Errata for Principles of Analysis

5/28/2019

- **47** The line in middle of page

$$\sigma(\mathcal{O}_1 \times \cdots \times \mathcal{O}_k) = \sigma(\mathcal{O}_1) \times \cdots \times \sigma(\mathcal{O}_k) = \mathcal{B}(\mathbb{R}^{d_1}) \otimes \cdots \otimes \mathcal{B}(\mathbb{R}^{d_k}).$$

  should read

$$\sigma(\mathcal{O}_1 \times \cdots \times \mathcal{O}_k) = \sigma(\mathcal{O}_1) \otimes \cdots \otimes \sigma(\mathcal{O}_k) = \mathcal{B}(\mathbb{R}^{d_1}) \otimes \cdots \otimes \mathcal{B}(\mathbb{R}^{d_k}).$$

- **65** In line 7 from the top, the sentence beginning "Then the collection" should be replaced by the following:

  If $H^\varepsilon \subseteq H$ is a closed interval with $\lambda(H^\varepsilon) > \lambda(H) - \varepsilon$, then the collection of intervals $\operatorname{int} H_j^\varepsilon$ is an open covering of H^ε, so there exists an $m \in \mathbb{N}$ such that

$$H^\varepsilon \subseteq \operatorname{int} H_1^\varepsilon \cup \cdots \cup \operatorname{int} H_m^\varepsilon \subseteq H_1^\varepsilon \cup \cdots \cup H_m^\varepsilon.$$

- **65** In Exercise 1.72, $\mathbb{R}$ should be replaced by $\mathbb{R}^d$.
- **77** Line 5 from top should read

$$\mathcal{F} := \Big\{E \subseteq X : T_i^{-1}(E) \in \bigcap\nolimits_{i \in \mathfrak{I}} \mathcal{F}_i\Big\}.$$

- **79** To Proposition 2.2.2 add the sentence: Let μ be a measure on $\mathcal{F}$.
- **84** Theorem 2.3.3 should begin: Let $(X, \mathcal{F}, \mu)$ be a measure space.
- **97** Line 14 from top: delete "the".
- **98** Replace Exercise 3.11 with the following:

  Let $f \geq 0$ be Lebesgue integrable on $\mathbb{R}^d$ and suppose that f is not zero a.e. Prove that the series $\sum_{n=1}^\infty n^{-p} f(nx)$ converges a.e. to an integrable function on $\mathbb{R}^d$ iff $p > 1 - d$.
- **107** In last line of Exercise 3.36, switch lower and upper integrals and delete $d\lambda$ in the notation.
- **111** Replace Ex. 3.57 by the following: Let $f_n : X \to [0, \infty)$ be integrable, $f_{n+1} \leq f_n$ a.e. for all n, and $f_n \to 0$ on X. Show that $\sum_n (-1)^{n+1} f_n$ is integrable and that $\displaystyle\int \Big(\sum_n (-1)^{n+1} f_n\Big)\, d\mu = \sum_n (-1)^{n+1} \int f_n \, d\mu.$
- **124** In line one of the proof of 4,1,2, the word "holds" should be replaced by the phrase "is zero".
- **130** In line 9 of proof of 4.2.2, $\lambda(U_k \setminus C_k)$ should be $\lambda^d(U_k \setminus C_k)$.
- **130** In line 8 of proof of 4.2.3, $2M^p$ should be $(2M)^p$.
- **131** Last sentence of the statement of Theorem 4.3.1 should read: In particular, $f_n \overset{L^\infty}{\to} f$ implies that $f_n \overset{\text{a.u.}}{\to} f$.
- **142** Exercise 5.3. Last equation should read $\mu^-(E) = -\mu(E \cap Q^c)$.

