Errata for Real Analysis Text

11/13/2018

- **7** Exercise 1(e): Add the condition $c \neq 0$.
- **20** Example 1.5.4: $B(n+1)^3 - n^2$ should be $B(n+1)^2 - n^2$.
- **34** Exercise 9: Add the condition $k > 1$.
- **37** Exercise 9: Add the condition $a_1 \geq \sqrt{r}$.
- **51** Theorem 3.1.11: In statement, replace the inequality $|x - y| < \delta$ by $|x - a| + |y - a| < \delta$. In proof, delete phrase "$|a_n - b_n| \to 0$ so".
- **64** Line 2: delete the word "below".
- **66** Exercise 3.4.5: Omit part (b) and delete solution on 526.
- **66** Exercise 3.4.7(f): Change $2x^2 - 5$ to $2x^2 - 4.8$.
- **66** Exercise 3.4.8: Change $\mathbb{R}$ to $(0, +\infty)$.
- **82** Exercise 4.2.7: Add phrase "on the interval $(-\pi/2|a|, \pi/2|a|)$" at end.
- **100** Exercise 4.5.21: Replace the interval $(0, 1)$ by $(-1, 1)$.
- **109** Last line of proof of 5.1.7: Last phrase should read 'infimum over $\mathcal{P}$ and then the supremum over $\mathcal{Q}$".
- **109** Line 7 from bottom: Replace $\overline{S}(f, \mathcal{P}_\varepsilon) - \overline{S}(f, \mathcal{P}_\varepsilon) < \varepsilon$ by $\overline{S}(f, \mathcal{P}_\varepsilon) - \underline{S}(f, \mathcal{P}_\varepsilon) < \varepsilon$.
- **110** Line 5 from bottom: Replace $\Delta x_j < r$ by $\Delta x_j = r$.
- **121** In second sentence of the proof of (b), replace $F = f' = G$ by $F' = f = G'$..
- **124** Example 5.3.4: Add "for $k > 1$" after "We show that".
- **125** Example 5.3.5: Last term in square brackets should be $\dfrac{6}{5^4}$.
- **125** Example 5.3.5: Table caption should read "...$\int (x+1)^3 e^{5x}\, dx$..."
- **126** Exercise 5.3.2: Replace the phrase "Prove that $f' \in \mathcal{R}_a^b$ " by "Prove that the continuous extension of f' to $[a, b]$ satisfies".
- **133** Exercise 5.5.1: Integral coefficients a, b on the right should be $\sqrt{a}, \sqrt{b}$.
- **133** Exercise 5.5.7: Second integral on the right should be $\int_c^b g$.

- **146** Theorem 5.7.9: Limit is from the left: $\lim_{x\to b-} f(x)/g(x)$.
- **147** Theorem 5.7.11: b should be ∞ throughout.
- **149** Exercise 5.7.4: add the phrase "and $\int_1^\infty f$ and $\int_1^\infty g$ converge"
- **151** Exercise 5.7.27: g should be bounded.
- **155** Equation (5.31): $S(f', \mathcal{P}, \boldsymbol{\xi})$ should be $S(|f'|, \mathcal{P}, \boldsymbol{\xi})$.
- **157** Line 3: Remove the paragraph starting with "Conversely," and add "Conversely, if f is continuous at 1, then for small $\|\mathcal{P}\|$ the quantities $f(\xi_n)$ approximate $f(1)$, so $f \in \mathcal{R}_0^1(w)$."
- **175** Exercise 6.2.12: Start the problem with the sentence "Let $0 < a_n < r$, where r is sufficiently small."
- **186** Exercise 3: Add the hypothesis $b_n \downarrow 0$.
- **187** Exercise 4(r): Replace $(1)^n$ by $(-1)^n$.
- **193** Definition 7.1.1: Limit should be $f = \lim_n f_n$.
- **194** Example 7.1.6(e): Replace $\max\{|a|, ||b\}$ by $n^{-1}\max\{|a|, |b|\}$.
- **195** Theorem 7.1.9(d): Convergence should read $\dfrac{1}{g_n} \to \dfrac{1}{g}$.
- **198** Exercise 7.1.8: Delete last sentence (regarding example).
- **199** Exercise 7.1.16: Replace (b)^{s} by (b).
- **210** Exercise 12: Intervals should be closed as well.
- **212** Theorem 7.4.2: Replace $c_n > 0$ by $c_n \neq 0$.
- **244** Example 8.3.7: The inequality defining A should be $0 < x \leq 2/\pi$.
- **264** Inequality (8.6) should read $\rho(f_n(x_n), f_n(a)) \geq \varepsilon$.
- **270** Corollary 8.7.6: Last line of proof: Change A to B.
- **271** Theorem 8.7.8: Change α to φ in proof.
- **293** Exercise 9.1.3(f): The two part definition should be for $x \neq y$ and $x = y$.
- **296** Third line from top: Change ≤ 1 to $\leq \|T\|$.
- **304** Exercise 4: Replace right side by $\left[r\dfrac{\partial z}{\partial x}\right]^2 + \left[r\dfrac{\partial z}{\partial y}\right]^2$.
- **305** Exercise 9: $\nabla(f \circ \alpha) = \alpha'$ should read $(\nabla f) \circ \alpha = \alpha'$.

