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Statistical methods for assessing the fairness of the allocation of shares in initial public offerings

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In the late 1990s the stocks of technology companies, especially those involved with the Internet, were highly profitable. Customers of investment firms requested many more shares of new issues or initial public offerings (IPOs) than were available. Some customers allegedly made questionable arrangements with brokers for favourable treatment. This article describes statistical measures for assessing the fairness of an investment firm's or broker's allocations of IPO shares to their customers. One approach compares the success rates of different groups of customers, e.g. those alleged to be favoured with those of other groups. The second method incorporates the prior business each customer gave the firm into the comparison as an allocation criterion. This metric is an objective quantification of the fact that it is economically sensible for a firm to allocate more IPO shares to its 'best' customers.

Keywords: initial public offering; fair allocation; discrimination; expectancy analysis; success rate comparison.

1. Introduction

The allocation system used by many major investment firms when they sold shares in initial public offerings (IPOs) to their customers has come under legal scrutiny.[1] Recently, the Securities and Exchange Commission (SEC) has proposed new rules to govern the allocation of IPO shares in the future.[2] Some of the practices used by investment firms that are considered questionable by the regulators should be mentioned. The first, called laddering, required recipients of these coveted IPO shares to support the market by purchasing more shares of the IPO later, after it was listed on an exchange. Secondly, the research analysts at some investment banks are alleged to have issued overly optimistic reports on the prospects of these companies. Finally, investment banks allegedly demanded inflated commissions from recipients of IPO shares, i.e. the banks insisted on 'sharing the profits' of their customers in the form of extra commissions.

This paper originated from a regulatory case concerning the last issue. The authors were hired to provide statistical analyses for a law firm representing a small investment firm accused of 'sharing

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[1] See GASPARINO, C., SCHROEDER, M. AND KRANHOLD, K. (19 December 2000) SEC targets IPO process with probes. *Wall Street Journal*, p. C1.

[2] See http://www.sec.gov/news/press/2004-145.htm.

profits' with some customers, primarily managers of hedge funds, because these customers paid 'inflated' commissions on other trades made through the firm on or near days they received IPO shares. For instance, the complaint alleged that one broker had learned from a particular customer that his general practice was to give commission business amounting to between 30% and 40% of the profits he made from IPO shares to the firm that allocated the shares to him. It then gave some examples where the customer paid substantially more than the usual rate of commissions, in terms of cents per share rather than the commonly used percentage of the trade's value, on other trades made on days he received IPO shares.

The case involved both legal and statistical issues. Prior to the complaint, a commission was deemed to be 'excessive' if it exceeded 5%. Although the published rule does state that commissions less than 5% might also be judged excessive, no alternative guideline is provided.[3] Since almost all the so called inflated or excessive commissions were less than 5%, indeed, less than 3%, the propriety of the regulators changing the criteria for commissions to be deemed excessive *without* conducting the standard rule-making procedures, including publication in the Federal Register and allowing for public comment, was a major legal issue.

An important statistical issue concerns the fairness of the IPO allocation process used at the firm. If there were a set of 'profit sharing' customers, one would expect that they would be favoured in the allocation process. Thus, one would expect that their requests for IPO shares would be granted more often than those of other customers. Section 3 is devoted to describing the methods used to compare the success rates of the alleged 'profit sharers' with those of all other customers, a group of customers who were more comparable to the alleged profit sharers as they were quite active in the IPO market and 'retail' customers. It is known (Ljungqvist & Wilhelm, 2002) that allocation policies of most firms favoured the institutional investors. Furthermore, the Chairman[4] of the SEC from 1993 to 2001 has stated that it is reasonable for a firm to allocate shares to its 'best' customers, especially those who had given it substantial business prior to the market and IPO 'boom'. Therefore, a measure of the fairness of the IPO allocations in a particular IPO needs to incorporate the previous business of the customers who requested shares in that IPO to determine which ones received 'less' than their appropriate share and which customers were 'favoured'. A proposed measure of the fairness of the allocations of IPO shares based on comparing the actual number of shares the alleged profit sharers received to the number they would have been expected to receive based on their fraction of the business all requesting customers gave the firm during the previous 12 months is presented in Sections 4 and 5. The translation of this measure in terms of the 'selection ratio' used to evaluate an employment practice for a possible disparate impact is presented in Section 6. It will be seen that requests for IPO shares from the group of alleged profit sharers did not have a statistically significantly higher success rate than that of other customers and the selection ratio measure indicates that the profit sharers were actually disadvantaged in the allocation process.

[3] Rule National Association of Securities Dealers (NASD) IM-2440 states the five percent guideline.

[4] Arthur Levitt, the SEC Chairman from 1993 to 2001, stated in an interview he gave to Martin Smith, producer of Frontline, a Public Broadcasting System (PBS) program, that 'There's no reason why an investment firm shouldn't allocate new issues or hot issues, or whatever you want to call them, to their best customers.' More recently, SEC Commissioner Atkins also said that it is reasonable for an investment firm or broker to consider the 'business relationship' in their allocation decision. He is quoted in the article by MATHEWSON, J. (13 October 2004) US SEC proposes new initial public offering rules. *Bloomberg News*.

2. Further background

No specific SEC rules govern the process of allocating shares in a securities offering. Instead, broker-dealers, investment advisers and other regulated individuals and entities must comply with the general antifraud and manipulation provisions of the federal securities laws.[5] Investment advisers must also allocate shares in a way that satisfies the statutory fiduciary duty imposed by the Investment Advisors Act.

