

Analyzing Distances in a Frontal Plane

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Abstract

In exploration of the process whereby horizontal and vertical components are extracted from distances between pairs of points, participants made speeded *absolute distance judgments* – deciding whether horizontal distances between pairs of points in a frontal plane exceeded a criterion distance. Judgments reflected the horizontal and vertical distances between the points. Three accounts of the results were considered. 1) Only the *overall distance* between two points was directly available to judgment processes; the horizontal or vertical distance between the points was available only to the extent that the horizontal and vertical positions of the points were differentially weighted prior to the assessment of overall distance. 2) The perceptual effects of positions on the horizontal and vertical dimension were collapsed onto a composite dimension. 3) The decision criterion for the distance judgment considered both the horizontal and vertical distance between the points. In support of the overall distance account: 1) performance in the distance judgment task was facilitated by repetition of the same overall distance from trial to trial, but not by repetition of the distance on the composite dimension, or by repetition of horizontal or vertical distance; 2) differential weighting that the overall distance account predicted for the absolute distance judgments was reflected in concurrent relative distance judgments pitting horizontal against vertical distance, counter to the composite dimension account, which sees the horizontal and vertical dimensions as collapsing onto the composite dimension in balanced, symmetrical fashion, and counter the decisional account, given that no criterion is required for the relative judgment.

When an object layout is understood *allocentrically* - without reference to the observer's vantage point, the relative locations of the objects are given by *exocentric distances* between the objects and *exocentric directions* of the objects relative to one another. Distance information is at least as important as direction information in understanding a layout (Waller, Loomis, Golledge, & Beall, 2000). The distance between two points is thought to be perceived in terms of the *visual angle* formed when the points are intersected by vectors originating at the observer (Foley, Ribeiro, & Da Silva, 2004). A polar coordinate system is implicit in the visual angle conception.

Although exocentric distances are *perceived* in terms of the visual angle, they are often *organized in memory* in terms of an *allocentric reference frame*. Such a frame comprises a pair of orthogonal axes, one of which gives a *reference direction* (Mou, McNamara, Valiquette, & Rump, 2004). The frame provides a basis for encoding the locations of objects in a layout and for updating those locations over movement (Mou, Liu, & McNamara, 2009). To understand the distance between two objects in terms of an allocentric reference frame, the observer must often extract *orthogonal components*, corresponding to the axes of the frame, from that distance (Mou et al., 2009). Figure 1 illustrates this idea. Notice that, whereas a polar coordinate system is implicit in the assessment of the distance between two objects, a rectangular coordinate system is implicit in the extraction of orthogonal components from that distance.

Insert Figure 1 about here

How are the orthogonal components corresponding to the axes of a reference frame extracted from the distance between two objects? Such problems have attracted interest because observers often do not solve them cleanly (Maddox & Dodd, 2003). An observer is not, that is, guided solely by the psychological dimension of interest in solving such a problem. This sort of imprecision has been described as *dimensional*

interaction (Melara, 1989, 1992). Dimensional interaction may be fostered when orthogonal components are extracted from distances because the process requires reconciliation of the polar and rectangular coordinate systems.

Traditionally, questions of dimensional interaction have been studied with speeded classification tasks (Lockhead, 1972, 1979; Kemler Nelson, 1993). Recently such a task has been used to explore how horizontal and vertical components are extracted from distances in a frontal plane (Dopkins, 2005). On each trial of the *complex distance task*, small circles appear at two locations on a computer screen. Across trials, circles appear at a *task array* of locations, with adjacent locations being separated by the same physical distance on the horizontal and vertical dimension of the array. The participant is required to indicate whether the test locations are a specified distance apart on one of the dimensions of the array. A response-signal controls the interval between the presentation of the circles and the participant's response. Thus, the error-rate data are of greatest interest.

Dopkins (2005) tested participants in the complex distance task with a 7 column X 3 row task array. Participants were to indicate on each trial whether the test locations were two or fewer horizontal positions apart (See Figure 2). The error rate in Dopkins' (2005) task increased with increases in the horizontal distance between the test locations for pairs of locations that were two or fewer horizontal positions apart ($2 \geq$ pairs) and decreased with increases in horizontal distance for pairs of locations three or more horizontal positions apart ($3 \leq$ pairs). Of greater interest, the error rate increased with increases in vertical distance for $2 \geq$ pairs and decreased with increases in vertical distance for $3 \leq$ pairs¹. Thus, although the task required an assessment of the *focal* horizontal distance between the test locations, performance also reflected the *non-focal* vertical distance between the locations. Of particular interest were the relative sizes of

the horizontal and vertical distance effects. The $2 \geq$ pairs were most important to this comparison because the horizontal and vertical distance between the test locations varied through the same three levels (0,1,2 positions), making possible a matched test of the relative sizes of the two distance effects. When the response interval was 400 ms, the error rate for $2 \geq$ pairs depended to an equivalent degree on horizontal and vertical distance. When the response interval was 1000 ms, the error rate for $2 \geq$ pairs depended to a greater degree on horizontal than on vertical distance.

Insert Figure 2 about here

The present study proposes an interpretation for Dopkins' (2005) non-focal distance effect and offers support for this interpretation. To motivate the interpretation, notice that the task of Dopkins (2005) can be performed in two ways. On one hand, responses can be based on the horizontal distance between the test locations, as the instructions indicate. Alternatively, responses can be based on the *overall* distance between the test locations. Performance in the latter case depends on how the horizontal and vertical dimensions are weighted in the calculation of overall distance. With the two dimensions weighted equally, the overall distance is at most 2.8 (positions) for $2 \geq$ pairs and at least 3 for $3 \leq$ pairs. Thus, the task can be accurately performed, but the required discrimination is more difficult than if responses are based on horizontal distance (in which case the overall distance is at most 2 for $2 \geq$ pairs and at least 3 for $3 \leq$ pairs).² To the degree that the vertical dimension is weighted less heavily than the horizontal dimension, the maximum overall distance for $2 \geq$ pairs decreases toward 2 and the discrimination becomes easier.

In explaining the non-focal distance effect, we start from the following premises:

a) the distance between the test locations in the complex distance task is assessed on the basis of the visual angle between them, in terms of an implicit system of polar

coordinates, centered at the observer (Foley et al., 2004); b) the horizontal and vertical positions of the test locations are assessed on the basis of the *frontal/horizontal/median frame*, in terms of an implicit system of rectangular coordinates, also centered at the observer (Matin & Li, 1995). Given these assumptions, we suggest that, because polar coordinates do not acknowledge the horizontal and vertical dimensions, the focal horizontal or vertical distance between the test locations cannot be directly assessed under the conditions of the complex distance task. Only the overall distance between the test locations can be assessed. As a consequence, the task is performed on the basis of overall distance. However, because the horizontal and vertical *positions* of the test locations can be assessed, in terms of rectangular coordinates, the focal and non-focal positions of the locations may be differentially weighted, *before* the overall distance between the locations is assessed, to reduce the contribution of non-focal distance to overall distance (Maddox, 1992; Nosofsky, 1986, 1987). Such *position weighting* occurs because it makes the required discrimination easier. For Dopkins (2005), such weighting occurred at the long but not the short response interval.

