SPRING 2005

LOGIC SEMINAR

Friday, February 4, 2005
2:30-3:30 p.m.
Old Main (1922 F Street), Room 104
Speaker: Jennifer Chubb, GWU
Title: *Maximal sets*
Abstract: Maximal sets have the "thinnest" possible complements among computably enumerable sets. More precisely, a set M if maximal if it is computably enumerable, co-infinite, but its complement can not be split into two infinite pieces by a computably enumerable set containing M. We will present the e-state construction of a maximal set, which is a very elegant infinite injury priority construction. We will also discuss the importance of maximal sets in the study of the lattice of computably enumerable sets.

Friday, February 11, 2005
2:30-3:30 p.m.
Old Main (1922 F Street), Room 104
Speaker: Eric Ufferman, GWU
Title: *Immune relations on computable structures*
Abstract: Consider an infinite co-infinite new relation on the domain of a computable structure A. We establish a necessary and sufficient syntactic condition for the existence of an isomorphic copy of A in which the image of the relation is immune, that is, it does not contain any infinite computably enumerable subset.

Friday, February 25, 2005
2:30-3:30 p.m.
Old Main (1922 F Street), Room 104
Speaker: Andrei Morozov, Sobolev Institute of Mathematics (Russia) and GWU (Columbian Teaching Fellow)
http://www.math.nsc.ru/~asm256/
Title: *On the automorphism group of a countable dense linear order*
Abstract: The automorphism group of a countable dense linear order was extensively studied in the last twenty years by researchers in England, France, Germany, Russia, and US. I will present some results on the subgroup of computable automorphisms. In particular, I will prove that this group cannot be embedded into a computable group.

Friday, March 4, 2005
2:30-3:30 p.m.
Old Main (1922 F Street), Room 104
Speaker: Andrei Morozov, Sobolev Institute of Mathematics (Russia) and GWU (Columbian Teaching Fellow)
http://www.math.nsc.ru/~asm256/
Title: *Trees and automorphisms*

Abstract: We will present a construction that establishes a natural relationship between nontrivial automorphisms of computable models and infinite branches in trees. This result enables us to construct a computable model whose automorphism group has the cardinality of the continuum, but whose unique hyperarithmetical automorphism is trivial.

Friday, March 11, 2005
2:30-3:30 p.m.
Old Main (1922 F Street), Room 104
Speaker: Michael Moses, GWU
Title: *Questions I'd like answered* I*: A recursive version of Dushnik and Miller*
Abstract: In their classic paper (1940, *Bulletin of the AMS*), D & M construct a dense suborder of the reals that has no non-trivial self-embedding (using a non-recursive priority argument!) and observe that every countable LO has a non-trivial self-embedding. The recursive version is conjectured to be: Every recursive copy of a recursive LO *L* has a non-trivial, recursive self-embedding if and only if L contains a closed interval in which all "blocks" are of size less than some fixed k (a "block" being a maximal set of elements which are finitely far away from each other).

Many years ago, Rod Downey and I thought we had the result and, at the same time, so did Richard Watnick; both proofs turned out to suffer from the same error. Rod has shown that, if the result is true, then it is non-uniform and in fact has come to believe that the conjecture is false. I'll present the background results and techniques, where they fail and why, and suggest some perhaps fruitful ways forward.

RELEVANT PAPERS
• Downey and Moses: "On Choice Sets and Strongly Non-Trivial Self-Embeddings of Recursive Linear Orders," *Zeitschr. f. math. Logik und Grundlagen d. Math*, 1989.
• S. Schwarz: "Quotient lattices, index sets, and recursive linear orders," *Proc. AMS Summer Research Institute on Recursion Theory*, 1982.

Friday, March 25, 2005
10:30-11:30 a.m.
Old Main (1922 F Street), Room 104
Speaker: Russell Miller, Queens College, CUNY
http://qcpages.qc.cuny.edu/math/web/faculty/miller.htm
Title: *Spectrally universal structures*
Abstract: The random graph is a countable, computably presentable graph with properties quite similar to those of the countable dense linear order. We add to the list of these properties by proving that both these structures are spectrally universal: every countable graph G embeds into the random graph in such a way that the image of the embedding (as a relation on the random graph) has the same spectrum as the structure G itself, and similarly for countable linear order embedding into the countable dense linear order. We give additional results which, combined with a theorem of Hirschfeldt, Khoussainov, Shore, and Slinko, show that the possible spectra of countable structures in finite languages are precisely the spectra of unary relations on the random graph.

Both the random graph and the countable dense linear order are Fraïssé limits, for the classes of finite graphs and finite linear orders, respectively. Our current work (with Csima and Montalban) on these topics attempts to generalize the above results to arbitrary Fraïssé limits of classes of finite or finitely generated structures.

This is joint work by Valentina Harizanov and the speaker.

