Errata for Real Analysis Text

11/13/2018

- **7** Exercise 1(e): Add the condition \( c \neq 0 \).
- **20** Example 1.5.4: \( B(n + 1)^3 - n^2 \) should be \( B(n + 1)^2 - n^2 \).
- **34** Exercise 9: Add the condition \( k > 1 \).
- **37** Exercise 9: Add the condition \( a_1 \geq \sqrt{r} \).
- **51** Theorem 3.1.11: In statement, replace the inequality \( |x - y| < \delta \) by \( |x - a| + |y - a| < \delta \). In proof, delete phrase “\(|a_n - b_n| \to 0\) so”.
- **64** Line 2: delete the word “below”.
- **66** Exercise 3.4.5: Omit part (b) and delete solution on 526.
- **66** Exercise 3.4.7(f): Change \( 2x^2 - 5 \) to \( 2x^2 - 4.8 \).
- **66** Exercise 3.4.8: Change \( \mathbb{R} \) to \((0, +\infty)\).
- **82** Exercise 4.2.7: Add phrase “on the interval \((-\pi/2|a|, \pi/2|a|)\)” at end.
- **100** Exercise 4.5.21: Replace the interval \((0, 1)\) by \((-1, 1)\).
- **109** Last line of proof of 5.1.7: Last phrase should read “infimum over \( P \) and then the supremum over \( Q \)”.
- **109** Line 7 from bottom: Replace \( \mathcal{S}(f, P_\varepsilon) - \mathcal{S}(f, P_\varepsilon) < \varepsilon \) by \( \mathcal{S}(f, P_\varepsilon) - \mathcal{S}(f, P_\varepsilon) < \varepsilon \).
- **110** Line 5 from bottom: Replace \( \Delta x_j < r \) by \( \Delta x_j = r \).
- **121** In second sentence of the proof of (b), replace \( F = f' = G \) by \( F' = f = G' \).
- **124** Example 5.3.4: Add “for \( k > 1 \)” after “We show that”.
- **125** Example 5.3.5: Last term in square brackets should be \( \frac{6}{5\pi} \).
- **125** Example 5.3.5: Table caption should read “\( \ldots \int (x + 1)^3 e^{5x} dx \ldots \)”.
- **126** Exercise 5.3.2: Replace the phrase “Prove that \( f' \in R^b_a \)” by “Prove that the continuous extension of \( f' \) to \([a, b]\) satisfies”.
- **133** Exercise 5.5.1: Integral coefficients \( a, b \) on the right should be \( \sqrt{a}, \sqrt{b} \).
- **133** Exercise 5.5.7: Second integral on the right should be \( \int_c^b g \).
• 146 Theorem 5.7.9: Limit is from the left: \( \lim_{x \to b^-} f(x)/g(x) \).

• 147 Theorem 5.7.11: \( b \) should be \( \infty \) throughout.

• 149 Exercise 5.7.4: add the phrase “and \( \int_1^\infty f \) and \( \int_1^\infty g \) converge”

• 151 Exercise 5.7.27: \( g \) should be bounded.

• 155 Equation (5.31): \( S(f', \mathcal{P}, \xi) \) should be \( S(|f'|, \mathcal{P}, \xi) \).

• 157 Line 3: Remove the paragraph starting with “Conversely,” and add “Conversely, if \( f \) is continuous at \( 1 \), then for small \( \|\mathcal{P}\| \) the quantities \( f(\xi_n) \) approximate \( f(1) \), so \( f \in \mathcal{R}^1_0(w) \).”

• 175 Exercise 6.2.12: Start the problem with the sentence “Let \( 0 < a_n < r \), where \( r \) is sufficiently small.”

• 186 Exercise 3: Add the hypothesis \( b_n \downarrow 0 \).

• 187 Exercise 4(r): Replace \((1)^n\) by \((-1)^n\).

• 193 Definition 7.1.1: Limit should be \( f = \lim_n f_n \).

• 194 Example 7.1.6(e): Replace \( \max\{|a|, |b|\} \) by \( n^{-1} \max\{|a|, |b|\} \).

• 195 Theorem 7.1.9(d): Convergence should read \( \frac{1}{g_n} \to \frac{1}{g} \).

• 198 Exercise 7.1.8: Delete last sentence (regarding example).

• 199 Exercise 7.1.16: Replace \( (b) \# \) by \( (b) \).

• 210 Exercise 12: Intervals should be closed as well.

