

**PRINCIPLES of REAL ANALYSIS**  
**MEASURE, INTEGRATION, FUNCTIONAL ANALYSIS, APPLICATIONS**

**0 Preliminaries**

- 0.1 Sets
- 0.2 Algebraic Structures
- 0.3 Metric Spaces
- 0.4 Normed Linear Spaces
- 0.5 Topological Spaces
- 0.6 Continuity in Topological Spaces
- 0.7 Normal Topological Spaces
- 0.8 Compact Topological Spaces
- 0.9 Totally Bounded Metric Spaces
- 0.10 Equicontinuity
- 0.11 The Stone-Weierstrass Theorem
- 0.12 Locally Compact Topological Spaces
- 0.13 Spaces of Differentiable Functions
- 0.14 Partitions of Unity
- 0.15 Connectedness

**I Measure and Integration**

**1 Measurable Sets**

- 1.1 Introduction
- 1.2 Measurable Spaces
- 1.3 Measures
- 1.4 Complete Measure Spaces
- 1.5 Outer Measure and Measurability
- 1.6 Extension of a Measure
- 1.7 Lebesgue Measure
- 1.8 Lebesgue-Stieltjes Measures
- \*1.9 Some Special Sets

**2 Measurable Functions**

- 2.1 Measurable Transformations
- 2.2 Measurable Numerical Functions
- 2.3 Simple Functions
- 2.4 Convergence of Measurable Functions

### **3 Integration**

- 3.1 Construction of the Integral
- 3.2 Basic Properties of the Integral
- 3.3 Connections with the Riemann Integral on  $\mathbb{R}^d$
- 3.4 Convergence Theorems
- 3.5 Integration against a Product Measure
- 3.6 Applications of Fubini's Theorem

### **4 $L^p$ Spaces**

- 4.1 Definition and General Properties
- 4.2  $L^p$  Approximation
- 4.3  $L^p$  Convergence
- \*4.4 Uniform Integrability
- \*4.5 Convex Functions and Jensen's Inequality

### **5 Differentiation**

- 5.1 Signed Measures
- 5.2 Complex Measures
- 5.3 Absolute Continuity of Measures
- 5.4 Differentiation of Measures
- 5.5 Functions of Bounded Variation
- 5.6 Absolutely Continuous Functions

### **6 Fourier Analysis on $\mathbb{R}^d$**

- 6.1 Convolution of Functions
- 6.2 The Fourier Transform
- 6.3 Rapidly Decreasing Functions
- 6.4 Fourier Analysis of Measures on  $\mathbb{R}^d$

### **7 Measures on Locally Compact Spaces**

- 7.1 Radon Measures
- 7.2 The Riesz Representation Theorem
- 7.3 Products of Radon Measures
- 7.4 Vague Convergence
- \*7.5 The Daniell-Stone Representation Theorem

## II Functional Analysis

### 8 Banach Spaces

- 8.1 General Properties of Normed Spaces
- 8.2 Bounded Linear Transformations
- 8.3 Concrete Representations of Dual Spaces
- 8.4 Some Constructions
- 8.5 Hahn-Banach Extension Theorems
- \*8.6 Applications of the Hahn-Banach Theorem
- 8.7 Baire Category in Banach Spaces
- \*8.8 Applications
- 8.9 The Dual Operator
- 8.10 Compact Operators

### 9 Locally Convex Spaces

- 9.1 General Properties
- 9.2 Continuous Linear Functionals
- 9.3 Hahn-Banach Separation Theorems
- \*9.4 Some Constructions

### 10 Weak Topologies on Normed Spaces

- 10.1 The Weak Topology
- 10.2 The Weak\* Topology
- 10.3 Reflexive Spaces
- \*10.4 Uniformly Convex Spaces

### 11 Hilbert Spaces

- 11.1 General Principles
- 11.2 Orthogonality
- 11.3 Orthonormal Bases
- 11.4 The Hilbert Space Adjoint

### 12 Operator Theory

- 12.1 Classes of Operators
- 12.2 Compact Operators and Operators of Finite Rank
- 12.3 The Spectral Theorem for Compact Normal Operators
- \*12.4 Hilbert-Schmidt Operators
- \*12.5 Trace Class Operators

### 13 **Banach Algebras**

- 13.1 Introduction
- 13.2 Spectral Theory
- 13.3 The Spectrum of an Algebra
- 13.4 Gelfand Theory
- \*13.5 The Non-unital Case
- 13.6 Operator Calculus

### 14 **Miscellaneous Topics**

- 14.1 Weak Sequential Compactness
- 14.2 Weak Compactness in  $L^1$
- 14.3 Convexity and Compactness
- 14.4 Extreme Points
- 14.5 Applications of the Krein-Milman Theorem
- 14.6 Vector-Valued Integrals
- 14.7 Choquet's Theorem

## III **Applications**

### 15 **Distributions**

- 15.1 General Theory
- 15.2 Operations on Distributions
- 15.3 Distributions with Compact Support
- 15.4 Convolution of Distributions
- 15.5 Tempered Distributions
- 15.6 Sobolev Theory

### 16 **Analysis on Locally Compact Groups**

- 16.1 Topological Groups
- 16.2 Haar Measure
- 16.3 Some Constructions
- 16.4 The  $L^1$ -Group Algebra
- 16.5 Representations
- 16.6 Locally Compact Abelian Groups

### 17 **Analysis on Semigroups**

- 17.1 Semigroups with Topology
- 17.2 Weakly Almost Periodic Functions

- 17.3 Almost Periodic Functions
- 17.4 The Structure of Compact Semigroups
- 17.5 Strongly Almost Periodic Functions
- 17.6 Semigroups of Operators

## **18 Probability Theory**

- 18.1 Random Variables
- 18.2 Independence
- 18.3 Conditional Expectation
- 18.4 Sequences of Independent Random Variables
- 18.5 Discrete-Time Martingales
- 18.6 General Stochastic Processes
- 18.7 Brownian Motion
- 18.8 Stochastic Integration
- 18.9 An Application to Finance

## **IV Appendices**

### **A Change of Variables Theorem**

### **B Separate and Joint Continuity**