Because we seek to model these ideas, we introduce General Recognition Theory (GRT), which provides a framework for understanding dimensional interaction in recognition/categorization judgments. A key virtue of GRT is that it separates the perceptual and decisional aspects of such judgments. In GRT, a stimulus consists of multiple *components*, each of which corresponds to a *level* on a physical dimension. The perceptual *effect* of a stimulus on a given occasion is represented as a point in a psychological space, the dimensions of which correspond to the physical dimensions in terms of which the stimulus is defined. The perceptual effect of the stimulus across occasions is represented as a multivariate probability distribution, the variables of which correspond to the dimensions of the space. A judgment regarding the effect of a

stimulus on a given occasion is based on a *decision bound* that divides the space into sectors associated with different judgment outcomes (Ashby & Perrin, 1988).

For modeling purposes, we state our Interaction in Distance Extraction (IE) Hypothesis as follows: The perceptual effect of each location in the complex distance task is represented across trials as a multivariate normal distribution in a Euclidean psychological space³. Under the hypothesis, the test locations for the task are represented in terms of polar coordinates for the purposes of distance assessment and in terms of rectangular coordinates for the purposes of position weighting. For simplicity in our modeling, we represent the locations in terms of rectangular coordinates for both processes (notice that this simplification does not affect the predictions of the hypothesis). Thus, the dimensions of the space record perceived focal and non-focal position. The response on a given complex distance trial is generated by: 1) sampling from the distributions for the test locations, 2) computing the overall (Euclidean, unsigned) distance between the sampled points and 3) comparing that distance to a criterion value. Before the overall distance between the points is computed, the coordinates of the points on the dimension of perceived non-focal position may be adjusted to reduce the difference between the points on that dimension⁴.

Other accounts may be offered, however, for the non-focal distance effect. One possibility is that the effect reflects dimensional interaction in the process whereby the test locations are perceived rather than in the process whereby the horizontal and vertical distances between them are extracted. This account holds, in contrast to the IE Hypothesis, that the horizontal or vertical distance between the test locations can, in principle, be independently assessed. The account holds, however, that the horizontal or vertical distance between the locations cannot, in practice, be independently assessed because the perceptual effect of a given horizontal or vertical position is not

the same at all vertical or horizontal positions. In more detail, we formulate the account in terms of *mean shift integrality (MSI)* (Ashby & Maddox, 1994) - the most common conceptualization of dimensional interaction in the perceptual component of the GRT⁵ (Kadlec & Hicks, 1998; Kingston & MacMillan, 1995; Kingston, MacMillan, Dickey, & Thorburn, 1997; MacMillan & Ornstein, 1998). Figure 3 demonstrates degrees of MSI for a 7 column X 3 row task array such as Dopkins (2005) used. Figure 3A shows a representation of the array in the absence of MSI. Each point in the figure corresponds to a location in the array. Figure 3B shows the array under the form of MSI that has typically been studied in past work. A location with a given horizontal position has a greater value on the dimension of perceived horizontal position the greater its vertical position. However, an increase on the dimension of horizontal position produces a greater increase on the dimension of perceived horizontal position than does an increase on the dimension of vertical position. Figure 3C shows the array under what we will term absolute MSI. The perceptual effects of increases on the dimension of horizontal and vertical position are indistinguishable. The mental representation of the task array collapses the horizontal and vertical dimensions of the array onto the minor diagonal of the psychological space. Most of the points in the resulting representation correspond to multiple locations. To explain the results of Dopkins (2005), we would assume that MSI was absolute at the short but not the long response interval.

More precisely, the Interaction in Perception (IP) Hypothesis holds that the perceptual effect of each location in the complex distance task is represented across trials as a multivariate normal distribution in a Euclidean psychological space the rectangular dimensions of which correspond to perceived focal and non-focal position. The response on a given trial is generated by 1) sampling from the distributions for the test locations, 2) computing the focal (unsigned) distance between the sampled points

and 3) comparing that distance to a criterion value. The space in which the locations are represented is characterized by MSI that varies in degree.

Insert Figure 3 about here

A third possibility is that the non-focal distance effect reflects dimensional interaction in the decision component of the complex distance task. This account holds, in contrast to the IP Hypothesis, that the horizontal and vertical positions of a location in the task can be independently perceived. The account holds, in contrast to the IE Hypothesis, that the horizontal and vertical distance between a pair of test locations can be independently assessed. The account holds, however, that decisions in the task are based on the non-focal as well as the focal distance between the test locations. The importance of considering dimensional interaction in the decision component of a task has been emphasized in recent work (Mack, Richler, Gauthier, Palmeri, 2011; Silbert & Thomas, 2013; Wenger & Ingvalson, 2003). To explain the results of Dopkins (2005) in terms of such an interaction, we would assume that decisions depended equivalently on horizontal and vertical distance at the short interval and on horizontal distance to a greater degree than vertical distance at the long interval.

More precisely, the fundamental stimulus entities are the combinations of (unsigned) focal and non-focal distance between the pairs of test locations. Over trials, the perceptual effect of each *distance combination* is represented as a multivariate normal distribution in a Euclidean psychological space the rectangular dimensions of which correspond to perceived focal and non-focal distance. The response on a given trial is determined by 1) sampling from the distribution for the relevant distance combination and 2) comparing the sampled point to a decision bound. Figure 4 illustrates this Interaction in Decision (ID) Hypothesis in terms of Dopkins' (2005) task⁶.

Insert Figure 4 about here

Experiment 1

The present study sought support for the IE Hypothesis in contrast to the IP and ID Hypotheses. Experiment 1 sought support for the IE Hypothesis from the pattern of sequential dependencies in the complex distance task. The important data came from crucial *trial pairs* on each of which the participant responded, in succession, to a *preceding trial* and a *target trial*. Performance on the target trial was of primary interest. The rationale was as follows: Under each hypothesis, complex distance responses are determined by a process in which *critical values*, derived from the test locations, are compared with a criterion. Under a given hypothesis, performance on a given target trial should be facilitated if the values that are critical under that hypothesis are similar on the preceding trial and the target trial. Because the critical values differ under the different hypotheses, the hypotheses make different predictions as to the crucial trial pairs for which facilitation should be observed on the target trial.