Generally, an NASD member may not sell shares in a 'hot issue'[6] to (i) persons associated with the member or another broker-dealer or their family members, (ii) persons who were finders with respect to the public offering, (iii) persons acting in a fiduciary capacity (such as underwriter's counsel) to the managing underwriter and (iv) senior officers of entities such as banks, investment companies, investment advisory firms or other institutional types of accounts and/or others who are involved in or influence the buying and selling of securities for those types of entities (Meshulam, 2000).

NASD Conduct Rule 2110 requires member firms to observe just and equitable principles of trade. For broker-dealers, investment advisers and others subject to regulation by the SEC, favouring particular groups or individuals when allocating shares of hot issues may violate the antifraud provisions of the federal security laws. The SEC has deemed it fraudulent to disproportionately allocate hot issues to a particular group of accounts without disclosure of that practice.[7]

In this case, the regulatory branch of the NASD identified several groups of accounts as profit sharers at different times during the investigation and pre-trial period. The penultimate such set of accounts was selected by a new criterion[8] for excessive commissions introduced by the NASD in this lawsuit.[9] Trades were categorized as inflated rate commission trades if their share volume was 10 000 or more with commission of 20 or more cents per share.[10] Also, for the accounts with such trades, trades of 1000 or more shares with commission of 75 or more cents per share were also considered inflated. These criteria were met by about 30 accounts of the firm. NASD decided to drop several of these because their 'trading pattern' differed from the other members of this group.[11] The final set of customers will be referred to as the FG group in this paper.

3. Success rate comparisons

In both the complaint and pre-hearing brief, the regulators presented only one statistic: 'every customer who had paid commissions of 20 cents per share or more on trades of 10 000 shares or more during the relevant period was a recipient of shares in at least one hot IPO'. Such commissions

[5] See e.g. 'Allocation of New Issues of Securities', 1994 WL 744595 (SEC 18 October 1994).

[6] An offering is considered 'hot' if its opening price exceeds its offering price.

[7] See: In the Matter of McKenzie Walker Management, Inc., 62 SEC Docket 1010, 1996 WL 396091 (16 July 1996); In the Matter of Account Management Corporation, 60 SEC Docket 962, 1995 WL 579449 (29 September 1995); In The Matter of John Guira, Securities Exchange Act Release No. 34-25115, 1987 SEC LEXIS 3201 (13 November 1987)(default); In the Matter of Shearson Lehman Brothers, Inc, and Stein, Roe & Farnham, 49 SEC 619, 1986 WL 732253 (24 September 1986).

[8] SEC or NASD rules prior to this case precluded brokerage firms from charging commissions in excess of 5%, such commissions were considered excessive. See *supra n.* 3.

[9] Unlike many brokerage houses, this firm did not have a schedule for commissions for trades of various sizes. From its inception, this firm allowed its customers to place their own commissions on their trades. As a consequence of the deregulation of the securities industry, this practice was perfectly legal.

[10] Pre-hearing brief on page 5.

[11] From a statistical viewpoint this is important as it indicates that the regulator used the same data to both formulate and test its hypotheses. See also the last paragraph in Section 7.

were considered excessive by the regulators. As stated in Section 2, the final group of alleged profit sharers, termed FG, consisted of slightly fewer accounts that met this criterion. This statistic was *not* compared to the corresponding one either for other customers or a set of similar customers. Furthermore, since the customers who made trades of 10 000 shares or more tended to be reasonably large, they were likely to be more active in the IPO market and to request shares in more IPO deals than other customers. Hence, even if they had a higher probability of receiving shares in *at least one* hot IPO, it does not mean that their requests for shares in a particular IPO met with greater success than other customers who made *fewer* requests.

To demonstrate that the statistical data showing that all of the FG customers received shares in at least one hot IPO does not prove profit sharing, this rate was compared to the data for all other customers. As 103 of the other 109 customers or 94.5% of them also received shares in at least one hot IPO, Fisher's exact test showed that the difference between the two 'at least one hot IPO' proportions was not statistically significant (p-value $= 0.5986$).

To determine whether the requests for IPO shares from the group (FG) of profit sharing customers were granted more frequently than the requests from other customers, several comparisons were made. First, the average success rate of the FG was compared to that of all other customers, the retail customers and a group of more active customers. This last group consisted of about 30 customers who had requested shares in at least eight offerings, the minimum number of requests made by a member of the FG, and had not been on any list of potential profit sharers transmitted by the regulators to the defendant firm.[12] It will be called the 'minimum request' (MR) group. Similar to the FG group, all customers in the MR group received shares in at least one hot IPO.

3.1 *Statistical comparison of the success rate distributions of the FG to that of all other customers*

In Table 1 we report the mean and standard deviation of the success rates of the FG customers alleged to be profit sharers by the regulatory body and the corresponding statistics for all other customers. Notice that the average success rates of the two groups are virtually identical (89.1% vs. 88.2%) so a formal test of significance is not really needed. Since the standard deviation of the success rates of the FG customers is smaller than that of all other customers, the distribution of the success rates of the individual FG customers was compared to that of all others by the Wilcoxon test.[13] This test showed that the probability (PROB) that a randomly chosen customer from the FG

[12] As noted previously, the membership of the FG changed several times during the proceedings. At first, it consisted of about 30 customers who had placed a commission of 20 cents per share or more on a trade of 10 000 or more shares. Then some customers who 'flipped' their IPO shares and put a commission of $1.00 or more on the sale were added. Later, these customers and several others of the original group were dropped. These changes indicate that the regulators were examining the data before formulating their hypothesis about which customers were profit sharing. Statistically, this is a questionable practice as many statistical tests, some implicitly, are being made on the data and it is well known that when almost any data set is studied one will 'see' a pattern. See KAYE, D. H. & FREEDMAN, D. A. (2000) Reference guide to statistics. *Reference Manual on Scientific Evidence*, 128. Washington D.C.: Federal Judicial Center; GOOD, P. I. & HARDIN, J. W. (2003) *Common Errors in Statistics (and How to Avoid Them)*. New York: John Wiley & Sons. ('Patterns in data can suggest, but cannot confirm hypotheses unless these hypotheses were formulated before the data was collected').