Friday, April 1, 2005
2:30-3:30 p.m.
Old Main (1922 F Street), Room 104
Speaker: Reed Solomon, University of Connecticut
http://www.math.uconn.edu/~solomon/
Title: *Highness and almost everywhere domination*
Abstract: Dobrinen and Simpson recently introduced the computability theoretic notion of almost everywhere (a.e.) domination and made several conjectures about the relationship between highness and a.e. domination. In this talk, I will survey some recent joint work with Stephen Binns, Bjorn Kjos-Hanssen and Manny Lerman concerning the relationship between highness and a.e. domination.

Friday, April 8, 2005
2:30-3:30 p.m.
Old Main (1922 F Street), Room 104
Speaker: Wesley Calvert, University of Notre Dame
http://www.nd.edu/%7Ewcalvert/
Title: *Internal and external complexity among computable structures*
Abstract: I will address two related questions which arise in computable model theory. First, given a computable structure, what is its simplest description, up to isomorphism? This question deals with internal complexity, or description. The second question, which deals with external complexity, or comparison, is the following: Given some class of computable structures, how difficult is it to distinguish nonisomorphic members?

In particular, let *K* be a class of computable structures, and let *I(K)* denote the set of indices for members of *K*. We write *I(A)* for the set of indices for a structure *A*. Write *E(K)* for the set of ordered pairs from *I(K)* which index isomorphic members of *K*. Now, if *A* is computable, *I(A)* is Σ^1_1, and if *I(K)* is hyperarithmetical, then *E(K)* is Σ^1_1.

Often when *E(K)* is complete at some level (for instance, Π^0_3, this completeness is witnessed by *I(A)* for some *A* in *K*. In this talk I will describe some data suggesting what such witnesses might look like. Structures considered will include vector spaces, Abelian *p*-groups, models of an Ehrenfeucht theory, ordered fields, and others.

Friday, April 22, 2005
Logic/Topology Seminar
2:30-3:30 p.m.
Old Main (1922 F Street), Room 104

Speaker: Tomek Bartoszynski, Boise State University and NSF
http://math.boisestate.edu/~tomek/
Title: *Hausdorff ultrafilters*
Abstract: Ultrafilters on omega, particularly those with some additional properties, were studied by set theorists and topologists for quite a while. Existence of many of these special ultrafilters cannot be proved in set theory alone, and it requires some additional assumptions. My talk is concerned with an example of this phenomenon, namely with the notion of Hausdorff ultrafilter which arose from studying problems in nonstandard analysis.

An ultrafilter U on ω is called Hausdorff if for any two functions f, g in ω^ω, there exists X in U such that either $f[X]$ is disjoint with $g[X]$ or $f(n)=g(n)$ for n in X.

Theorem: (Bartoszynski, Shelah)
It is consistent that there are no Hausdorff ultrafilters.

OTHER LOGIC TALKS

Mathematics Colloquium
Friday, February 4, 2005
1:00-2:00 p.m.
1957 E Street, Room 212
Speaker: Andrei Morozov, Sobolev Institute of Mathematics (Russia) and GWU (Columbian Teaching Fellow)
http://www.math.nsc.ru/~asm256/
Title: *Symmetry and computability*
Abstract: The mathematical notion of computability was formulated in the first half of the XXth century. The study of this notion and its relationship to other basic mathematical notions is one of the most important tasks for modern mathematics.

I will present a survey of results on the properties of the computability related to symmetry. Symmetries of mathematical objects are usually expressed by means of their automorphisms. I will mainly discuss the results on automorphisms of computable structures, and groups of computable permutations.

Mathematics Colloquium
Friday, April 1, 2005
1:00-2:00 p.m.
1957 E Street, Room 212
Speaker: Reed Solomon, University of Connecticut
http://www.math.uconn.edu/~solomon/
Title: *Computability theory and algebraic structures*
Abstract: Computability theorists have developed powerful techniques for studying computational properties of sets of natural numbers. Many of these methods can be used in the context of countable algebraic structures once they have been suitably coded into

the natural numbers. In this talk, I will discuss some of the basic questions considered in computable algebra and will use several simple examples to illustrate the main methods. The talk does not require any previous knowledge of logic or computability theory.

Mathematics Colloquium
Friday, April 8, 2005
1:00-2:00 p.m.
1957 E Street, Room 212
`Speaker:` Wesley Calvert, University of Notre Dame
http://www.nd.edu/%7Ewcalvert/
`Title:` *Comparing classes of structures*
`Abstract:` The problem often arises to classify some set of mathematical objects (vector spaces, fields or groups of special kinds, etc.) up to isomorphism, or some other invariants. The invariants should be objects that we understand well, and the process assigning them should be something we understand well.

