:00 in Corcoran Hall 309, 2.0 hours. – (to be confirmed) Final Exam: Thursday, 20 Dec at 09:30 in Corcoran Hall 309, 2.5 hours. – (to be confirmed)

Typical workload for this course: 10-12 hours weekly, plus lectures and surgery hours – at least 8 hours.

Web-site: http://home.gwu.edu/~hgrie/lectures/math-methods18/math-methods18.html for up-to-date information, .pdf-files of Problem Sheets, a manuscript, suggested reading, corrections, etc.

Prerequisites: Advanced undergraduate Mathematical Methods, Mechanics, Electrodynamics and Quantum Mechanics. Experience of vectors, matrices, multi-dimensional differentiation and integration, ordinary differential equations, complex numbers, etc., comparable to suggested Refs. [Sha, except Sects. 6 (complex analysis) and 10.5-7 (partial diff. eq.'s)], [Boa, roughly chapters 2-8], [BF, roughly chapters 1 and 3]. **Coordinated** with *PHYS 6120: Classical Mechanics* (Haberzettl). **Co-requisite:** *PHYS 6130: Computational Physics I* (Haberzettl/Griesshammer).

Goals/Learning Objectives: Introduction to the mathematical methods of Theoretical Physics with many examples and applications to Physics problems. Focus on skill-building. No formal but "heuristic" proofs. Students will demonstrate proficiency with fundamental methods of Mathematical Physics; apply their knowledge to solve a wide range of problems in modern Physics, with special relevance to parallel and future lectures of the graduate curriculum; compare and evaluate problem solving strategies, interpreting their respective value and limitations; identify common mathematical tools and patterns for very different Physics problems; be able to study more advanced and specialised mathematical techniques on their own and as their research will necessitate.

The tools and tricks discussed form the indispensable back-bone of the graduate curriculum. We do thus not aim fro a comprehensive and systematic study of techniques – given the vastness of the field, that would be futile. Rather, we explore an "alternative narrative": the mathematical perspectives of Physics research.

Outline of Contents, in thematic order only; durations are estimated and likely to change.

1. Qualitative Methods (1.5 lectures)

Mental math and oft-forgotten tools – dimensional analysis – universal scaling relations and mechanical similarity – arbitrariness of systems of units – the Natural System of Units

2. Science (and Not Art) of Approximations (3 lectures)

Taylor series and importance of harmonic oscillators – asymptotic series, Stirling's formula – non-degenerate perturbation theory

3. <u>Calculus of Variations</u> (1.5 lectures)

Optimisation and extremisation – functionals and functional derivatives – fixed-endpoint and flexibleendpoint variations – Euler-Lagrange equations – invariants – Rayleigh-Ritz variational principle

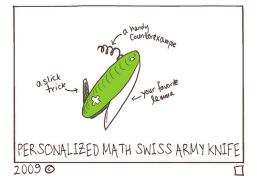


Figure 1: The personalized math swiss army knife comes with everything you could ever need to tackle even the toughest math problems! If you order now for the low-low price of $\$e^{2\times 2-\sqrt{4}}$ (plus a mere $\$\phi^{\pi}$ in shipping and handling), then you'll even get a free physics attachment, complete with hand-waving arguments, heuristic examples, and special tools specifically designed with a physicist in mind! You'll be solving via separation of variables and first order Taylor expansions in no time! Call now, before its too late! [Robert Coyne 2009]

4. Vector Spaces and Scalar Products (2 lecture)

Abstract vector spaces with metric and scalar product – abstract, contra- and co-variant (dual) vectors – bra-ket notation – operators as abstract matrices – some special operators

5. Tensor Calculus and Tensor Analysis (3 lectures)

Differentiation and integration in orthogonal, curvilinear coordinates – gradient, divergence, curl, Laplacean in 3 and any dimensions – integration theorems of Gauß and Stokes – Helmholtz' fundamental theorem

6. Functional Analysis: Hilbert Spaces and Operators (3 lectures)

Recap vector spaces and matrices – function space, bras and kets – orthonormal bases of functions – Lesbegue integral – Generalised functions and Dirac's δ -distribution – Fourier transform – operator theory: Hermitean, self-adjoined and unitary operators – spectral decomposition – projectors – pitfalls