• 212 Theorem 7.4.2: Replace \( c_n > 0 \) by \( c_n \neq 0 \).

• 244 Example 8.3.7: The inequality defining \( A \) should be \( 0 < x \leq 2/\pi \).

• 264 Inequality (8.6) should read \( \rho(f_n(x_n), f_n(a)) \geq \varepsilon \).

• 270 Corollary 8.7.6: Last line of proof: Change \( A \) to \( B \).

• 271 Theorem 8.7.8: Change \( \alpha \) to \( \varphi \) in proof.

• 293 Exercise 9.1.3(f): The two part definition should be for \( x \neq y \) and \( x = y \).

• 296 Third line from top: Change \( \leq 1 \) to \( \leq \|T\| \).

• 304 Exercise 4: Replace right side by \( r \left[ \frac{\partial z}{\partial x} \right]^2 + \left[ \frac{\partial z}{\partial y} \right]^2 \).

• 305 Exercise 9: \( \nabla (f \circ \alpha) = \alpha' \) should read \( (\nabla f) \circ \alpha = \alpha' \).
• 325 In Equation (9.26) replace $h^{m_1} \cdots h^{m_n}$ by $h_1^{m_1} \cdots h_n^{m_n}$.

• 329 Exercise 7: $P(x \pm 1)$ should read $P(x \pm 1, y \pm 1)$.

• 350 2nd line: Inequality should read $|J_k| < |I_k| + \varepsilon/2^k$.

• 351 Exercise 3: Rephrase as follows:
Let $\mathcal{U}$ be the collection of all bounded open subsets of $\mathbb{R}$ and $\mathcal{K}$ the collection of all compact sets. Prove that $\lambda^*(A)$ equals each of the following:

(a) $\inf \left\{ \sum_j \lambda^*(U_j) : U_j \in \mathcal{U} \text{ and } \bigcup_j U_j \supseteq A \right\}$.

(b) $\inf \left\{ \sum_j \lambda^*(K_j) : K_j \in \mathcal{K} \text{ and } \bigcup_j K_j \supseteq A \right\}$.

• 352 Replace line 16 from top by: Since $(C \setminus E) \cup (C \setminus E^c \setminus F) = C \setminus (E \cup F)$, by (10.4),

• 353 Replace the dyadic expansions .00220... and .22202... in Figure 10.3 by .0220... and .2202..., respectively.

• 354 Replace the dyadic expansions .00220... and .22202... below Figure 10.4 by .0220... and .2202..., respectively.

• 354 Line 12 from bottom: Replace “(Figure 11.2)” by “(Figure 10.3)”.

• 364 Middle: Replace $A := \bigcup_{n=1}^\infty A_k$ by $A := \bigcup_{k=1}^\infty A_k$.

• 364 First line of Proposition 10.5.12: replace “be” by “is”.

• 377 Ex. 2: Replace 2/3 by 3/2.

• 378 Ex. 11: Replace $\leq$ by $\geq$.

• 388 Begin Eqn. (11.11) with $\int_{\mathbb{R}^n} f(z, y) d\lambda(z, y) =$

• 517 Sec. 1.2.1(f): Replace $bc(bd)^{-1}$ by $bc(bd)^{-1}$.

• 518 Ex. 10: Replace $x = a - b$ by $x = (a - b)/2$.

• 520 Sec. 2.1, Ex. 3: Replace min by max.

• 521 Sec. 2.1, Ex. 17: Replace $a^n$ by $a_n$.

• 521 Sec. 2.1, Ex. 22: Replace $djx$ by $d + jx$.

• 531 Sec. 4.5, Ex. 1(s): Limit = $+\infty$ in all cases.

• 541 The solution to 6(a) should read: Diverges for all $p$.

• 552 Sec. 8.3, Ex. 3(b): Add the point $(1, 0, 1)$. 

• **558** Sec. 9.2, Ex. 3: Equation should be $\nabla(\psi^{-1}) = -\psi^{-2}\nabla\psi$.

• **561** Ex. 9.7.1(b): Last term should be $\frac{\partial^3 f}{\partial y^3} (dy)^3$.