[13] The Wilcoxon test was used as the success rates of the customers not identified by the Department of Enforcement (DOE) of NASD as giving commissions of 20 cents or more on trades of at least 10 000 shares did not follow a normal distribution (p-value of the Wilk–Shapiro test of normality was less than 0.001). The value Z reported is the normal approximation to the distribution of the Wilcoxon test statistic. The Wilcoxon test is a non-parametric alternative to the two-sample t-test which is based solely on the relative order or ranks of the observations from the two samples when they are considered a single sample. The Wilcoxon test is used to infer whether the distribution of the first population is the same as the distribution of the

TABLE 1 *Comparison of the success rates of the FG customers with that of all other customers who requested IPO shares*

Customer group	Average success rate	Standard deviation	Wilcoxon (PROB, Z-value)	p-value of the Wilcoxon test
FG	0.8913	0.0814	PROB = 0.291 $Z = -3.19$	<0.0014
All other customers	0.8818	0.2426	N/A	N/A

Note: The Z-value is the normal form of the Wilcoxon statistic. When the sample sizes are large enough for the normal approximation to the distribution of the statistic to be applicable, one can use the usual critical values, e.g. 1.96 for a 0.05 level test, to determine statistical significance.

group had a higher success rate when it requested shares in an IPO than a randomly chosen other customer (one not in the group identified by the regulators) was 0.291, statistically significantly less than 0.50, its expected value if the two distributions were the same.[14] Considering both the similarity of the average success rates and the comparison of the success rate distributions, there is no statistical evidence that IPO requests from the FG customers identified in the complaint received more favourable treatment from the firm.

3.2 *Two statistical comparisons of the comparative success of the FG and MR customers*

In the previous comparisons of the groups of customers alleged to be profit sharers with all other customers who requested IPO allocations, retail customers and very small institutional customers are included. Thus, they might not be considered 'comparable' to the more active FG group.[15] The success rates of the two groups were compared by the same methods described in Section 3.1. The results, given in Table 2, show that the success rates of the two types of customers were similar

second and is designed to detect whether the members of the first population are generally 'larger' than those in the second. A careful description of the test is given in Hollander & Wolfe (1999) and Lehmann (1975). The application to data in actual legal cases is described in Gastwirth (1988).

[14] While one would not expect that every customer would have identical success rates, if the FG customers identified in the complaint were being favoured, one would expect that most of them would have higher success rates than those of other customers. Conversely, if membership in either the DOE/NASD group of FG customers or the remaining group of customers who were not so identified by DOE/NASD had no effect on their chances of successfully requesting shares in an IPO, one would expect the distribution of success rates to be the same in both groups of customers. The Wilcoxon statistic tests the hypothesis that the two distributions of success rates are the same by comparing the success rate of each FG customer with the success rate of each customer in the other group. Then the proportion, P, of these comparisons in which the FG customer had the higher success rate is calculated. Intuitively, if the success rate distributions of the two groups is the same, i.e. members of each group had the same chance of having a particular success rate, one would expect that in about half of the comparisons, the customer in the FG group would have the higher success rate while the customer in the other group would have the higher success rate in the remaining (again about one-half) comparisons. The fact that the proportion, P, of comparisons in which the FG customer, is less than 0.5 (one-half) indicates that their distribution of success rates was shifted downward relative to the success rate distribution of the non-FG customers.

[15] The issue of ensuring that the groups being compared are similar with respect to other factors related to the response of interest, here obtaining some shares of a requested IPO, occurs quite often in equal employment cases. See *Mozee* v. *Am. Comm. Marine Serv. Co.*, 940 F.2d 1036,1045 (7th Cir. 1991) and *Keystone Coal* 151 F.3d at 1109 for discussions concerning the factors or variables required to determine a suitable pool for comparison purposes.

TABLE 2 *Summary statistics and results of comparing of the success rates of the FG and MR-30 customers with the Wilcoxon test*

Customer group	Average success rate	Standard deviation	Wilcoxon (PROB, Z-value)	p-value of the Wilcoxon test
FG	0.8913	0.0814	PROB = 0.548	0.5218
MR-30	0.8573	0.1365	$Z = 0.6406$	

TABLE 3 *Regression equation predicting the number of successful requests of MR-30 and FG customers*

Successes =	−2.54773 Intercept	+0.94415 Requests	+3.34420 House	
	(0.7545)	(0.0214)	(0.7358)	Standard errors
	−3.38	44.19	4.54	t-statistic
	0.0014	<0.0001	<0.0001	p-value

(89.1% vs. 85.7%). Furthermore, the Wilcoxon test for a difference in the distribution of the success rates in the two groups is not statistically significant.