7. Partial Differential Equations and Green's Functions (4.5 lectures)

Recap Ordinary Differential Equations – Green's functions/operators and boundary value problems – potential theory (Poisson equation) – Legendre polynomials, spherical harmonics and other complete orthonormal sets of functions – spherical multipoles – Sturm-Liouville theory – integral transforms (Fourier, Laplace, ...) – integral equations

8. Complex Analysis (4.5 lectures)

Theory of analytic functions – complex differentiation and integration – poles and cuts – Laurent series – Cauchy's theorem and residue calculus – principle value prescription and dispersion relations – analytic continuation – special functions

9. Groups and their Representations (5 lectures)

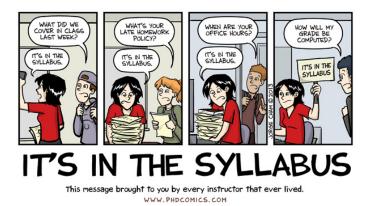
Matrix and abstract groups – Abelian and non-Abelian groups – discrete groups – continuous (Lie) groups, algebrae, generators – rotations: SO(3) and SU(2) – representations – irreducible tensors

Style: "Commenting lecture with student participation", i.e. focus on central points to guide and assist you in exploring relevant literature. The home-page lists strongly suggested reading to efficiently prepare and in particular follow-up on course material. Most important is the link to a manuscript of the lecture in my (illegible) hand-writing, see p. 4. Read over the manuscript before class and grasp the essential points. I will assume you have read this material before each lecture, and will familiarise yourself with its formal aspects after each lecture. The better prepared you are, the more we can focus on discussing your questions,

problems, observations. The class becomes more interactive and thus more fun – and therefore you learn more. Study details of the manuscript after the lecture, as starting point for your own literature research using good books like those recommended for particular subjects in the "Suggested Reading" column below. Some people prefer to have a print-out of the manuscript in class, so that they do not have to write down all details but can highlight and annotate important points.

We will *not* repeat in class the contents of textbooks, but add different perspectives, seek a deeper understanding of the underlying concepts and develop skills by addressing problems as a group. I present not all the mathematical tools you will ever need – that would be impossible –, but provide an introductory overview with a personal selection of topics which are frequently used in modern research. *Proofs will largely be deferred to books. Rigorous proofs are left to Mathematicians.* Some topics are *not covered thoroughly enough* in class, some may be hard to find in textbooks (see notes on the bibliography), and others are only addressed in the homework. The "lectures" are only a first guide to study the mathematical methods outlined more rigorously in books, e.g. in those listed below. You should ask yourselves the type of questions that lead to developing and understanding the key concepts and the skills of scientific reasoning. I as teacher can assist, guide, motivate, trigger and speed up your studies, but learning is an active process which takes place within you more than in the lecture hall. Its difference to research is mostly that when doing research, you learn what is not yet found in textbooks.

I encourage you strongly to ask questions and initiate discussions in class and during Surgery hours at all times. Think of lectures rather as "tutorial" or "studio" than a fixed set of hours in which I talk and you listen. If I cannot give you a satisfactory answer right away, I will come back to you, and you should continue asking until you are satisfied. If you find discussion in class or Surgery hard to follow, see me instantly!



Grading policy: The course will be graded on an absolute scale. The final grade is a sum of:

- Exercises/Homework (20% of total grade; need 50% to qualify for exam): weekly, details below;
- Mid-Term Exam (40% of total): closed-book, but a sheet with relevant formulae is provided; 2.0 hours;
- Final Exam (40% of total): closed-book, but a sheet with relevant formulae is provided; 2.5 hours.

In order to pass, you need at least 60% of all points *and* at least 50% of the points available in each of the three components *separately*. In particular, you need at least 50% of all points in all Problem sheets together (not per sheet!). 80% is an excellent score, and 90% has not been achieved yet. I do not post scores on the web. If you have questions or comments on your grade or your overall score, please see me. For your protection, the exams are closed-book. A sheet with some possibly relevant mathematical formulae will be provided by me, several days before the exam. If you have understood the contents of formulae and

practised enough examples, you will not even bother to consciously memorise anything.

