

GWU Department of Mathematics
Topics in Logic: Axiomatic Set Theory
CRN 44734 Math 6720: Topics in Logic
Spring 2019
MW 3:45–5:00p.m.
Phillips Hall (801 22nd Street), Room 730

- **Professor**

Valentina Harizanov

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Office Hours: M 5:15–6:45pm

W 12:30–2:00pm

At other times by appointment.

- **Mathematics department**

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- **Description**

Naive set theory was founded by Georg Cantor who defined a set as a “collection into a whole of definite, distinct objects of our intuition or our thought.” However, this definition allows the existence of some unusual sets that lead to paradoxes. The paradoxes leave set theorists with the task of determining which properties do define sets. Unfortunately, Kurt Gödel’s results indicate that a complete answer to this question is not even possible. Therefore, axiomatic set theory attempts a less lofty goal. It formulates some of the relatively simple properties

of sets, used by mathematicians, as axioms. Within this axiomatic system, practically all notions of contemporary mathematics can be defined and their properties can be derived. In this sense the axiomatic set theory serves as a foundation of mathematics.

Cantor proved that there are infinitely many infinities. He showed that, while there are as many rational numbers as natural numbers, there are more real numbers than natural numbers. Is there an intermediate infinity? A negative answer to this question is known as the *continuum hypothesis*. In 1963, Paul Cohen obtained a surprising result, which was “rather unsatisfactory to an average mathematician,” by establishing that the continuum hypothesis is *independent*, that is, neither provable nor refutable from the usual set-theoretic axioms. Another mathematical principal which the usual set-theoretic axioms fail to settle is the *axiom of choice*. The independence results use the *forcing technique* for which Cohen won the Fields Medal.

- **Textbook**

Set Theory by K. Kunen, Elsevier (series: Studies in Logic and the Foundation of Mathematics, vol. 102).

Supplementary book: *Classic Set Theory* (for Guided Independent Study) by D. Goldrei, Chapman & Hall/CRC.

Other reading material will be provided in class.

- **Required background**

Math 2971 or an equivalent.

Math 6720 can be taken for credit repeatedly.

Advanced **undergraduate** students may also take this course for credit.

- **Learning goals**

As a result of completing this course students should be able to:

1. Establish foundations of mathematics based on axiomatic set theory.
2. Apply axioms of Zermelo-Frankel set theory to develop familiar mathematical concepts and results;

3. Design and analyze various set-theoretic models satisfying certain axioms or their negations;
4. Discuss and establish the independence results.

- **Grading**

Based on class participation (10%); take-home assignments (50%), midterm project (20%), take-home final exam (20%), and their in-class presentations.

- **Take-Home Assignment Policy**

Working in groups to discuss problems on the take-home assignments is not only permitted but is encouraged. However, the final submitted answers must be your own work, composed by you without copying from any source. You will also be asked to give in-class presentation of the solutions to selected problems in the assignments.

- **Attendance/Absence Policy**

Students are expected to attend all lectures, and are responsible for learning all material covered in lectures. If you must be away from class on a given day, please contact the instructor before the class and explain the need for your absence. In such a situation, your absence can be excused. Work missed owing to excused absence will not count against a student's grade. (Still, students are responsible for learning the material that is covered, even if they cannot be in class.)

- **Work Outside Class**

More than $2/3$ of the time you devote to this class should take place outside the classroom. Even the best students in the class should plan on spending an average of at least 6 hours a week on assignments and other studying. Some students may need to spend more time in order to master the material.

- **Blackboard**

Announcements and relevant information will be posted on blackboard: <https://blackboard.gwu.edu/>

Use your Net Id and password to access blackboard.

- **Academic Integrity Code**

Academic dishonesty is defined as cheating of any kind, including misrepresenting one's own work, taking credit for the work of others without crediting them and without appropriate authorization, and the fabrication of information. For the remainder of the code, see

<https://studentconduct.gwu.edu/code-academic-integrity>

- **Support for Students Outside the Classroom**

Academic Support Services: <https://advising.columbian.gwu.edu/academic-support>

Disability Student Services

Any student who may need an accommodation based on the potential impact of a disability should contact the Disability Support Services office at 202-994-8250 in Rome Hall, Suite 102, to establish eligibility and to coordinate reasonable accommodations. For additional information please refer to: <https://disabilitysupport.gwu.edu/>

Mental Health Services: 202-994-5300

The University's Mental Health Services offers 24/7 assistance and referral to address students' personal, social, career, and study skills problems. Services for students include: crisis and emergency mental health consultations, confidential assessment, counseling services (individual and small group), and referrals. For additional information see: counselingcenter.gwu.edu/

- **University Policy on Religious Holidays:**

<https://registrar.gwu.edu/university-policies#holidays>

Students should notify faculty during the first week of the semester of their intention to be absent from class on their day(s) of religious observance.

Faculty should extend to these students the courtesy of absence without penalty on such occasions.

- **Emergency Preparedness:** www.gwu.edu/~gwalert