There were some differences between the FG and MR accounts, e.g. most of the FG were customers of the individual brokers while most of the MR were house customers and two of the MR were retail customers rather than institutional customers. To control for the possible effect of these differences, a regression analysis[16] of the number of successful requests as a function of the group membership, number of requests and being retail or house customer was undertaken.

The first regression equation[17] run showed that only two of the four factors had significant effect on the number of successful requests a customer had were the *number* of requests it made and whether it was a *house* account. Being a member of the MR (or FG) group or being a retail customer is *not statistically significantly* related to a customer's successful requests. The simpler regression equation, reported in Table 3, which predicts the number of successful requests for a customer from the number of requests it made and whether it was a house account, shows that these two factors are

[16] Regression analysis is any statistical method where the expected value of a random variable is predicted given the values of other variables. Often, one needs to identify the model that best fits the observed data (e.g. see Weisberg, 1985). In this case, a linear model of three relevant variables (group membership, number of requests and whether the account is retail or house) was initially used to predict expected number of granted requests.

[17] For completeness, we report the results of the full regression demonstrating that membership in the FG group of customers is not a statistically significant predictor of the number of successful requests for shares in an IPO a customer had during the review period. Only predictors whose t-statistics are statistically significant, i.e. the associated p-value is *less than* 0.05, are considered statistically significant. Even if one relaxed the 0.05 level criterion to 0.10, neither group membership nor being a retail customer would have had a statistically significant additional effect on the number of successful requests once the number of requests and status as a customer of the house or a broker were considered.

Successes =	−2.6481 Intercept	+0.9315 Requests	+3.6499 House	+0.8950 FG	+0.4057 Retail	
	(0.8064)	(0.0256)	(0.7972)	(0.7296)	(1.7880)	Standard errors
	−3.28	36.42	4.58	1.23	0.23	t-statistic
	0.0018 ∗∗	<0.0001 ∗∗	<0.0001 ∗∗	0.2254	0.8214	p-value

very strongly related to a customer's success as the adjusted R^2 was 0.9738. The adjusted R^2 is a statistic for measuring the proportion of the variability in the number of successes explained by the fitted model and also accounts for the number of predictor variables used. In this case, this means that 97.4% of the variability in the number of successful requests of these two groups of customers is explained by the number of requests they made and whether or not they were a house account.[18]

The standard errors of the estimated coefficients and the corresponding values of the t-statistic that tests whether the coefficient is zero are below the estimated coefficients. The last line reports the p-value corresponding to the t-statistic. The p-values of the tests of the hypotheses that the coefficients for the number of requests or of being a house customer are zero (no effect) are both <0.0001. Such a small p-value indicates that both factors are significantly related to the number of successful requests for shares in the IPO a customer obtained.

Although being a retail (non-institutional) customer did not have a statistically significant effect on the number of successful requests for IPO shares a customer made once the number of requests and whether the customer dealt with a broker or the house on the regression, at the trial the comparability of the MR group was criticized[19] because more of its customers dealt with the house and two were retail. Nonetheless, as an extra check, we deleted the two retail customers from the MR and reran the analyses described previously. The basic conclusions did not change; i.e. after accounting for the effect of being a house customer and the number of requests for IPO shares a customer made, the success rates of the FG and MR-2 were the same. Group membership was not a significant variable in the regression analysis while being a house customer and the number of requests remained significant.[20]

3.3 *Two statistical analyses comparing the success rate of the FG customers to that of the firm's retail customers*

As noted in Section 1, at most Wall Street investment houses, retail customers typically were noticeably less successful in obtaining shares in an IPO. Therefore, if the FG customers were being favoured by the firm being scrutinized, one would expect that they would have a higher success rate than the retail customers. To examine whether the FG and retail customers were treated the same by the firm or the FG had greater success, we first compared their success rates and distributions of success rates that was done in Section 3.1. Then we restricted our analysis to the IPO deals for which customers from both the FG and retail groups requested shares.[21] In Table 4 we report the success rates of the retail and FG customers.

Without any statistical analysis, it seems clear that IPO requests from retail customers were treated at least as well as those from the alleged profit sharers. If one assumes that whether or not a request for IPO shares is granted is statistically independent of the success of other requests, the

[18] It is worth noting that values of $R^2$of this magnitude do not occur very often in practice. Thus, this equation fits the data very well.

[19] The expert for NASD made these criticisms in his direct testimony. He did not offer a quantitative assessment of their potential impact on the analyses presented here.

[20] As the fitted regression when the two retail customers were deleted from the original MR group, referred to as the MR-2, did not differ much from the original one reported *supra n*. 17 it is omitted.

[21] The number of shares for each IPO the firm participated in varied widely and in a few offerings the firm had less than 1000 shares to allocate. Only customers in the FG group requested shares in these deals so retail customers were not in competition for these shares.