Exercises/Homework: distributed Wednesdays after class on the web, due the following Wednesday at 16:00 in my pigeon-hole for paper-submissions or electronically to my email (.pdf-file only). Late homework is graded as zero points, unless you notify me before the due date with reproducibly legitimate reasons (e.g. illness). Handwritten solutions must be on 5x5 quadrille ruled paper; electronic solutions must be in .pdf format. Use of a "lab-book" or "journal" for homework is strongly encouraged and may become mandatory if homework presentations are overly untidy to the point to become indecipherable.

You may use a symbolic programming language like Mathematica, but do not dependent on it. It is useful for some things (like plots), but you will work with pen and paper in exams. If you use it for part or all of your assignment, you *must* submit a paper printout or .pdf file of the code you used, with all results, and with all your documentation or comments. The course website contains Mathematica files used in the lecture, but they are not as well documented as I expect yours to be. You can supplement your code by a write-up (.pdf or quadrille paper), and vice versa.

Graded solutions are **returned and discussed during Surgery hour on Friday**. Typically, problem sheets contain a mix of detailed and only outlined questions, with up to 30 points per sheet. Some problems require numerics or graphics programmes (Maple, Mathematica, Fortran, Assembler, C(++), etc.).

Some of the projects of *PHYS 6130: Computational Physics I* will be chosen from material in this course, and hence will need some "real" programming. These do not count towards your grade in Math. Methods; see separate announcement and later in the course.

It is a fact of life that if you score more than 60% of the homework points, you will most likely perform well in exams and the qualifying. Some exam questions will almost certainly be based on homework solutions.

While it is necessary to have the correct answer for full credit, it is not sufficient. Indeed, it may serve you only one point. What you hand in should be a tidy and efficiently short presentation of your results and how they come about, which can be understood and reproduced by your peers. Imagine it is not homework, but a research problem whose solution you are asked to explain to your peers. I neither encourage nor discourage you to submit solutions electronically. But if you do so, work with a good drawing programme like xfig or gimp (freeware) for sketches. Electronic submission is no excuse for leaving out sketches.

I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework. Homework serves several purposes, e.g.: expand and solidify your math "tool-chest", deepen your understanding by applying what you learned, and cover topics of relevance which are not discussed in the lecture. To preclude a common mis-conception: A substantial portion of the problems addresses questions in which you have to expand on what was covered in class or the manu-script. A good book can help you to get inspired, but make sure that the solution is yours alone. "Practise-problems" are in the minority, so you should practise additional problems on your own, e.g. using a good book.

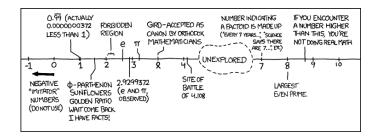
I encourage you to form study groups to discuss the reading and attack Problem sheets as team. Nothing helps you understand better than interacting with your peers. However, practise additional problems alone to make sure that you do not become dependent on the others.

You can best study and check your progress if you present results and problems with selected exercises in the seminar-style Surgery hours. Your discussion of solutions, problems and comments shape them. As integral part of the lecture, I encourage you to attend them regularly. There is no better preparation for the exams. Surgery is my prime tool to gauge our progress and revisit material which is not fully digested yet.

Lecture Manuscript A scanned version of a chapter-by-chapter manuscript can be found by following the links of chapter headings in the Class Schedule and Contents Sections on the web-site. The files are in .djvu-format, at present the most condensed way of storing scanned images: 50 scanned pages translate into 1.2 GBytes of bitmap, 50 MB .pdf or 4.7 MB .djvu. The freeware djvu reader "djvulibre" for all operating systems is available at http://djvu.sourceforge.net/, or as add-on to every decent Linux distribution.

Caveat: Warning and Disclaimer These are my notes for preparing the class, in my handwriting. While considerable effort has been invested to ensure the accuracy of the Physics presented, this manuscript bears only witness of my limited understanding of the subject. I am most grateful to every reader who can point out typos, errors, omissions or misconceptions. Maybe over the years, with lots of student participation, this can grow into something remotely useful.

The manuscript only intends to ease the pain of preparing and following the lecture. It does not replace the thorough study of textbooks (see note on the Style of the Lecture, p. 2). The manuscript is not intended to be comprehensible, comprehensive – or even useful. It is certainly not legible. Your mileage will vary. This manuscript is not useful or relevant for exams of any kind.



