

**Additional Practise Sheet: Functional Analysis**

Completely voluntary.

If you want, we can discuss your solutions in the **Final Question Time of the semester**.

No extra points are awarded – the values are only meant as grade of difficulty here.

1. DIRAC'S  $\delta$ -DISTRIBUTION (**9P**): Prove all of its properties on the handout.
2. FOURIER TRANSFORMS (**9P**): Prove all of its properties on the handout.
3. (**3P**) Consider a Gaussian,  $f(x) = \exp -x^2/(2\sigma_x^2)$ , with width  $\sigma_x$ . When you calculate its Fourier transform, you will find that you get another Gaussian in momentum space, but with a different width  $\sigma_k$ . What is it? Make a connection to HEISENBERG'S UNCERTAINTY RELATION when you interpret  $f(x)$  as a wave-packet in coordinate space.