In more detail, the preceding trial and target trial for the crucial trial pairs were always of the $2 \geq$ type. The crucial trial pairs were of 14 types. The distance combination on the target trial was $\{2,0\}$ for 7 of the trial-pair types and $\{0,2\}$ for 7 of the trial-pair types. To generate the 14 trial-pair types, each of the two types of target trial was paired with seven types of preceding trial, corresponding to the seven possible distance combinations that produced $2 \geq$ trials, aside from that of the target trial. Predictions were made under the three hypotheses as to which of the crucial trial-pair types would produce the most facilitation, and thus the lowest error rates on the target trial. To simplify the predictions, the 400 ms response interval was used, so that perceived horizontal and vertical distance would be interchangeable (as in Dopkins (2005)). The predictions of the hypotheses are summarized in Table 1 with respect to trial-pair types for which the distance combination on the target trial was $\{2,0\}$ ⁷. In the first line under

each hypothesis, the table shows the predicted ordering of trial-pair types, where the types are identified in terms of the distance combination on the preceding trial. In the second and third lines for each hypothesis the table shows, for each subset of trial-pair types, the idealized critical values on which responses to the preceding trial and the target trial would be based. These values are idealized in that they assume complete veridicality and no variability in the mental representation of the task array. In the fourth line for each hypothesis the table shows, for each subset of trial-pair types, the differences between those critical values⁸. (The rest of the table will be discussed later).

The predictions of the hypotheses had the following rationale. IE Hypotheses: The critical value would be the perceived overall distance between the test locations. IP Hypothesis: The critical value would be the perceived horizontal distance between the test locations. However, horizontal distance would not, in practice, be capable of assessment independently of vertical distance because the perceptual effect of a given horizontal position would not be the same at all vertical positions. When MSI was absolute, as it would have to be to account for the perceived interchangeability of horizontal and vertical distance, the mental representation of the task array would collapse the horizontal and vertical dimensions. Pairs of test locations that were separated by the same sum of horizontal and vertical distance would be associated with the same perceived horizontal distance. ID Hypothesis: The critical values would be the perceived horizontal and vertical distances between the test locations.

Insert Table 1 about here

Method

Participants. The 75 participants were drawn from an undergraduate psychology course. They participated in fulfillment of a course requirement.

Stimuli. Each test circle was 3 mm in diameter. Circles in horizontally and vertically adjacent locations were separated by 8 mm (from edge to edge). The participant sat approximately 60 cm from the computer screen. Thus each test circle subtended a visual angle of approximately .29 degrees and circles in adjacent locations were separated by a visual angle of approximately .76 degrees.

Design. A total of 600 trials were presented in the experiment. Eight versions of each of the 14 crucial trial-pair types described earlier were presented. Thus, a total of 112 (2 (target trial type) X 7 (preceding trial type) X 8 (repetition)) crucial trial pairs were presented (for a total of 224 trials). In addition, 112 pairs of successive 3< trials (thus, a total of 224 trials) were presented (recall that the preceding trial and target trial were always of the 2> type on the crucial trial pairs). The distance combination on each of the trials in each of these pairs was sampled randomly from the set of 12 possible 3< distance combinations. In addition, 152 filler trials were presented. The distance combination on each of these filler trials was sampled randomly from the set of 20 (8 2> and 12 3<) possible distance combinations. On average, then, 285 2> trials were presented ($224 + 8/20 \times 152$). On average, 315 3< trials were presented ($224 + 12/20 \times 152$). The particular test locations for a given trial were sampled randomly from those that were consistent with the distance combination in effect on the trial.

Procedure. At the beginning of each trial, "Ready" appeared on the computer screen. When the participant pressed the space bar of the computer, "Ready" disappeared and a pair of circles appeared. Four hundred ms after the circles appeared, four asterisks appeared at the bottom of the screen. The participant was instructed a) to indicate whether or not the test locations were two or fewer horizontal positions apart, b) to use the "B" and "N" keys to indicate positive and negative responses respectively, c) to respond concurrently with the appearance of the asterisks. If the interval between the appearance

of the circles and the participant's response was less than 400 ms, the message "TOO FAST" appeared at the bottom of the screen after the participant's response and remained there until the participant pressed the space bar. If the interval was greater than 650 ms, the message "TOO SLOW" appeared in the same manner. When the participant made an error, a message appeared to that effect, after the message, if any, regarding response speed. The message remained on the screen until the participant pressed the space bar. At the beginning of every 6th trial, a message appeared asking the participant to press the space bar to see the task array. When the participant pressed the space bar, circles simultaneously appeared in the twenty-one test locations. The array of circles remained on the screen until the participant pressed the space bar.

Results

Table 2 presents the error-rate and response-time data for the experiment, collapsed across participants, as a function of the horizontal and vertical distance between the test locations. To assess the relationship between error rate and response time, a correlation was computed, for each participant, between error rate and mean response time across the 20 distance combinations. Across participants, the average correlation was .389. Thus, there was no speed-accuracy trade-off. Focusing primarily on the error-rate data is warranted. An equal variance Gaussian signal detection analysis of the error-rate data, treating $2 \geq$ trials as signal and $3 \leq$ trials as noise, found d' and c to be 1.83 and .095 respectively. Thus, performance was moderately accurate and demonstrated a slight bias toward '3<' responses.

Examination of Table 2 suggests that the pattern of Dopkins (2005) was replicated. A generalized linear model analysis⁹ found that error rate for $2 \geq$ trials increased with increases in horizontal ($Z = 12.36$, $SE = .04$, $p < .0001$) and vertical ($Z = 12.67$, $SE = .04$, $p < .0001$) distance. The effects of horizontal and vertical distance for

$2 \geq$ trials were of roughly equivalent size. At the same time the error rate for $3 \leq$ trials decreased with increases in horizontal ($Z = 7.70$, $SE = .05$, $p < .0001$) and vertical ($Z = 6.54$, $SE = .02$, $p < .0001$) distance.

Insert Table 2 about here

Our goal in the experiment was to distinguish the contending hypotheses in terms of their proposed *processes*, and the critical values that these processes compared, as revealed in sequential dependencies on a crucial subset of trials. As a precondition to doing this, we modeled performance across all trials with GRT to establish the *representations* underlying the proposed processes. We made an attempt, for each hypothesis, to find the parameters that allowed the hypothesis to best fit the average numbers of correct and incorrect responses, across participants, for each distance combination (as are reflected in Table 2), assuming the same series of trials as participants experienced. We fit the three hypotheses with a simplex procedure, using the likelihood objective (Busemeyer & Diederich, 2010). Details regarding the specific models that were fit are given in the appendix. The fit to the three hypotheses was good: IE Hypothesis: $G^2 = 27.23$; IP Hypothesis: $G^2 = 32.70$; ID Hypothesis: $G^2 = 32.60$. Tables 3-5 give the predicted error rates for each hypothesis under the best fitting parameters. Tables 9 and 10 describe the parameters and give the best-fitting values.