• **566** Ex. 10.3.4: Replace $n$’s by $k$’s.
Solution Manual

- 4 Indices $j$ and $k$ in problem 7 should start at 0.
- 19 Should have $a_n := (1 - 1/n^2)^n = (1 - 1/n)^n(1 + 1/n)^n$.
- 20 Answer to 1(b) should be $-1, 0, 2$.
- 26 Replace the solution to 3.1.10 with the following:
  Assume $\lim_{x \to c} f(x)$ does not exist. Suppose first that $\lim_{x \to c^-} f(x)$ does not exist. Then, by the Cauchy criterion for functions, there exist sequences of points $x_n, y_n < c$ with $x_n, y_n \to c$, and $\epsilon > 0$ such that $|f(x_n) - f(y_n)| \geq \epsilon$ for all $n$. Construct a subsequence $\{x'_n\}$ of $\{x_n\}$ such that $x'_n \uparrow c$. Let $\{y'_n\}$ denote the corresponding subsequence of $\{y_n\}$, and choose a subsequence $\{y''_n\}$ such that $y''_n \uparrow c$. Let $z_n$ be any sequence with $z_n \downarrow c$. Then not both $f(x''_n) - f(z_n) \to 0$ and $f(y''_n) - f(z_n) \to 0$. This proves the assertion for the case that $\lim_{x \to c^-} f(x)$ does not exist. The proof for $\lim_{x \to c^+} f(x)$ is similar. Finally, suppose $\lim_{x \to c^-} f(x)$ and $\lim_{x \to c^+} f(x)$ both exist but are unequal. Then choose any $a_n \uparrow c$ and $b_n \downarrow c$.
- 31 Delete (b). Replace the solution of (a) with the following:
  For example on an interval $((k + 1)^{-1}, k^{-1})$, let $a_n$ strictly decrease to $(k + 1)^{-1}$, where $a_1 = 1/k$. Consider the polygonal graph obtained by connecting $(a_1, 0)$ to $(a_2, 1)$ to $(a_3, 0)$, etc.
- 45 Replace solution to Exercise 4.5.3(c) by the following:
  $a_n = \frac{(1 + 1/n)^n - e}{1/n}$ is of the form $0/0$ hence has the same limit as
  
  $-n^2 \frac{d}{dn} (1 + 1/n)^n = (1 + 1/n)^n \left[n/(1 + 1/n) - n^2 \ln(1 + 1/n)\right]$.

  The first factor tends to $e$. The second factor may be written

  \[
  \frac{(n + 1)^{-2} - \ln(1 + 1/n)}{n^{-2}},
  \]

  which has the same limit as

  \[
  -(n + 1)^{-2} + n^{-2}/(1 + n^{-1}) \rightarrow -1/2.
  \]

  Therefore, $a_n \rightarrow -e/2$.
- 55 Replace solution to Exercise 8 by the following:
  (a) Let $x_0$ be a continuity point of $f$ with $f(x_0) > 0$. Choose $\delta > 0$ such that $f(x) > f(x_0)/2$ for $x_0 - 2\delta < x < x_0 + 2\delta$. Let $g$ be a continuous function that has value 1 on $[x_0 - \delta, x_0 + \delta]$, 0 on the complement of $[x_0 - 2\delta, x_0 + 2\delta]$ and linear in between. Then $\int_a^b fg > 0$.
  (b) Similar to (a).
• **57** In 5.3.2, delete phrase "The desired result follows by summing on $j$."

• **72** The solution to 6(a) should read: Diverges for all $p$.

• **88** The numerator in (i) should be $(1 - x^2/n^2)$.

• **157** Replace solution in 4(b) by the following:
This follows by taking $K = C_r(x_0)$ in (a) and noting that by continuity $f(x_K) \to f(x_0)$ as $r \to 0$.

• **110** Replace solution of 14(b) by the following:
Let $x \in X$ and $r > 0$. Suppose that $B_r(x) \setminus \{x\}$ contains only finitely many $a_n$, say $a_{n_1}, \ldots, a_{n_k}$. Set $s = \min\{\|x - a_{n_1}\|, \ldots, \|x - a_{n_k}\|\}$. Then $s > 0$ so and $B_s(x) \setminus \{x\}$ is a nonempty open set disjoint from $\{a_1, a_2, \ldots\}$ contradicting the hypothesis. Therefore, $B_r(x)$ contains infinitely many $a_n$.

• **127,8** Solution to 4(b) should read
$$dg_x(h) = \frac{\|x\|^2 h - (2x \cdot h)x}{\|x\|^4} \text{ and } dg_x(x) = -\frac{x}{\|x\|}. $$

• **130** Second line of solution to 11:
Switch the matrices $[z_u, z_u]$ and $[z_x, z_y]$ in the first equality.

• **151** Ex. 10.3.4 : Replace $n$’s by $k$’s.