- **325** In Equation (9.26) replace $h^{m_1} \cdots h^{m_n}$ by $h_1^{m_1} \cdots h_n^{m_n}$.
- **329** Exercise 7: $P(x \pm 1)$ should read $P(x \pm 1, y \pm 1)$.
- **350** 2nd line: Inequality should read $|J_k| < |I_k| + \varepsilon/2^k$.
- **351** Exercise 3: Rephrase as follows:
  Let $\mathcal{U}$ be the collection of all bounded open subsets of $\mathbb{R}$ and $\mathcal{K}$ the collection of all compact sets. Prove that $\lambda^*(A)$ equals each of the following:
  (a) $\inf \left\{ \sum_j \lambda^*(U_j) \,:\, U_j \in \mathcal{U} \text{ and } \bigcup_j U_j \supseteq A \right\}$.
  (b) $\inf \left\{ \sum_j \lambda^*(K_j) \,:\, K_j \in \mathcal{K} \text{ and } \bigcup_j K_j \supseteq A \right\}$.
- **352** Replace line 16 from top by: Since $\big(C \cap E\big) \cup \big(C \cap E^c \cap F\big) = C \cap (E \cup F)$, by (10.4),
- **353** Replace the dyadic expansions .00220 . . . and .22202 . . . in Figure 10.3 by .0220 . . . and .2202 . . ., respectively.
- **354** Replace the dyadic expansions .00220 . . . and .22202 . . . below Figure 10.4 by .0220 . . . and .2202 . . ., respectively.
- **354** Line 12 from bottom: Replace "(Figure 11.2)" by "(Figure 10.3)".
- **364** Middle: Replace $A := \bigcup_{n=1}^{\infty} A_k$ by $A := \bigcup_{k=1}^{\infty} A_k$.
- **364** First line of Proposition 10.5.12: replace "be" by "is".
- **377** Ex. 2: Replace 2/3 by 3/2.
- **378** Ex. 11: Replace $\leq$ by $\geq$.
- **388** Begin Eqn. (11.11) with $\int_{\mathbb{R}^n} f(\mathbf{z}, \boldsymbol{y})\, d\lambda(\boldsymbol{z}, \boldsymbol{y}) =$
- **517** Sec. 1.2.1(f): Replace $bc(bd^{-1})$ by $bc(bd)^{-1}$.
- **518** Ex. 10: Replace $x = a - b$ by $x = (a - b)/2$.
- **520** Sec. 2.1, Ex. 3: Replace min by max.
- **521** Sec. 2.1, Ex. 17: Replace a^n by a_n.
- **521** Sec. 2.1, Ex. 22: Replace djx by $d + jx$.
- **531** Sec. 4.5, Ex. 1(s): Limit $= +\infty$ in all cases.
- **541** The solution to 6(a) should read: Diverges for all p.
- **552** Sec. 8.3, Ex. 3(b): Add the point $(1, 0, 1)$.

- **558** Sec. 9.2, Ex. 3: Equation should be $\nabla(\psi^{-1}) = -\psi^{-2}\nabla\psi$.
- **561** Ex. 9.7.1(b): Last term should be $\dfrac{\partial^3 f}{\partial y^3}\,(dy)^3$.
- **566** Ex. 10.3.4: Replace n's by k's.

Solution Manual

- **4** Indices j and k in problem 7 should start at 0.
- **19** Should have $a_n := (1 - 1/n^2)^n = (1 - 1/n)^n(1 + 1/n)^n$.
- **20** Answer to 1(b) should be -1, 0, 2.
- **26** Replace the solution to 3.1.10 with the following:

  Assume $\lim_{x\to c} f(x)$ does not exist. Suppose first that $\lim_{x\to c^-} f(x)$ does not exist. Then, by the Cauchy criterion for functions, there exist sequences of points x_n, $y_n < c$ with x_n, $y_n \to c$, and $\varepsilon > 0$ such that $|f(x_n) - f(y_n)| \geq \varepsilon$. for all n. Construct a subsequence $\{x'_n\}$ of $\{x_n\}$ such that $x'_n \uparrow c$. Let $\{y'_n\}$ denote the corresponding subsequence of $\{y_n\}$, and choose a subsequence $\{y''_n\}$ such that $y''_n \uparrow c$. Let z_n be any sequence with $z_n \downarrow c$. Then not both $f(x''_n) - f(z_n) \to 0$ and $f(y''_n) - f(z_n) \to 0$. This proves the assertion for the case that $\lim_{x\to c^-} f(x)$ does not exist. The proof for $\lim_{x\to c^+} f(x)$ is similar. Finally, suppose $\lim_{x\to c^-} f(x)$ and $\lim_{x\to c^+} f(x)$ both exist but are unequal. Then choose any $a_n \uparrow c$ and $b_n \downarrow c$.
- **31** Delete (b). Replace the solution of (a) with the following:
  For example on an interval $[(k+1)^{-1}, k^{-1}$, let a_n strictly decrease to $(k+1)^{-1}$, where $a_1 = 1/k$. Consider the polygonal graph obtained by connecting $(a_1, 0)$ to $(a_2, 1)$ to $(a_3, 0)$, etc.
- **45** Replace solution to Exercise 4.5.3(c) by the following:
  $a_n = \dfrac{(1+1/n)^n - e}{1/n}$ is of the form $\frac{0}{0}$ hence has the same limit as

  $$-n^2 \frac{d}{dn}(1+1/n)^n = (1+1/n)^n \left[n/(1+1/n) - n^2 \ln(1+1/n)\right].$$

  The first factor tends to e. The second factor may be written

  $$\frac{(n+1)^{-1} - \ln(1+1/n)}{n^{-2}},$$

  which has the same limit as

  $$\frac{-(n+1)^{-2} + n^{-2}/(1+n^{-1})}{-2n^{-3}} \to -1/2.$$

  Therefore, $a_n \to -e/2$.
- **55** Replace solution to Exercise 8 by the following:
  (a) Let x_0 be a continuity point of f with $f(x_0) > 0$. Choose $\delta > 0$ such that $f(x) > f(x_0)/2$ for $x_0 - 2\delta < x < x_0 + 2\delta$. Let g be a continuous function that has value 1 on $[x_0 - \delta, x_0 + \delta]$, 0 on the complement of $[x_0 - 2\delta, x_0 + 2\delta]$ and linear in between. Then $\int_a^b fg > 0$.

  (b) Similar to (a).

- **57** In 5.3.2, delete phrase "The desired result follows by summing on j."
- **72** The solution to 6(a) should read: Diverges for all p.
- **88** The numerator in (†) should be $(1 - x^2/n^2)$.
- **157** Replace solution in 4(b) by the following:
  This follows by taking $K = C_r(\boldsymbol{x}_0)$ in (a) and noting that by continuity $f\big(\boldsymbol{x}_K\big) \to f(\boldsymbol{x}_0)$ as $r \to 0$.
- **110** Replace solution of 14(b) by the following:
  Let $\boldsymbol{x} \in \mathcal{X}$ and $r > 0$. Suppose that $B_r(\boldsymbol{x}) \setminus \{\boldsymbol{x}\}$ contains only finitely many $\boldsymbol{a}_n$, say $\boldsymbol{a}_{n_1}, \ldots, \boldsymbol{a}_{n_k}$. Set $s = \min\{\|\boldsymbol{x} - \boldsymbol{a}_{n_1}\|, \ldots, \|\boldsymbol{x} - \boldsymbol{a}_{n_k}\|\}$. Then $s > 0$ so and $B_s(\boldsymbol{x}) \setminus \{\boldsymbol{x}\}$ is a nonempty open set disjoint from $\{\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots\}$ contradicting the hypothesis. Therefore, $B_r(\boldsymbol{x})$ contains infinitely many $\boldsymbol{a}_n$.
- **127,8** Solution to 4(b) should read
  $dg_{\boldsymbol{x}}(\boldsymbol{h}) = \dfrac{\|\boldsymbol{x}\|^2\boldsymbol{h} - (2\boldsymbol{x} \cdot \boldsymbol{h})\boldsymbol{x}}{\|\boldsymbol{x}\|^4}$ and $dg_{\boldsymbol{x}}(\boldsymbol{x}) = -\dfrac{\boldsymbol{x}}{\|\boldsymbol{x}\|^2}$.
- **130** Second line of solution to 11:
  Switch the matrices $[z_u, z_u]$ and $[z_x, z_y]$ in the first equality.
- **151** Ex. 10.3.4 : Replace n's by k's.