TABLE 4 *The number and proportion of requests for IPO shares from retail and FG customers that were successful*

Type of customer	Successes	Requests	Percent successful
Retail	143	146	97.95
FG	937	1046	89.58
Total	1080	1192	90.60

retail customers had a statistically significant higher rate of success than the FG at the 0.05 level (the p-value of Fisher's exact test[22] equals 0.0004). In light of the general experience of retail customers who requested IPO shares during the 1999–2000 boom period, this result is quite surprising. Moreover, the assumption that the success of each request is independent of other requests is questionable because the firm allocates shares IPO by IPO and has a limited number of shares to allocate.[23]

To address the concerns expressed above, we conducted a more refined analysis by examining the success rates IPO by IPO using the Cochran–Mantel–Haenszel (CMH) test.[24] The procedure considers only offerings for which there were requests from both groups. It ignores those IPO deals where both groups have a 100% success rate as there is clearly no difference in their success rates. There were 13 IPO deals for which at least one customer who requested the IPO did not receive shares and both retail and FG customers asked for shares. The resulting test statistic, in the normal or Z form, is -0.0842 and the two-sided p-value $= 0.9329$, far from the 0.05 cut-off. Indeed, a Z-value so close to zero, the expected value of the statistic when the null hypothesis is true, indicates that after accounting for the variation in the level of interest the two customer groups had in the individual offerings, their success rates were equal. The reason for this is that the retail customers tended to ask for shares when the firm had a comparatively large number to allocate, while the FG and other institutional customers tended to request shares for more offerings.

4. Expectancy analysis

The firm had over 300 accounts[25] during the period we considered. Of these, over 100 accounts participated in IPO allocations, roughly a third were retail and the remainder were institutional accounts. Institutional accounts generated the lions share (about 99%) of commission business for the firm in the year prior to the period the firm was allegedly involved in profit sharing. As noted in Section 3, virtually all accounts that requested IPO shares received at least one allocation.

[22] Fisher's exact test is a statistical test used to determine if there is a difference between two proportions conditional on the total number of 'successes' in both groups. Examples of the use of the test can be found in Agresti (1990) at pp. 59–62 and Gastwirth (1988) at pp. 217–221.

[23] Technically, this introduces a negative correlation between the number of shares one customer can receive and the number available to other customers who requested shares in the same IPO. Fisher's exact test also assumes that the requests of a customer for different IPO deals have the same probability of success even though the number of IPO shares the firm had to allocate and the number of customers requesting them varied substantially.

[24] This is the standard method for combining the results of several 2×2 tables reporting the success rates of two groups. See, e.g. SIMONOFF, J. C. (2003) *Analyzing Categorical Data*. New York: Springer; AGRESTI, A. (1996) *An Introduction to Categorical Data Analysis*. New York: John Wiley & Sons. The method was accepted for use in equal employment cases in *Hogan* v. *Pierce*, 31 FEP Cases 115 (D.D.C. 1983) and the data from the case are analyzed by the CMH test in GASTWIRTH, J. L. (1984) Statistical methods for analyzing claims of employment discrimination. *Ind. Labor Rel. Rev.*, **38**, 75–84. Further discussion of the method is given by FINKELSTEIN, M. O. & LEVIN, B. (2001) *Statistics for Lawyers*, 2nd edn.

[25] Several of the accounts belonged to firm employees who could not participate in IPO allocations.

To our knowledge, the firm did not have a set of criteria or rules for determining which accounts were eligible to receive shares in an IPO. Also, and most importantly for this part of the analysis, the firm did not utilize an objective formula for allocating hot issue shares among eligible accounts, although it did consider a customer's historical business relationship.[26] This practice is not in conflict with any general SEC and NASD guidelines that existed at the time or even today. Even the recent IPO Advisory Committee report did not wish to restrict the underwriter's exercise of its allocation discretion.[27]

In the absence of an objective rule for distributing IPO shares to IPO participant accounts, we were faced with the problem of defining 'fair' IPO share allocation to the set of accounts that exhibited interest by placing a request. The decision process in such a case is probably based on a multitude of factors that attempt to quantify how 'good' a customer is for the firm. Some measurable factors that could be used are time length of a customer's business relationship with the firm, amount of business an account generated for the firm over a period of time and how active the account has been. While the potential of a customer to generate future business is also a reasonable decision factor, it is somewhat subjective; especially as the firm had not established any criteria for assessing the business potential of their customers.

The data on how long many customers had been with the firm were not very accurate. In part this was due to the fact that some seemingly new customers were old customers of individual brokers who followed them when they moved from other brokerage firms.

On the other hand, we had very reliable trading data of all firm customers over a period of 2 years from which we could easily extract the commission business the customers gave to the firm through agency trades. Since an investment firm would naturally allocate more shares to customers who had given it more business than to other customers, the relationship between the allocations of shares in all IPO deals and the previous year's commission business customers gave the firm was studied.[28] We focused on one year's prior business since it would account for varying trading patterns of different customers. The management of the firm confirmed that most brokers would have a good idea of the business customers generated during the previous year.

In the complaint, the NASD regulators appeared to evaluate customers on the basis of whether they gave high commissions in terms of the cents per share they put on trades made at the firm. We started by investigating whether aggregate historical commission business would be a good predictor of IPO allocations especially in comparison to cents per share. We compared the fraction of IPO shares allocated to customers requesting those shares to the 12-month average of the cents per share they put on trades and the corresponding average of their total commission business.

Since neither the average cents per share a customer gave during the previous year nor the total commission they gave during the same time were normally distributed,[29] the Spearman correlation

[26] Indeed, one of the joint stipulations stated that it was permissible for a firm to take commission business into account in the IPO allocation process.

[27] See New York Stock Exchange/NASD IPO Advisory Committee: Report and Recommendations (May 2003) at p. 13 (stating that unless an allocation process involves prohibited conduct such as spinning or an unlawful quid pro quo, the underwriter may allocate IPO shares as it chooses).

[28] Two of the joint stipulations in the case are relevant here. The first is cited, *supra n.* 26, and the second states 'It was common practice for firms to take true commission business into account in making IPO allocation decisions during the time period covered by the Complaint'.