Insert Tables 3-5 about here

It is worth noting that the predictions of the IP Hypothesis departed systematically from the observed results. Notice that the IP Hypothesis predicted equivalent error rates for all trials on which the horizontal and vertical distance between the test locations summed to the same value (these are trials corresponding to sets of cells parallel to the minor diagonal of Table 4). This prediction followed under the

hypothesis because, when MSI is absolute, as it must have been to account for the interchangeability of perceived horizontal and vertical distance in Experiment 1, the mental representation of the task array collapses the horizontal and vertical dimensions. Pairs of test locations that are separated by the same sum of horizontal and vertical distance are associated with the same perceived horizontal and vertical distance. Given that performance in the task depends on perceived horizontal distance, performance should be equivalent for all such pairs. In fact, however, error rates were not equivalent for such trials. For example, the error rate (.31) for trials on which the distance combination was {3,1} differed from the error rate (.17) for trials on which the distance combination was {4,0} ($Z = 7.14$, $SE = .09$, $p < .0001$). Similarly, linear trends were present across the error rates for trials on which the distance combinations were {3,2}, {4,1}, and {5,0} ($Z = 5.23$, $SE = .04$, $p < .0001$) and trials on which the distance combinations were {4,2}, {5,1}, and {6,0} ($Z = 3.48$, $SE = .04$, $p < .0005$).

Having found the representational parameters that allowed each hypothesis to best fit the aggregate data for the experiment, we used these parameters, along with simple, plausible, processing assumptions, to simulate predictions regarding the sequential manipulation. Specifically, we used each hypothesis to simulate error rates on the target trial for the different types of crucial trial-pair, under the assumption that performance on a given target trial would be facilitated if the critical values for the preceding trial and target trial were similar. In detail, we simulated error rates, assuming the same series of trials as participants experienced, under the assumption that the standard deviations of the *critical distribution(s)* (corresponding to the test locations under the IE and IP Hypotheses and to the distance combination for the test locations under the ID Hypothesis) for a given target trial was/were reduced by a proportion r if the critical values on the preceding trial and target trial differed by less than a criterion

value d . We varied r and d informally for each hypothesis until the ordering of error rates across subsets of trial-pair types matched as closely as possible the ordering given by the idealized analysis. In Table 1, the fifth line of the section for each hypothesis shows the mean error rate that was simulated across the pair types in each subset of pair types and the complementary subset (that is, the subsets of pair types for which the distance combination on the target trial was $\{0,2\}$). For the most part, the ordering of the simulated error rates matched the ordering given by idealized predictions.

In Table 1, the sixth line of the section for each hypothesis shows the mean error rate that was observed across each subset of the crucial trial-pair types and the complementary subset. We will refer to the trial-pair types in terms of the distance combination on the preceding trial. The observed error rates were ordered in fairly good agreement with the predictions of the IE Hypothesis, with many of the observed differences being statistically significant. In keeping with the predictions of the hypothesis, the error rate (.164) for pair type $\{0,2\}$ was significantly lower than the error rate (.191) for trial-pair types $\{0,1\}$ and $\{1,0\}$ ($Z = 2.29$, $SE = .01$, $p = .02$), the error rate (.203) for trial-pair type $\{2,2\}$ ($Z = 2.69$, $SE = .01$, $p = .007$), and the error rate (.188) for trial-pair type $\{1,1\}$ ($Z = 1.76$, $SE = .01$, $p = .08$ (significant with one-tailed test)).

The observed error rates agreed fairly well with the predictions of the IP Hypothesis, with one of the observed differences being statistically significant (the error rate (.178) for trial-pair types $\{0,2\}$ and $\{1,1\}$ was significantly lower than the error rate (.203) for trial-pair type $\{2,2\}$ ($Z = 2.02$, $SE = .07$, $p = .04$)). However, whereas the IP Hypothesis predicted no difference between trial-pair types $\{0,2\}$ and $\{1,1\}$, as has been noted, the error rate for trial-pair type $\{0,2\}$ was significantly lower than the error rate for trial-pair type $\{1,1\}$, in accordance with the predictions of the IE Hypothesis.

Finally, the observed error rates did not agree with the predictions the ID Hypothesis. For example, whereas the hypothesis predicted that the error rate for trial-pair type {0,2} would be higher than that for any other trial-pair type, in fact the error rate (.164) for this trial-pair type was lower than that for any other trial-pair type. And whereas the hypothesis predicted that the error rate for trial-pair type {2,2} would be among the lowest for any trial-pair type, in fact the error rate (.203) for this trial-pair type was higher than that for any other trial-pair type. The ID Hypothesis might be able to accommodate the low error rate for pair type {0,2} because the assumed decision bound is attuned to perceived horizontal and vertical distance and thus to the perceived overall distance; facilitation might therefore be expected with repeated testing of the same overall distance. However, the hypothesis would predict that, if facilitation occurs with repeated testing of the same overall distance, facilitation should also occur with repeated testing of the same horizontal and vertical distance. Whereas overall distance is not explicitly represented in the model, horizontal and vertical distance is explicitly represented (See Figure 4).

Discussion

The results that Dopkins (2005) observed at the short interval were replicated. Perceived horizontal and vertical distance were essentially interchangeable for ≥ 2 trials. More important, the error rates for the target trials of the crucial trial pairs were as predicted by the IE Hypothesis.

Experiment 2

Experiment 2 sought further support for the IE Hypothesis in terms of evidence for the position weighting that the hypothesis posits. Dopkins (2005) observed larger effects of focal than non-focal distance in the complex distance task using a long response interval. Under the IE Hypothesis, such differential distance effects imply

position weighting. Pilot findings suggested that such differential distance effects also occur in the complex distance task when a relatively high level of distance exists between the locations in the task array. The IE Hypothesis can accommodate the position weighting implied by these pilot findings if we assume that the positions of locations are more accessible and amenable to weighting as the locations are further apart. Experiment 2 sought to confirm these pilot findings and to gather evidence of the position weighting that they imply.

Participants performed a complex distance task with a task array in which the locations were substantially further apart than in Experiment 1 or Dopkins (2005). Whereas some participants compared the test locations on the horizontal dimension, using a 7 column X 3 row task array as in Dopkins (2005), other participants compared the locations on the vertical dimension, using a 7 row X 3 column task array. After making the absolute distance judgment required by the complex distance task, participants, on some trials, made a relative distance judgment, deciding whether the focal or the non-focal distance between the test locations was greater. It was expected, in light of the relatively high level of distance between the locations in the task array, and the aforementioned pilot findings, that the focal distance effect would be greater than the non-focal distance effect in the absolute distance judgment. The question of interest was whether this differential distance effect reflected position weighting, as was predicted by the IE Hypothesis. To answer this question we asked whether focal distances tended to be perceived as greater than non-focal distances in the relative distance judgment. We assumed that the relative distance judgment would be based on an assessment of the orientation of the implicit segment joining the test locations. Whereas the IE Hypothesis predicted an advantage for focal distance in the relative distance judgment, as a reflection of position weighting, the IP and ID Hypotheses did

not. The rationale was as follows:

IE Hypothesis: To explain the focal distance advantage in the absolute distance judgment, the hypothesis would hold that the focal dimension was weighted more heavily than the non-focal dimension. It was assumed that the dimensional weighting in place for the absolute distance judgment would be maintained for the relative distance judgment. Thus focal distance would have an advantage. IP Hypothesis: To explain the focal distance advantage in the absolute distance judgment, the hypothesis would hold that MSI was partial. Under partial MSI, the two dimensions would be affected symmetrically. To see this, notice, in Figure 3B, that the points for locations with complementary combinations of vertical and horizontal position are equally far from the minor diagonal. This point is made more formally in Table 6 which gives average distance, on the dimensions of perceived focal and non-focal position, in an idealized IP representation, under partial MSI ($\sigma = .5$, for an explanation of σ see Appendix), between the means of the distributions for all pairs of test locations associated with each combination of focal and non-focal distance on 2 trials. Similarly symmetrical patterns were predicted under different partial degrees of MSI. Because partial MSI underlies the perception of the test locations, and because the locations must be perceived for the relative as well as the absolute distance judgment, partial MSI would be reflected in the relative as well as the absolute distance judgment. Thus focal distance would have no advantage. ID Hypothesis: To explain the focal distance advantage in the absolute distance judgment, the hypothesis would hold that the decision bound took focal distance into account to a greater degree than non-focal distance. But the decision bound for the absolute distance judgment would play no role in the relative distance judgment. Thus focal distance would have no advantage.

Insert Table 6 about here

Method

Participants. The 96 participants were drawn from the same population as the participants for Experiment 1.

Stimuli. Each test circle was 7 mm in diameter. Pairs of circles in adjacent locations were separated by 22 mm (from edge to edge) on the horizontal and vertical dimensions. The participant sat approximately 60 cm from the computer screen. Thus each test circle subtended a visual angle of approximately .67 degrees and pairs of circles in adjacent locations were separated by a visual angle of approximately 2.1 degrees.

Design. Eight groups of participants were tested. Participants in Groups 1-4 made absolute judgments of horizontal distance (2 or fewer vs. 3 or more horizontal positions) with respect to locations in a 7 column by 3 row array. Participants in Groups 5-8 made absolute judgments of vertical distance (2 or fewer vs. 3 or more vertical positions) with respect to locations in a 7 row by 3 column array. In the relative distance judgment, participants in Groups 1 and 5 responded to the prompt 'Horizontal Greater,' participants in Groups 2 and 6 responded to the prompt 'Vertical Greater,' participants in Groups 3 and 7 responded to the prompt 'Horizontal Smaller,' and participants in Groups 4 and 8 responded to the prompt 'Vertical Smaller.'

A total of 420 trials were presented in the experiment. The Crucial Trials were those on which the distance combination was either {2,1} or {1,2}. These trials were important for testing the experimental hypothesis because the levels of focal and non-focal distance complemented one another. Each type of Crucial Trial was presented 48 times. On 40 of each type of Crucial Trial, the absolute distance judgment was followed by a relative distance judgment. Each of the other 18 types of trial (20 types of trial could be distinguished on the basis of the focal/non-focal distance between the test locations) was presented 18 times. On 10 of each of these other types of trial for which focal

distance was not equivalent to non-focal distance, the absolute distance judgment was followed by a relative distance judgment. The particular test locations on a given trial were sampled randomly from those that were consistent with the values of focal and non-focal distance in place for the trial. In sum, 204 $2 \geq$ trials were presented ($48 + 48 + 108$ (6 [the number of remaining $2 \geq$ distance combinations aside from $\{2,1\}$ and $\{1,2\}$] X 18)). In addition, 216 $3 \leq$ trials were presented (12 [the number of $3 \leq$ distance combinations] X 18). The trials were presented in 21 blocks of 20 each.

Procedure

The procedure was the same as for Experiment 1 except in the following respects: a) the task array was presented at the beginning of each block of 20 trials; b) on trials requiring a relative distance judgment, the judgment was prompted by the message 'Horizontal Greater', 'Vertical Greater', 'Horizontal Smaller', or 'Vertical Smaller', which appeared on the screen after the response speed and accuracy feedback (if any) for the absolute distance judgment. The participant responded 'yes' or 'no' to the prompt. A response signal was not used for the relative distance judgment. Feedback was provided on the accuracy of the relative distance judgment.

Results

Table 7 presents the error-rate and response-time data for the absolute distance judgments, collapsed across participants, as a function of the focal and non-focal distance between the test locations. To assess the relationship between error rate and response time, correlations were computed between error rate and response time as in Experiment 1. Across participants, the average correlation between error rate and response time was .309. Thus, there was no speed-accuracy trade-off. Focusing primarily on the error-rate data is warranted. An equal variance Gaussian signal detection analysis of the error-rate data, treating $2 \geq$ trials as signal and $3 \leq$ trials as

noise, found d' and c to be 1.51 and .15 respectively. Thus, performance was again moderately accurate and again demonstrated a slight bias toward '3<' responses.

Examination of Table 7 suggests a replication of the pattern that Dopkins (2005) observed at the 1000 ms interval. A generalized linear model analysis found significant effects of focal distance ($2 \geq$ trials: $Z = 10.17$, $SE = .04$, $p < .0001$; $3 \leq$ trials: $Z = 8.37$, $SE = .03$, $p < .0001$). The analysis found significant effects of non-focal distance ($2 \geq$ trials: $Z = 7.22$, $SE = .03$, $p < .0001$; $3 \leq$ trials: $Z = 3.67$, $SE = .03$, $p < .0002$). As was expected on the basis of the pilot findings, the effect of focal distance effect was larger than the effect of non-focal distance in the data for $2 \geq$ trials.

Insert Table 7 about here

The data for the relative distance judgments were of primary interest. Here, the data for trials on which focal distance was less than 3 were most useful because focal and non-focal distance varied through the same three levels (0,1,2) on these trials; thus, the levels of focal and non-focal distance were matched. Table 8 presents the data for these trials as a function of the focal and non-focal distance between the test locations. Notice that two complementary sets of trials are presented in the table. For each combination of focal and non-focal distance in the set of trials that is presented below the main diagonal, there is a complementary combination of non-focal and focal distance in the set of trials that is presented above the main diagonal. The overall distance in the two sets is matched. In the Focal Set, which is presented below the diagonal, focal distance is greater than non-focal distance. In the Non-focal Set, which is presented above the diagonal, non-focal distance is greater than focal distance. As can be seen in the table, the error rate was greater for the Non-focal Set than for the Focal Set (.41 vs. .35) ($Z = 2.34$, $SE = .06$, $p < .01$). Response time, however, did not differ in the Focal and Non-focal sets, (1200 vs. 1139) ($F(1,95) = 2.39$, $MSe = 225,704$).

The difference between the two sets was reflected in the results for the Crucial Trials. Error rate ($Z = 2.0$, $SE = .02$, $p < .05$) and response time ($F(1,95) = 5.72$, $MSe = 109,249$) were greater for trials on which focal distance was 1 and non-focal distance was 2 than for trials on which focal distance was 2 and non-focal distance was 1.