Some Suggested Reading

There is no required reading for this course. You will not be able to find all aspects of the lecture explained well in only one textbook. Moreover, it is an essential part of the learning process to view the same topic from different angles, i.e. to use different textbooks. As [Nea p. vii] writes: "It is always useful to get a second viewpoint because it's commonly the second one that makes sense – in whichever order you read them." Here is a list of those which I found most useful. If you discover others, tell me. I will bring most of the books listed to the first lecture. I suggest to wait with buying for a while before you decide which book suits your style best. Advanced graduate students are an excellent source for recommendations as well. The web-site lists recommended readings for each lecture.

An asterisk * indicates titles we have on semi-permanent loan in the Graduate Office Corcoran 3XXX, via Course Reserve at Gelman Library. Be social.

Books on which the course is (mostly) based – I often prefer combining [Sni] and [SG].

- [AW] * G.B. Arfken, H.J. Weber and F.E. Harris: Mathematical Methods for Physicists; 7th ed., Academic Press 2012, ca. US\$110 (cheaper as international student edition). "Standard text" for this kind of course. Contains all aspects like an encyclopedia, but sometimes not very pedagogical. Good reference for research.
- [Nea] J. Nearing: Mathematical Tools for Physics; Dover Publications 2010, ca. US\$20; .pdf version at http://www.physics.miami.edu/~nearing/mathmethods/. Like [Sni], an excellent undergraduate text driven by physical insight which can serve as start for your studies of more advanced texts like [SG]. Tell me what you think about it.
- [Sni] * R. Snieder and K. van Wijk: A Guided Tour of Mathematical Methods: For the Physical Sciences; 3rd ed., Cambridge University Press 2015, ca. US\$75. Like [Nea], a new and readable approach which is driven by physical insight. Sometimes not deep enough for graduates, but excellent start point for more advanced texts like [SG].
- [SG] * M. Stone and P. Goldbart: Mathematics for Physics; Cambridge University Press 2009, ca. US\$75; also online at http://webusers.physics.illinois.edu/~goldbart/PostScript/MS_PG_book/bookmaster.pdf, free. Like [Nea] new and readable approach which is driven by physical insight. Sometimes too formal and specialised, but many students find it very useful when combined with [Sni] or [Nea].
- [GR] I.S.S. Gradshteyn and I.M. Ryzhik (A. Jeffrey and D. Zwillinger, eds.): Table of Integrals, Series, and Products; 8th ed., Academic Press 2015, ca. US\$75. Not a book to study, but best reference for functions, their integrals and properties. On my desk; heavily used by theorists. The integral table for adults, rated "R" for Required.

Further books useful for this course

- [Cho] T.L. Chow: Mathematical Methods for Physicists: A concise introduction; Cambridge University Press 2000, ca. US\$75. Placed between undergraduate and beginning graduate level; covers some topics which are usually left out. Sometimes not deep enough for us. Tell me what you think of it.
- [RHB] * K.F. Riley, M.P. Hobson and S.J. Bence: Mathematical Methods for Physics and Engineering: A Comprehensive Guide; 3rd ed., Cambridge University Press 2006, ca. US\$75. Many lecturers base their course on this book as pedagogical alternative to [AW]. A Students Solutions Manual for evennumbered solutions comes separately and is very valuable for self-study, ca. US\$25.

More texts with many of the topics which are also prerequisites of or covered by this course

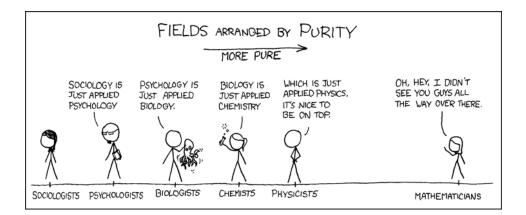
- [Boa] M.L. Boas: Mathematical Methods in the Physical Sciences; 3rd ed., John Wiley 2005, ca. US\$120.
- [Has] S. Hassani: Mathematical Methods for Students of Physics and Related Fields; Springer 2000, ca. US\$110. Likes mathematical abstractions.
- [Sha] * R. Shankar: Basic Training in Mathematics A Fitness Program for Science Students; Springer 2008, ca. US\$54. An excellent self-study guide of the material which we take for granted in this course – except for Sects. 6 (complex analysis) and 10.5-7 (partial diff. eq.'s).