[29] The data are shown not to be normally distributed in Gel *et al.* (2005).

coefficient[30] was used to assess the strength of the relationship between the number of shares allocated and both of these variables. To test whether the two Spearman correlations are the same or one is statistically significantly larger than the other, we applied the sign test to the differences between the correlation between total commission during the previous 12 months and the shares in the IPO allocated to customers requesting them and the correlation between average cents per share of the commissions given by these customers and the allocation of shares in the IPO they received. The result showed that in 52 of the 57 IPOs the firm had shares to allocate, the correlation between total commissions given during the previous 12 months was the larger correlation. In the other five offerings, the two correlations were equal. This is a highly significant result (p-value < 0.000001), which is equivalent to a difference of more than 7 standard deviations indicating that the actual allocations of IPO shares were much better predicted by the previous year's total commissions than the average cents per share of those commissions. In fact, the average Spearman correlation between IPO shares allocated and total commissions in all 57 offerings was 0.58 while the average Spearman correlation between shares allocated and average cents per share was only 0.04.

Having established that historical commission is a good predictor for IPO share allocation, we turned to defining a fair allocation rule for IPO shares based on aggregate commission business a customer paid the firm over the 12 months prior to the month an IPO opened. We defined as fair or expected allocation share for each customer who participated in an IPO deal to be an IPO share proportional to the customer's aggregate commission business over the year prior to the month the IPO was introduced to the market.

For each offering, the expected number of shares a customer who expressed interest would receive was determined by its fraction of the total prior business given the firm among the customers who were interested. Prior business refers to the commissions the customer[31] gave the firm over the 1-year period prior to the month of the offering. The commission amounts for each customer were calculated at the beginning of each month, i.e. the total of all commissions given the firm during the previous 12-month period was calculated. Commissions given during the month in question were not included in calculating the expected number of shares a customer should have had received. This was done because the data the brokers could examine only reported account business through the end of the previous month. Once this expected fraction, F, was determined, the expected number of shares the customer should receive is just F times the number of shares the firm had to allocate. For example, if the firm had 20 000 shares to allocate and a customer's fraction F of prior business over the last year was 0.25, the customer would be expected to receive 5000 shares.

[30] From WEISSTEIN, E. C. (2005) *Spearman Rank Correlation Coefficient*. From *MathWorld–A Wolfram Web Resource*. http://mathworld.wolfram.com/SpearmanRankCorrelationCoefficient.html.

A nonparametric (distribution-free) rank statistic proposed by Spearman in 1904 as a measure of the strength of the associations between two variables. The Spearman rank correlation coefficient can be used to give a correlation coefficient estimate and is a measure of monotone association that is used when the lack of normality of the data make Pearson's correlation coefficient misleading. The Spearman rank correlation coefficient is defined by

$$r' \equiv 1 - 6 \sum \frac{d^2}{N(N^2 - 1)'},$$

where d is the difference in statistical rank of the corresponding variables.

For further discussion and illustrative examples see Sprent & Smeeton (2001) at pp. 242–246.

[31] Some clients had several related accounts, e.g. a trading account and a retirement account and typically bid for shares for just one of those accounts. To calculate their prior business we combined all accounts relating to the customer.

TABLE 5 *Calculation of expected number of LNUX shares allocated to FG customers and comparison with actual*

	Number of received LNUX shares	Fraction of aggregate prior year business	Expected number of LNUX shares	Difference (received–expected)
FG	19 300	0.532	62 700 × 0.532 = 33 368.33	−14 068.33
All other accounts	43 400	0.468	62700 × 0.468 = 29 331.67	14 068.33
Total	62 700	1.0	62 700	0

To assess whether the special clients received more than their fair share, for each offering we summed the expected fractions (Fs) of those special clients who requested shares to determine their total, say FT. The expected number of shares the special clients should receive, assuming an allocation proportional to prior business is FT times the number of shares the firm had to allocate. To continue the example, if FT were 0.60, e.g. three special clients with F's of 0.25, 0.20 and 0.15 asked for shares, the special clients would be expected to receive $0.6 \times 20\,000 = 12\,000$ shares of the offering. By comparing the actual number of shares to the expected number of shares, we can assess whether the special clients received more or less than their fair share. This is accomplished by calculating the difference between the actual and expected numbers of shares for each offering and summing over all IPO deals. The results can be summarized by comparing the actual fraction of shares received by the special clients in the offerings of interest to their fraction expected in those deals.[32]

To further illustrate how the calculations are carried out, we present the following example using the VA LINUX IPO that opened on 9 December 1999. The firm had a total of 66 000 shares of VA LINUX to distribute to the 68 customers who requested shares. Seven of these customers, mostly retail, were new to the firm in the sense that they had not executed any trades in the year prior to December 1999. They were excluded from the computations, as they would be expected to receive zero shares since they had no business in the prior 12 months.[33] There remained 62 700 shares to allocate to the remaining 61 customers. For these accounts, Table 5 summarizes our calculations.

Table 6 reports the number of shares *in all 57 IPO* deals the FG customers, alleged to be profit sharers, received and the number of shares they would have been expected to receive on the basis of their fraction of the commissions received by the firm in the previous 12 months.

As the total number of IPO shares allocated to all customers with prior business was 3 118 025, the actual and expected percentages of these offerings allocated to these accounts are given below.