Insert Table 8 about here

Discussion

These results support the IE Hypothesis, which predicted an advantage for focal distance over non-focal distance in the relative distance judgment. The results offer no support for the IP and ID Hypotheses, which predicted no such advantage. To allay any concerns that the results somehow reflect the unequal extension of the task array along the focal and non-focal dimensions, we note that we have replicated the results using a 6 X 6 task array.

General Discussion

We wish to understand how components corresponding to the orthogonal axes of an allocentric reference frame are extracted from distances between the objects in a layout. We have addressed this question with a task in which participants assess the horizontal and vertical distance between pairs of locations in a frontal plane. We have found that, when processing is speeded, performance in the task depends on the non-focal as well as focal distance between the test locations. To account for these results, we suggest that, under speeded conditions, neither the horizontal nor the vertical distance between a pair of locations is directly available; only the overall distance between the locations is available. The horizontal or vertical distance between the locations can be assessed, however, to the extent that the horizontal and vertical positions of the locations are weighted prior to the assessment of overall distance so that horizontal or vertical distance makes a disproportionate contribution to overall distance.

We have contrasted our account of the non-focal distance effect (the IE Hypothesis) with accounts that attribute the effect to dimensional interaction in the perception of the test locations (the IP Hypothesis) and in the decision component of the complex distance judgment (the ID Hypothesis). The contrast between the IE and the IP Hypothesis can be best seen in the way the two accounts construe the absence of dimensional interaction. Under the IE Hypothesis, dimensional interaction is absent when the non-focal dimension is given no weight at all – when the focal dimension is chosen over the non-focal dimension. Under the IP Hypothesis, dimensional interaction is absent when MSI is absent. No choice of dimension is required. The IE Hypothesis differs from the ID Hypothesis primarily in that it attributes the relative influence of focal and non-focal distance to perceptual as opposed to decisional processes. In Experiment 1 we showed that the pattern of sequential dependency in complex distance data could be accommodated better under the IE Hypothesis than under the IP and ID Hypotheses. In Experiment 2 we observed relative judgment effects that were consistent with the IE Hypothesis and not the IP and ID Hypotheses.

In addition, we note that the IE Hypothesis is simply more plausible than the ID Hypothesis. To explain the non-focal distance effect, the IE account proposes that only the overall distance between the test locations is available to perceptual processes. This is plausible given that selective access of focal distance requires reconciliation of polar and rectangular coordinate systems. To explain this effect, the ID account proposes that decision factors consistently set the decision bound in the complex distance task so as to be sensitive to non-focal as well as focal distance. It is difficult, however, to understand what these factors might be, given that the task is more easily performed on the basis of focal distance alone, with no consideration of non-focal distance.

Generality. Further work is needed to establish the boundary conditions of the

non-focal distance effect and our interpretation of it. One question is whether the IE account generalizes to situations in which processing is not speeded. In such situations, observers may use other means to assess horizontal and vertical distance. For example, to find the horizontal or vertical distance between two points, observers may mentally project one of the points onto a line running parallel to the horizontal or vertical axis of the prevailing reference frame and through the other point¹⁰. Regardless of the relevance of the IE account to non-speeded situations, speed is necessary in enough situations that the present results are probably of general relevance.

Implications for the perception of distance and position. We suggest that the horizontal or vertical distance between two points cannot be directly assessed under speeded conditions because distance is assessed on the basis of visual angle, in terms of implicit polar coordinates, which do not acknowledge the horizontal and vertical dimensions. We suggest that, under speeded conditions, horizontal and vertical distance can only be assessed through the weighting of position information. By implication, horizontal and vertical position are available under such conditions, although horizontal and vertical distance are not.

Implications for attention. Precedent exists for the idea that the dimensions of *psychological* space can be weighted to reflect the requirements of the current task. This sort of dimension weighting has been proposed to explain performance in object-judgment, visual search, and category learning tasks (Kumada, 2001; Muller & O'Grady, 2000; Nosofsky, 1986, 1987). By suggesting that the dimensions of *physical* space can be also weighted to reflect the requirements of the current task, the present results contribute to an emerging view of physical space as subject to attentional manipulation (Liverence & Scholl, 2011).

In conclusion, we have presented evidence that, under speeded conditions, the

horizontal or vertical distance between two locations in a frontal plane can only be assessed to the extent that the horizontal or vertical positions of the locations are weighted more heavily than the vertical or horizontal positions, so that horizontal or vertical distance makes a disproportionate contribution when the overall distance between the locations is assessed.

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Notes

1. Notice that the effect of vertical distance cannot reflect simple confusion as to the identity of the dimension of comparison. If the vertical distance effect had reflected such confusion, then the error rate for $2 >$ trials should have increased at the same rate with increases in vertical distance for all values of horizontal distance and the error rate for $3 <$ trials should have decreased at the same rate with increases in vertical distance for all values of horizontal distance.
2. Although performance in Dopkins (2005) task was quite inaccurate, because participants were required to respond very quickly, the task for the experiment could, in principle, have been performed accurately.
3. Controversy exists as to the geometry of visual space and in fact as to whether visual space can be understood in terms of a consistent geometry (Foley, 1964, 1972; Foley et al., 2004; Indow, 1991, 1997; Koenderink, Van Doer, Kappers, & Todd, 2002; Luneburg, 1947, 1950; Wagner, 1985). For the purposes of understanding performance in the complex distance task, in which the test circles appear a short distance from the participant, in a frontal plane, we are probably safe in assuming a Euclidean space.
4. Given that the visual system can access the non-focal positions of the test locations for the purposes of dimension weighting, one might ask why the system cannot access the focal positions of the test locations and compute the focal distance between the locations from those positions. Evidently, the distance between the test locations, like exocentric distance in general, must be assessed on the basis of the visual angle. One possibility is that the locations are held in a visual buffer and that the distance between the locations is assessed in terms of this buffer once the positions of the locations are adjusted.