Classical texts which are also important as resource/reference

- [CH] R. Courant and D. Hilbert: Methods of Mathematical Physics (2 volumes); reprint by John Wiley 1991, ca. 2×US\$145.
- [MF] P.M. Morse and H. Feshbach: Methods of Theoretical Physics (2 volumes); priceless.

Reference Tables and Collections of Formulae for reference in the University library. You will not need this in the course, you might not even need it ever – but if you do, you will need it desperately.

- [NIST] F.W.J. Olver and D.W. Lozier (eds.; GW contribution): NIST Handbook of Mathematical Functions; Cambridge University Press 2010, ca. US\$95 (with CD-ROM). Not an integral table but a collection of techniques and properties (without proofs). While descriptions and formulae are surpassed by [GR], valuable for quick-references: plots of functions, descriptions of numerical techniques, and summaries of functions which Physicists use frequently but which Mathematicians do not consider to be fundamental, e.g. spherical harmonics and 3j, 6j, 9j-symbols of angular momentum coupling. Digital version at http://dlmf.nist.gov/.
 - [AS] M. Abramowitz and I.A. Stegun (eds.): Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables; Dover 1965 and later, ca. US\$35. Started as a log-table, precursor of [NIST].
 - [Bat] Bateman Manuscript Project (A. Erdelyi et al.): *Higher Transcendental Functions* (3 vol.); *Tables of Integral Transforms* (2 vol.). Collects properties and integrals of the weirdest functions imaginable.
 - [Obe] F. Oberhettinger (and others): Tables of Indefinite Integrals, Fourier, Laplace, Bessel, Mellin Transforms (4 volumes).
 - [Pol] A.D. Polyanin (and others): Handbooks of: Exact Solutions for Ordinary Differential Equations, Linear Partial Differential Equations for Engineers and Scientists, First Order Partial Differential Equations, Integral Equations (4 separate books). If it's not in there, it's nowhere.
 - [PB] A.P. Prudnikov and Yu.A. Brychkov (and others): *Integrals and Series* (5 volumes, plus a volume Integral Transforms of Generalized Functions). If it's not in there, it's nowhere.



A note on academic integrity: You like Physics, or you would not be here. Thus, it is trivial that you will abide by the GW Code of Academic Integrity in all graded work. An excerpt: "Academic dishonesty is defined as cheating of any kind, including misrepresenting one's own work, taking credit for the work of others without crediting them and without appropriate authorization, and the fabrication of information." For the remainder of the code, see: http://studentconduct.gwu.edu/code-academic-integrity. I will deal with violations according to the Code.

A breach of academic integrity is a serious issue. The Scientific Method relies on the faith of the scientific community that the findings of its members are not fraudulent or forged. Reserachers may be wrong or sloppy, but we inherently trust they try their best to do a good job. Every researcher builds that reputation during a whole academic life – in graduate school, postdoctoral research, and thereafter. In recommendation letters, the department, your thesis advisor and collaborators all put their reputation at stake to endorse you. They all trust you. When that trust is broken, the Scientific Community feels violated and offended. It takes the only and strongest remedy: ostricisation, i.e. banishment from the scientific discourse; see e.g. wikipedia articles "Jan-Hendrik Schön", "Victor Ninov", "Andrew Wakefield".

Academic integrity is at the heart of your credibility as scientist. It is your most valuable asset. Do not risk it.

You are encouraged to collaborate on your homework and even to be inspired by a good textbook, but make sure you have understood what you hand in as your solution. Do *not* offend your own (and my) intelligence by copying other people's work (especially without referencing). The web-site, all problems and solutions are for your personal use only. Do not pass solutions or problems on to any student who has not taken the course (yet). Do not accept or solicit solutions from students who have taken the course. Other examples of a breach of academic integrity include: to facilitate cheating or help others to cheat; to obtain information for homework, exams, presentations, etc., by means other than disclosed in your bibliography; to ask for or give any kind of factual information which is not in an exam but needed to solve the problem, no matter how insignificant it may seem, except if the examiner approves; etc.