[32] The expectancy analysis is based on the CMH methodology used in biostatistics and equal employment law where at each time an event of interest may occur (e.g. a promotion in an equal employment case), one compares the number of events occurring in the group of interest to their expected number. Then one sums all the differences between observed and expected events at these times to see whether they are in agreement or if fewer events than expected are occurring in the group one is studying. See *supra n.* 24 for references and illustrations of the method. A similar technique is called event analysis in the sociological literature.

[33] Of course, some of these customers might have had an account with the firm but they were not active in the year preceding the month of an IPO deal.

TABLE 6 *Comparison of actual and expected number of shares allocated to FG customers*[34]

	Actual number	Expected number	Difference
57 IPOs	1 279 775	1 806 585.992	−526 811

TABLE 7 *Comparison of actual and expected percent of shares allocated to FG customers*

	Actual percent of allocated shares received by the FG	Expected percent of shares for the FG	Shortfall
57 IPOs	41.04%	57.94%	−16.90%

In relative terms, the FG received 70.83% of the IPO shares they would have been expected to receive had shares been allocated in proportion to a customer's commissions during the previous 12 months.[35] To confirm that these results reflecting the allocation process of IPO shares were not unique to the review period, the same expectancy analysis was conducted for the 6 months subsequent to the review period. In that period, the FG received about 66.15% of their expected number of shares or equivalently, 33.85% fewer shares than would be expected on the basis of their business during the previous 12 months.

A similar analysis for the value of the IPO shares at the offering price was conducted. Again, the FG customers received noticeably less than their expected value of IPO shares. Since the numerical values of the percentage shortfall were similar to those in Table 7, the detailed results are omitted.

5. Sensitivity analyses related to the expectancy analysis

5.1 *First sensitivity adjustment*

There were five small IPOs for which only the FG customers requested shares. To be statistically conservative, we *subtracted* these shares from the expected number of shares the FG would receive but *kept* them in the actual number of shares they received. During the period in question the firm acquired several new accounts some of which were in the FG. As the first month's activity of new accounts would not appear in the previous 12-month commissions, shares allocated to new clients during their first month with the firm were omitted from the results in Table 6. To be statistically conservative, we added the IPO shares the FG new clients received during their first month to the previous total of shares the FG group received. The results of these adjustments are tabulated in Table 8.

[34]Customers who had not done any prior business with the firm during the previous 12 months were excluded from these calculations. The effect of this was quite small as about 96% of all IPO shares went to customers with prior business.

[35] This is the percentage of the expected number of shares that the FG customers actually received. For example, they received 41.04% of all IPO shares but on the basis of their prior commissions they would have been expected to receive 57.94% so they were only allocated $41.04/57.94 = 0.7083$ or 70.83% of their expected number.

TABLE 8 *Comparison of adjusted received and expected number of shares allocated to FG customers*

	Adjusted received number	Reduced expected number	Difference
57			−498 460.992
IPO	1 305 025	1 803 485.992	

TABLE 9 *Comparison of adjusted received and expected percent of shares allocated to FG customers*

	Adjusted percent of allocated shares received by FG customers	Reduced expected percent of shares for FG customers	Shortfall
57			
IPO	41.85%	57.84%	−15.99%

The corresponding adjusted percentages are given in Table 9.

After the two sensitivity adjustments that increased the number of shares allocated to the FG customers by 25 250 and reduced their expected number by 3100, 41.85% of the IPO shares went to the FG accounts in comparison to their adjusted expected percentage of 57.84%. On a relative basis, the FG received 72.36% of their expected number of shares or equivalently, 27.64% fewer shares than expected. Thus, even after conservative adjustments that decreased the 'shortfall' in shares received by the FG customers, they still received substantially fewer shares than would be expected based on their prior 12 month business.

5.2 *Second sensitivity adjustment*

For purposes of the regulators' analysis, an inflated rate transaction is an agency trade with (i) commissions of \$0.20 per share or more on 10 000 shares or more or (ii) commissions of \$0.75 or more on 1000 shares or more. We sought to examine the effect these two criteria would have on the number of shares the FG customers would be expected to receive had these rules been in existence at the time and enforced by the firm. We thus adjusted the prior business for all customers who had executed such trades in the 12-month period prior to the review period as well as during the review period as follows. All trades satisfying criterion (i) were assigned commissions calculated by setting cents per share (cps) to \$0.19. All trades satisfying criterion (ii) were assigned commissions by setting cps to \$0.74. All other trades, and hence commissions, were left intact.

Table 10 reports the number of IPO shares the special accounts received and the number of shares they would have been expected to receive on the basis of their fraction of the adjusted commissions they would have had given the firm in the previous 12 months. For each offering, the expected number of shares for the FG customers is determined from their fraction of the adjusted total commission business during the previous 12 months of all customers requesting that specific offering as described in Sections 4 and 5.

We also report the actual and expected percentages of the offerings in each category in Table 11.

TABLE 10 *Comparison of actual and adjusted expected number of shares allocated to the FG customers*[36]

	Actual number	Expected number	Difference
57 IPO	1 279 775	1 542 538.836	−262 763.836

TABLE 11 *Comparison of actual and adjusted expected percent of shares allocated to the FG customers*

	Actual percent of allocated shares received by FG customers	Adjusted expected percent of shares for FG customers	Shortfall
57 IPO	41.04%	49.47%	−8.43%

In relative terms, even after this adjustment of their prior business, the FG received 17.04% fewer IPO shares they would have been expected to receive had shares been allocated in proportion to a customer's *adjusted* commissions during the previous 12 months.[37]

A similar analysis treating each IPO share in terms of the value of the offering price was conducted. Again, the FG customers received 9.14% less than their expected value of IPO deals. Since the numerical values of the shortfall were similar, the detailed results are not reported.