5. Although other conceptions of dimensional interaction in perception have been proposed (e.g. Dzhamfarov, 2004), the MSI conception is attractive here because it gives a clear idea of how to model dimensional interaction in suprathreshold terms.
6. Can the IE and IP Hypotheses be instantiated with distance combinations as the fundamental stimulus entities? The IE Hypothesis can be instantiated this way, but the idea of position weighting is then difficult to instantiate. The IP Hypothesis cannot be instantiated with distance combinations as the fundamental entities. To see why, notice that, in this case, a decision bound would be necessary, as in the ID account, and notice that it would be impossible for the decision bound to change in a coherent way as the mental representation of the distance combinations changed from no MSI to complete MSI. For example, let us assume that the distance combinations for the complex distance task of Dopkins (2005) are represented without MSI. In this case, the representation of the distance combinations would be similar to what is shown in Figure 3A. Let us assume that the decision bound is a line perpendicular to the horizontal axis, at horizontal distance 2.5. If this decision bound is to change consistently with the rest of the representation as we move from the absence of MSI (Figure 3A) to increasing degrees of MSI (Figure 3B), it should go from being perpendicular to the horizontal axis to being more and more nearly parallel to the minor diagonal, until, with absolute MSI (Figure 3C), the decision bound is absorbed into the minor diagonal. However, intuition tells us that, for absolute MSI, the decision bound must be a single point on the minor diagonal. A further problem is that some distance combinations that were represented on opposing sides of the decision bound under the absence of MSI would end up being represented on the same side of the decision point under absolute MSI. Consider, for example, the following distance combinations from Dopkins (2005): a) {1,2}, b) {3,0}. Under the absence of MSI, these distance combinations would be on opposite sides of

a decision bound perpendicular to the horizontal axis at horizontal distance 2.5. Under absolute MSI, in contrast, these distance combinations would be represented in terms of the same point, and thus would be on the same side of the decision point. This follows because combinations of horizontal and vertical distance that sum to the same value should be equivalent in this case, because horizontal and vertical distance should be interchangeable under absolute MSI.

7. Complementary predictions held with respect to trial-pair types for which the distance combination on the target trial was {0,2}.
8. These predictions might be naturally extended to the non-ideal case by adding standard assumptions regarding variability in applications of GRT (Ashby & Lee, 1993).
9. We used a generalized linear model analysis, based on maximum likelihood estimation (GENMOD in SAS) because the error rates were low in some of the cells of the analysis and we did not feel that least-squares estimation (GLM in SAS) was appropriate. In all analyses we specified the identity linking function.
10. Notice that under such a process, the time required for a distance judgment, and thus the error rate in a response signal task, would increase with increases in the non-focal distance between the test locations for negative as well as positive trials, in contrast to the pattern that we have observed in the present study. Thus the process does not appear to have been used here.

Appendix

IE Hypothesis. The distributions for all locations with the same horizontal or vertical position had the same mean on the corresponding dimension of the psychological space. The distribution means for locations with horizontal and/or vertical position 1 were set to 1 on the dimension of perceived horizontal and/or vertical position. The means for locations with higher levels of horizontal and vertical position were determined by 8 parameters: d_{h12} , d_{h23} , d_{h34} , d_{h45} , d_{h56} , and d_{h67} , which gave the distances, on the dimension of perceived horizontal position, between the means of the distributions for locations having adjacent levels of horizontal position, and d_{v12} and d_{v23} , which gave the distances, on the dimension of perceived vertical position, between the means of the distributions for locations having adjacent levels of vertical position. The distributions for all of the locations had the same standard deviation, sd_h , on the dimension of perceived horizontal position and the same standard deviation, sd_v , on the dimension of perceived vertical position. The value of sd_v was set at 1. The covariance between values on the two dimensions of the space was 0 in the distributions for all of the locations. The response on a given trial was determined by sampling from the distributions for the test locations, computing the Euclidean distance between the sampled points, and comparing the distance to a criterion value. To model the dimension weighting assumed under the hypothesis, the distance on the dimension of perceived vertical position was weighted ρ in computing the distance between the sampled points. The criterion value of distance was c . In sum, the hypothesis had 11 degrees of freedom. The parameters for the hypothesis are summarized in Table 9. The best-fitting values for the data of Experiment 1 are given in Table 10.

Insert Table 9 about here

Insert Table 10 about here

IP Hypothesis. The means of the distributions for the locations were set in three stages: 1) a representation was constructed for the task array in the case of absolute MSI. 2) the representation for absolute MSI was used to construct a representation for complete absence of MSI. 3) the representation for complete absence of MSI was adjusted to reflect the actual degree of MSI that held for the dataset.

Constructing the representation for absolute MSI. The process is illustrated in Figure 5A. The psychological space in which the locations were represented was divided into four sectors by the lines ℓ and κ , which intersected at the origin of the space. The angles formed by the intersection of ℓ and κ had magnitudes α and β , reflecting the relative degree of anisotropy in the estimation of distances between positions on the horizontal and vertical dimensions. Because of the relationship $\alpha + \beta = \pi$, only one parameter (α) was needed to set the magnitudes of these angles. Only nine distributions were needed to represent the 21 locations because most distributions represented multiple locations. For example, the same distribution represented the locations [1,2] and [2,1]. The centers of the nine distributions were located on a single *composite axis* that was aligned with either ℓ or κ . Here we describe the case in which the composite axis was aligned with κ ; the process was essentially the same when the axis was aligned with ℓ . Figure 5A shows the centers of the nine distributions (labeled A – H) under complete MSI. Once the magnitudes α and β were set, only 8 more parameters were needed to set the centers of the nine distributions. The center of one of the distributions (labeled G in the figure) coincided with the origin of the psychological space (labeled L in the figure), and the centers of the rest of the distributions were determined by the parameters: d_{AB} , d_{BC} , d_{CD} , d_{DE} , d_{EF} , d_{FG} , d_{GH} , d_{HI} , which gave the distances between the centers of adjacent distributions on the composite axis.

Constructing the representation for complete absence of MSI. Place a copy of the segment AG so that it runs from **Y**, on κ , to **Z**, on ℓ , with **Y** and **Z** being to the left and right of **L**, respectively, and so that the angles **LYZ** and **LZY** both have the same value, δ , which is equivalent to $(180 - \beta)/2$. Multiply the lengths of the component segments in AG (that is, AB, BC, CD, DE, EF, and FG) by $\psi = (1 - \cos(\alpha))^{1/2} / (1 - \cos(\beta))^{1/2}$ to create AG'. We will refer to this in the discussion that follows as the *anisotropy transformation* (The anisotropy transformation, which follows from the cosine law, is necessary because, to the extent that there is anisotropy in the estimation of distances on the horizontal and vertical dimensions, so that α differs from β , the same distance on the composite axis must map into different distances on the horizontal and vertical dimensions of the psychological space). Place AG' so that it runs from **Y**, on κ , to **J**, on ℓ , with **Y** and **J** being below and above **L**, respectively, and so that the angles **LYJ** and **LJY** both have the same value, γ , which is equivalent to $(180 - \alpha)/2$. Assign the labels **U** and **O** to points B' and C' on AG'. Remove the points C' through G' from AG'.

Place a copy of segment CG so that it runs, parallel to **YZ**, from **V**, on κ , to **W**, on ℓ , and so that the angles **LWV** and **LWV** both have the value δ . Multiply the lengths of the component segments in CG (that is, CD, DE, EF, and FG) by ψ to create CG'. Place CG' so that it runs, parallel to **YJ**, from **V** on κ , to **K** on ℓ , and so that the angles **LVK** and **LKV** both have the value γ . Assign the label **P** to point D' on CG'. Remove the points E' through G' from CG'.

Place a copy of segment EG so that it runs, parallel to **VW**, from **Q**, on κ , to **R**, on ℓ , and so that the angles **LQR** and **LRQ** both have the value δ .