Noncompliance with these rules is a breach of integrity and will be dealt with accordingly. If you have any questions about what constitutes academic dishonesty, ask.

Absences and Excuses follow standard GW policy. It is your own responsibility to make sure you fulfil the criteria for passing, in particular that you get at least 50% of all the points available in all Problem sheets together (not per sheet). The only way around this criterion is to submit in writing documentation that you were unable to perform homework for more than half the semester due to reasons out of your control, as outlined in the GW policy on absences and excuses.

There will be no make-up exams. A missed exam will be dealt with case-by-case. Bring any potential conflicts or difficulties to my attention *before* the exam. If you miss an exam for some unexpected reason, it is your responsibility to notify in writing *within 24 hours* of the missed exam, or the grade will be zero for the missed exam. Absence for medical reasons must have formal, written documentation from the medical office providing care. DC traffic is no excuse, and no additional time will be provided for late-comers.

If you see a conflict between religious observances and the class and exam schedule, you will bring them to my attention in advance, in the first week of the semester. It is University policy to extend to these students the courtesy of absence without penalty on such occasions, including permission to make up examinations.

Recording Taking written notes in lectures has been the practise for centuries and is of course permitted. Indeed, I encourage you to at least have the manuscript printed out and annotate it vigorously during class. Taking notes makes sure your brain processes and stays awake.

However, *electric or electronic recording* of the lecture or any discussion in any way, shape or form (audio, video with or without sound, camera, photography, shellack, casette, dictaphone etc.) *needs my prior written permission*, as well as prior permission by all students. I consider failing to obtain permission as a breach of GW's Students Rights and Responsibilities and Academic Freedoms policies. It also infringes on my and GW's copyright. I will call GW police, refer violators to the appropriate authorities for sanctions, and advocate for the strongest-possible prosecution permitted by criminal and civil law.

Security In the case of an emergency, if at all possible, the class should shelter in place. If the building that

the class is in is affected, follow the evacuation procedures for the building. After evacuation, seek shelter at a predetermined rendezvous location.

Disability Support Services (DSS): http://counselingcenter.gwu.edu/ Any student who feels that man accommodation may be needed based on the potential impact of a disability should contact me privately to discuss specific needs. Please also contact the Disability Support Services office at 202-994-8250 in Rome Hall, Suite 102, to establish eligibility and to coordinate reasonable accommodations. For additional information, please refer to: http://disabilitysupport.gwu.edu/.

GW's Mental Health Services (202-994-5300) offers 24/7 assistance and referrals to address students' personal, social, career, and study skill problems. Services for students include:

Crisis and Emergency Mental Helth Consultations 202-994-5300 24 hours, not only for emergency.
Confidential assessment, counseling (individual & small group), referrals.



IT'S IN THE SYLLABUS

This message brought to you by every instructor that ever lived.

Some Hints for a Successful Graduate Life

This is by no means a complete list, but it helps me in my teaching and research. Use your own judgement!

Follow simple tricks to check that your homework solutions are focused, understandable and correct. You do not have to type your homework for this course using LATEX, but Dr. Haberzettl collected some simple but highly efficient tricks how to structure your solution, how to avoid errors and which cross-checks are very valuable. Citation with minor modifications:

- Indicate each problem number and the parts according to the original questions in your solutions. Follow them in sequence. If you do not have an answer for any part, write "no answer" or leave blank. Do not repeat the question.
- Structure your document by paragraphs; do *not* run all parts together in one big amorphous narrative. Be concise. When necessary for cross-referencing, number the equations. Clearly identify your answers to the specific questions of the problem.
- Clearly explain your notation, what you are doing, and why you are doing it. Otherwise, a teacher needs to guess and you have no recourse if that guess is wrong. "But it's obvious that I meant..." is not a valid excuse. Even a Professor of Physics is not psychic, and what is obvious to you may not be obvious to me.
- Equations that mix incompatible mathematical entities are wrong! For example, clearly distinguish between vectors and scalars. " $\vec{F} = ma$ " is wrong because it equates a vector to a scalar.
- Always check whether your results have the right units. This is a very simple, but extremely effective, method to avoid many mistakes. If you make a habit of it, it becomes subconscious. For example, "[kg]=[N m³]" must be wrong.