- **142** Exercise 5.4. Add the hypothesis $\{x\} \in \mathfrak{F}$.
- **150** Second sentence in argument of Case I: Change $\mathfrak{F}$ to $\mathscr{F}$.
- **151** Second sentence in argument of Case II: Change $\mathscr{F}$ to $\mathfrak{F}$.
- **161** In the proof of 5.5.4, line 7, the inequality should be $V_{\mathcal{P}}(f) \le V_{\mathcal{Q}}(f) + V_{\mathcal{R}}(f) + |f(x_{k+1}) - f(x_k)| \le T(x) + V_{(x,y]}(f)$. Also in line 9, the inequality should read $V_{\mathcal{Q}}(f) + V_{\mathcal{R}}(f) \le T(y)$.
- **161** In the statement of 5.5.6, add the hypothesis that f be real-valued.
- **164** Second to last sentence on page should read: Since m such sums comprise $V_{\mathcal{Q}}(f)$, $V_{\mathcal{P}}(f) \le V_{\mathcal{Q}}(f) \le m \le 1 + 2\ell/\delta$, the last inequality by (†).
- **208** The definition of C_1' should read $C_1' := \{f \in \mathscr{X}' : \|f\| \le 1\}$.
- **209** Switch $1/p$ and $1/q$ in the last expression of Exercise 8.42.
- **218** The definition of positive homogeneity should read $p(t\boldsymbol{x}) = tp(\boldsymbol{x})$, $t \ge 0$.
- **221** In the definition of bidual, replace $\boldsymbol{x} = (x_n)$ in line 3 by $\boldsymbol{x}$.
- **223** Last line of 8.62 proof: "function" should read "functions".
- **228** In the last sentence of Ex 8.71, remove the phrase "and $\mathscr{Y}$".
- **243** In the statement of Proposition 9.1.4 remove the word "balanced". Also, at the end of the statement, add the sentence,"If U is balanced, then p_U is a seminorm."
- **243** Replace the beginning of line 5 of the proof of Proposition 9.1.4 by "For homogeneity, let $c \in \mathbb{F}$, $c \ne 0$. If U is balanced, then $c^{-1}U = |c^{-1}|U$, hence". Replace line 8 by "Therefore, p_U is a Minkowski functional and a seminorm if U is balanced." Replace the last line of the proof by "By convexity, $\boldsymbol{x} = t\,\boldsymbol{x}/t + (1-t)\mathbf{0} \in U$".
- **259** In line 6, write "$y(1) = \operatorname{sgn} x_{n_1}$ and" before the equality $y(j) = \operatorname{sgn} x_{n_k}$.
- **261** In Ex. 10.5, replace $\sup \|f_n\|_\infty$ in (a) by $\sup \|f_n\|_p$.
- **263** In first line of proof of 10.2.5 replace "sequence" by "bounded sequence".
- **274** In second line of proof of 11.1.3 replace $\|t\boldsymbol{x} + \alpha\boldsymbol{y}\|$ by $\|t\boldsymbol{x} + \alpha\boldsymbol{y}\|^2$.
- **284** Third line from bottom: replace F_τ by $F_{\mathscr{T}}$.
- **286** Replace Ex. 11.25 by the following: Let $(X, \mathfrak{F}, \mu)$ be σ-finite, $\phi \in L^\infty(\mu)$, and let $M_\phi f := f\phi$ on $L^2(\mu)$. Show that the orthogonal decomposition $(\operatorname{cl}\operatorname{ran} M_\phi) \oplus (\operatorname{cl}\operatorname{ran} M_\phi)^\perp$ takes the form $f \mapsto f\phi + f(1-\phi)$ iff $\phi = \mathbf{1}_E$ for some $E \in \mathfrak{F}$. Show that in this case $\operatorname{ran} M_\phi$ is closed.
- **290** Sixth line from bottom: replace "all with $|c| = 1$" by "all c with $|c| = 1$"
- **291** Bottom line: replace $(\boldsymbol{x} \mid T_n\boldsymbol{y}) = (T_n\boldsymbol{x} \mid \boldsymbol{y}) = \frac{1}{4}\sum_{k=1}^4 i^k \left(T_n\boldsymbol{x} + i^k\boldsymbol{y} \mid T_n\boldsymbol{x} + i^k\boldsymbol{y}\right)$ by $(\boldsymbol{x} \mid T_n\boldsymbol{y}) = \left(\sqrt{T_n}\boldsymbol{x} \mid \sqrt{T_n}\boldsymbol{y}\right) = \frac{1}{4}\sum_{k=1}^4 i^k \left(T_n(\boldsymbol{x} + i^k\boldsymbol{y}) \mid \boldsymbol{x} + i^k\boldsymbol{y}\right)$
- **297** Last part of proof of 12.2.1: replace $\boldsymbol{y}_j$ by $\boldsymbol{x}_j$