In this talk, I will describe an ordering on classes of structures, in which "less" means "less difficult to classify." The method is based on a notion from computability theory. I will describe general properties of this ordering, and will show how several familiar classes are situated within it.

Mathematics Colloquium
Friday, April 29, 2005
1:00-2:00 p.m.
1957 E Street, Room 212
`Speaker:` Michele Friend, GWU
`Title:` *The next step for structuralism*
`Abstract:` Preist compares a Meinonginan philosophy of mathematics to Platonism and Fictionalism in the philosophy of mathematics.[1] I believe that a Meinongian philosophy of mathematics can be favourably compared to Shapiro's structuralism. In fact, I believe it is the next step for, what I am calling, 'Shapiro's structuralist programme'.

Shapiro's structuralism offers an overarching logic for mathematics. This is second-order logic together with the tools and concepts of model theory. The claim made by Shapiro is that second-order logic gives the expressive power to faithfully express most mathematical concepts.[2] The use of model theory provides the structuralist with a rigorous definition of structure. The idea being that mathematics is the study of structures, as opposed to the study of mathematical objects, as traditionally conceived by a platonist philosophy of mathematics. For Shapiro's structuralist, model theory allows us to individuate structures corresponding to mathematical areas of study, and allows us to rigorously compare structures to each other. As such, structuralism can be seen as a

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[1] Graham Priest "Meinongianism and the Philosophy of Mathematics" Philosophia Mathematica Series III Vol. 11, first issue. February 2003. pp. 3 – 15.
[2] Stewart Shapiro: Foundations Without Foundationalism; A Case for Second-Order Logic (Oxford Logic Guides 17) (Oxford: Clarendon Press, 1991).

programme which seeks to provide the ‘space of reason’ (logical and logistical framework) within which mathematics can take place.

Complimenting Shapiro’s pluralism, is a conscious conception of the philosopher’s role as that of providing a description of mathematics, as opposed to the more traditional philosophies of mathematics which tend towards being normative, or even prescriptive of mathematics.

In the paper, I propose to take the structuralist programme along one more step by taking pluralism very seriously and by taking the descriptive mode very seriously. The next step for the structuralist programme is to replace second-order logic with paraconsistent relevant logic and to replace models with possible worlds. Possible worlds then give us our structures. Structuralism is then underpinned by Meinongianism.

A Meinongian philosophy of mathematics has two important elements which are inter-connected. One is a notion of possible worlds, which, for Meinong, includes impossible worlds. The latter are made sense of through Meinong’s famous distinction between an object’s existing and an object having properties. Crucially, having properties does not entail existence. The other element concerns the logic which governs the universe, or frame, of possible worlds. For Priest, this is a relevant paraconsistent logic.

A mathematical area of study, then, is a possible world. The ‘access relation’ between worlds only exists if there is a basis for comparison which can be expressed in terms of relevance.

Moreover, if we want to describe mathematics, we have to include ‘failed’ mathematical systems. For, these are not only confined to the lonely attics of mathematical hacks. For example, Frege’s formal system, as outlined in the <u>Begriffsschrift</u> together with the <u>Grundgesetze</u> is a ‘trivial’ system, in the sense of contradictory. Frege’s formal system is a (great) failed system of mathematics; its failure does not make it non-mathematical, it makes it unsuccessful. Indeed, we might even say that it is mathematically impossible; but for all that, it is still highly interesting. Its inception did not cause all of mathematics to dissolve or disappear in a puff of logic. To account for our interest in Frege’s formal system we make use of the resources of paraconsistent logic. This allows contradictions without engendering triviality. That is, paraconsistent logic, amongst other things, blocks the classically valid inference: from a contradiction, anything follows (*Ex Falso Quod Libet*). If a relevant paraconsistent logic governs the universe of possible mathematical worlds, then the existence of a mathematical system containing a contradiction will not infect all the other worlds; but has the advantage of accounting for our capacity to discuss inconsistencies and contradictions, as part of mathematics.

In contrast to a Meinongian philosophy of mathematics, a structuralism wedded to model theory as the guide to what counts as a mathematical structure and what does not, has no place for discussing ‘mathematically impossible’ systems. For, these stand outside mathematics. There are no such models according to model theory (since a model proves consistency), and the underlying logic of model theory endorses *Ex Falso Quod Libet*. In contrast, a Meinongian philosophy of mathematics underpinned by a paraconsistent relevant logic to govern the relationship between worlds does allow us to discuss trivial systems, as a *bona fide* part of mathematics. Moreover, such a philosophy of mathematics can account for the possibility of rigorous development of many logical and mathematical formal systems inspired by Frege's formal system.

In summary, for reasons of taking the structuralist's pluralist and descriptive programme seriously, the next step for Shapiro's structuralist should be to underpin mathematics with a Meinongian philosophy of mathematics which uses a paraconsistent relevant logic to govern the relations between possible, and impossible, mathematical worlds.