6. Selection ratio analysis

To further appreciate the magnitude and importance of the shortfall in the number of IPO shares received by the FG customers identified as profit sharers by the regulator the selection ratio corresponding to the difference between the percentages of IPO shares received and expected for the FG group. A selection ratio is a statistical measure traditionally used in equal employment cases. It is the ratio of the pass rate of a minority group of job applicants to the pass rate of job applicants from a majority group and is used to assess whether a job requirement (e.g. attaining a certain score on a test) has a 'disparate impact' on minority applicants.[38] In that application, the government guideline is that selection ratios less than 0.80 or four-fifths (e.g. minority applicants receive passing scores on a test at a rate less than 80%, or four-fifths, the rate of Caucasians) suggest a disparate impact and indicate that the job requirement needs to be justified as job-related.

The selection ratio for the FG customers calculated from the data in Table 7 is 0.5053, well below the value of the government guideline, 0.80. This indicates that on the basis of their prior commission business, the FG customers were disadvantaged in the IPO allocation process used

[36] Customers who had not done any prior business with the firm were excluded from the calculations. The effect of this was quite small. See *supra n*. 34.

[37] This is the percentage of the expected number of shares that the FG customers actually received. For example, they received 41.04% of all IPO shares but on the basis of their prior commissions they would have been expected to receive 49.47% so they only were allocated $41.04/49.47 = 0.8296$ or 82.96% of their expected number.

[38] Mathematically, the selection ratio is simply the ratio of two proportions and is used in a number of applications. In epidemiology and medicine, the ratio of the proportion of individuals who are exposed to a toxic chemical who subsequently are diagnosed with an illness to the corresponding proportion of unexposed individuals is called the 'relative risk'. In the medical context, relative risks larger than 1.0 indicate that the chemical (e.g. smoking) is harmful while relative risks less than 1.0 indicate that the chemical (e.g. a vitamin or mineral) is protective. The four-fifths rule is stated in the Uniform Guidelines 29 C.F.R. 1607.4 (D) (1998).

at the firm. The selection ratio for the data in Table 9, where the alleged excessive or 'inflated' commissions were adjusted downward to comply with the 'new' criterion, is 0.711. Thus, even when the alleged inflated portion of all commissions during the period are removed from the calculation of a customer's prior business, on the basis of prior business the FG customers were noticeably disadvantaged in the IPO allocation process. Hence, the selection ratio interpretation of the results found in the expectancy analyses shows that the FG group was not favoured by the firm.

7. Discussion

While the IPO frenzy during the late 1990s was unusual, there remains a substantial IPO market as new companies grow and decide to go public. The methodology described in this paper should be useful both to regulators and individual firms who desire to check on the fairness of their IPO allocations. In particular, a firm can readily monitor its individual brokers to ensure that none of them is favouring a few customers by giving them more shares than their prior business merits. If a customer has an unusually high success rate or often is being allocated more shares than expected, the firm could then examine the pattern of commission business the customer gives to see whether it is concentrated near the time the customer received shares in an IPO.

The 'pattern' that the regulatory body in the case motivating this research used really does not capture profit sharing activity around the days of an IPO. Recall that only trades of 10 000 shares or more with commissions of 20 cents or more per share or trades of at least 1000 shares with commissions of 75 cents per share or more were considered inflated or excessive. Notice that a customer who traded 5000 shares and placed a commission of 50 cents per share would go undetected even though he/she gave a commission of $2500, which exceeds the $2000 commission on a trade of 10 000 at 20 cents per share. It would seem more appropriate to consider the total commission business near the receipt of an IPO compared to a customer's usual business[39] and also examine the ratio of these commissions to their potential profit from the IPO shares they were allocated. If there had been a formal or informal *quid pro quo* arrangement or understanding, these ratios would be expected to concentrate around the understood 'share'.

The above example also indicates that the definition of excessive or inflated commissions was not designed to detect profit sharing; rather it was determined after the regulator examined the data and believed that certain customers had placed one or both of the allegedly inflated trades on days they received shares in an IPO from the defendant firm. Since the pattern was already seen in the data, it was of course 'detected' when a formal statistical test was applied to the very same data. In statistical lingo they performed what is called 'data-snooping'. Data-snooping occurs when a given set of data is used more than once for purposes of inference or model selection. This leads to the possibility that any results obtained in a statistical study may simply be due to chance rather than to any merit inherent in the method yielding the results.[40] It is well known in the statistical literature that using the same data to both determine and validate a hypothesis is intellectually unsound.[41]

[39] This is not as simple as it might seem because IPO shares do not open on a random sample of days the stock exchanges are open. Rather, they are offered at times when the issuing company and underwriter believe they will be successful. These tend to be during periods of a rising market with corresponding high volume.

[40] From the London School of Economics Finance Glossary (http://www.lse.co.uk/financeglossary.asp?searchTerm=&iArticleID=1492&definition=data-snooping).

[41] See *Supra n.* 3, Kaye & Freedman (2000); Good & Hardin (2003, p. 16) ('*We must specify our alternatives before we commence an analysis*, preferably at the same time we design our study'.) (emphases added).

In order to avoid statistical pitfalls, one needs to test the hypothesis and validate their results using new data, a different aspect of the same data or at the very least to adjust one's findings for the effect of using the same data to both formulate and test one's hypothesis.

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