Attack your homework early. Don't postpone it to the last few days or even minutes.

Get a first impression. Explain to you (or, better, your peers) in your own words what the problem is about and what you are asked to determine. Avoid using formulae, focus on a "story". Sketch a figure explaining the physical situation.

Make a plan of attack. First think what solution you expect from your physical intuition. This can include a sketch of the expected solution. Then ponder over a good way to find the solution. This can take even an hour. Then take a deep breath. Then think again about the problem. Then solve. The time spent on first thinking about the solution is much shorter than the time wasted with abandoned attempts when you instantly start scribbling. In particular in exams.

Form teams (see above). Nothing helps one to understand better than discussing homework and lectures with peers. But practise additional problems alone in order not to become dependent on others.

Rank craftsmanship over ingenuity. You will be outstanding soon enough, but for now, continuous, solid work is more reliable than occasional sparks of brilliance.

Don't get nailed-down. Nobody requires you to find the *best/most elegant/fastest* solution. Any solution will do for a start. Once you have one, you can always look for a better one – if you have the time. When stuck, discuss with your peers (and consult the lecture and books). If you get very stuck, do another problem first. It's no use to get no problem done because you wasted all your resources on the first one.

Practise sketching and plotting. Discussions, sketches and plots are a must! Not only because the homework is full of these words, and you will loose a lot of points if you do not discuss, sketch and plot. But human beings are visual beings: We understand and recollect much better when we see a figure.

Assess your answer. Does the result make sense? Compare to limits in which the problem simplifies or in which you know the solution. Does the answer match your expectations (see "first impression")? If not, why not? Check that you answered all questions in the problem description.

Scrutinise your homework when it is returned to you and reproduce a correct solution. Clean up your notes. What did you not understand? What did you miss? Was there a faster way? Where are your strengths and weaknesses? You should spend at least an hour on that, as soon as possible. It will help you with the next homework set.

Work through each lecture on the day it is delivered. If you miss that, you will have a very hard time to understand the next lecture. In that context, "Tomorrow will be another day" is a very bad motto.

"Fill in the gaps" of the lecture. Spell out the details of proofs, make sure the signs and factors are correct, etc. That already gives you a lot of free practise in math, and makes sure your thinking and notes are up-to-date and correct. And you have a set of notes you understand when you come back after weeks or months or years, for exams or research.

Consult books (plural!) after you have reviewed the lecture. It will clarify things further, show you new and different perspectives, and deepen your understanding. I usually excerpt information which I found interesting in a book, in addition to lecture notes.

"It is always useful to get a second viewpoint because it's commonly the second one that makes sense – in whichever order you read them." [Nearing: Mathematical Tools for Physics, p. vii]

Look at the Physics behind a formula. Does it make "sense" from your physical intuition? Do you understand what it means? What are its limits, i.e. regimes where it becomes particularly simple to understand? What are its limitations, i.e. where does it not work? Explain it and its underlying principle to a peer or to an undergraduate, using no math. You will believe your most beautiful mathematical proof only if you can also give a good intuitive argument why the formula should be right.

Ask yourself: What is the hidden agenda behind this topic in the lecture, homework, etc.? What can I learn that goes beyond the straightforward application? Is there a greater principle involved which I can use in different contexts? Why is e.g. a proof presented this way? In which other fields could I use similar techniques/reasoning?

Talk with your lecturers. We post out office hours not out of courtesy, and we don't bite. If you don't come to me with your problems, how can we help you? we – for one – love discussing. Have no fear to overburden us. We will tell you when we have had enough.

Have a life outside Physics.

The University Counseling Center (UCC) assists you in addressing personal, social, career, and study problems that can interfere with your academic progress and success. Services for students include:

- Crisis Consultations at 202-994-5300 open day and night, not only for emergency.
- Confidential assessment, counseling services (individual and small group), and referrals:
- http://counselingcenter.gwu.edu/counseling
- Academic Support and Peer Tutoring Services: http://gwired.gwu.edu/counsel/AcademicSupport
- Podcasts and Self-Help: http://gwired.gwu.edu/counsel/PodCast,
- http://gwired.gwu.edu/counsel/OutreachSelfHelp

They are also very good when you need to review your habits, like learning and exam strategies. It's never too early to get help.