Multiply the lengths of the component segments in GI (that is, GH and HI) by ψ to create GI'. Place GI' so that G' is on ℓ , coincident with **Z**. Extend GI' until it intersects m at **N**, arranging **ZN** so that the angles **LZN** and **LNZ** both have the value γ . Assign the labels **X** and **T** to points H' and I' on GI'. Multiply the length of the segment GH by ψ to create GH'. Place GH' so that G' is on ℓ , coincident with **W**. Extend GH' until it intersects m at **M**, arranging **WM** so that the angles **LWM** and **LMW** both have the value γ . Assign the label **S** to point H' on GH'. Figure 5B shows the representation resulting from this process. Each location is now represented by a different distribution; the figure shows the points corresponding to the centers of the distributions. The location associated with each point is listed next to the point. Each point is labeled A – H to indicate the nature of its derivation relative to the representation for the case of complete MSI. Some of the points are also labeled **J-Z** for reference in later discussion.

Adjusting the representation for no MSI to reflect the actual degree of MSI that held for the data being fit. The process is illustrated in Figure 5C. Place the segments **YZ**, **VW**, and **QR** so that the angles **LZY**, **LWV**, and **LRQ** all have the value ζ , which is equivalent to $\delta + \sigma(\alpha - \delta)$, where δ is the value of angles **LZY**, **LWV**, and **LRQ** under no MSI (See Figure 5B), α is the value of those angles under absolute MSI (compare Figure 5B and Figure 5A), and σ gives the degree of MSI that actually held (the effect of this is to add to the value of each of the angles under no MSI a proportion of the difference between that value and the value of the angle under complete MSI). Place the segments **YJ**, **VK**, **WM**, and **ZN** so that the angles **YJL**, **VKL**, **MWL**, and **NZL** all have the value ϵ , which is equivalent to $\gamma + \sigma(\beta - \gamma)$, where γ is the value of angles **YJL**, **VKL**, **MWL**, and **NZL** under no MSI, β is the value of those angles under absolute MSI, and σ gives the degree of MSI that actually held (the rationale here is the same as was

described earlier for the calculation of ζ). To place the segments **YJ**, **VK**, **WM**, and **ZN**, it will be necessary to adjust their lengths to reverse the anisotropy transformation in proportion to the degree to which MSI held. Recall that the anisotropy transformation is used in creating the representation for the state of no MSI. It reflects the relative degree of anisotropy in the estimation of distances on the horizontal and vertical dimensions in the state of absolute MSI. If MSI is absolute, the horizontal and vertical dimensions are collapsed, so there is no anisotropy in the estimation of distances on the two dimensions. By the same token, the transformation must be reversed to the degree that MSI is absent. We illustrate the necessary length adjustment for segment **YJ**. On the basis of the segment **YZ** and the angles β and ζ , use the sine law to find the length of the segment **YL**. On the basis of the segment **YL** and the angles α and ϵ , use the sine law to find the *adjusted* length of the segment **YJ**. Adjust the lengths of the components of the *adjusted* **YJ** (**YU**, **UO**, and **OJ**) so that their proportions match the proportions on the components in the original **YJ**.

The distributions for the 21 locations had the same standard deviation, sd , on the two dimensions of the psychological space (separate standard deviations could not be used for the horizontal and vertical dimensions, as with the IP and ID Hypotheses, because of the way the two dimensions collapsed under MSI); the covariance between values on the two dimensions of the space was 0 in the distributions for all of the locations. The response on a given trial was determined, as was described earlier, by sampling from the distributions for the test locations, computing the distance between the sampled points on the dimension of perceived horizontal position and comparing that distance to a criterion value. The criterion value of distance was c . In sum, the hypothesis had 12 degrees of freedom. The parameters are summarized in Table 9. The best-fitting values for the data of Experiment 1 are given in Table 10.

Insert Figures 5A-5C about here

Because the MSI conception was not developed for use with pairs of stimuli, it must be adapted to that purpose. To explain effects of non-focal distance in the complex distance task, it is necessary that the representation of the task array collapse on a given trial toward the diagonal most nearly parallel to a hypothetical line connecting the points for the test locations. Thus, the composite axis was aligned with either ℓ or π on a given trial, in such a way as to always be roughly parallel to the line joining the test locations. Figure 6 demonstrates the reasons for this.

Insert Figure 6A-6C about here

ID Hypothesis. The distributions for all distance combinations with the same level of horizontal or vertical distance had the same mean on the corresponding dimension of the psychological space. The means for distance combinations with horizontal or vertical distance 0 were set to 0 on the dimensions of perceived horizontal or vertical distance. The means for distance combinations with higher levels of horizontal and vertical distance were determined by 8 parameters: d_{h01} , d_{h12} , d_{h23} , d_{h34} , d_{h45} , and d_{h56} , which gave the distances, on the dimension of perceived horizontal distance, between the means of the distributions for distance combinations having adjacent levels of horizontal distance, and d_{v01} and d_{v12} , which gave the distances, on the dimension of perceived vertical distance, between the means of the distributions for distance combinations having adjacent levels of vertical distance. The distributions for all of the distance combinations had the same standard deviation, sd_h , on the dimension of perceived horizontal position and the same standard deviation, sd_v , on the dimension of perceived vertical position. The value of sd_v was set at 1. The covariance between values on the two dimensions of the space was 0. The decision bound was quadratic

(Ashby, 1992; Ashby & Gott, 1998; Ashby & Maddox, 1992; Maddox, 2001, Maddox & Ashby, 1993), determined with the expression $V = c_1 (H - c_2)^2 + c_3$, where V and H are vertical and horizontal distance respectively (this rather than a more complex decision bound was used to equate as nearly as possible the numbers of parameters across hypotheses)). In sum, this account had 13 degrees of freedom. The parameters are summarized in Table 9. The best-fitting values for the data of Experiment 1 are given in Table 10.

Table 1. Experiment 1: Predictions and results for sequential analysis.

IE Hypothesis					
Predicted ordering	{0,2}	{1,2}	{1,1}	{2,2}	{0,1}
		{2,1}			{1,0}
Perceived overall distance - preceding trial	2	2.236	1.414	2.828	1
Perceived overall distance - target trial	2	2	2	2	2
Difference	0	.236	.586	.828	1
Simulated error rate	.141	.174	.226	.236	.228
Observed error rate	.164	.183	.188	.203	.191
IP Hypothesis					
Predicted ordering	{0,2}	{0,1}	{2,2}		
	{1,1}	{1,0}			
		{1,2}			
		{2,1}			
Perceived horizontal distance - preceding trial	2	1 or 3	4		
Perceived horizontal distance – target trial	2	2	2		
Difference	0	1	2		
Simulated error rate	.111	.147	.166		
Observed error rate	.178	.187	.203		
ID Hypothesis					
Predicted ordering	{1,0}	{2,2}	{1,1}	{0,1}	{0,2}
	{2,1}			{1,2}	
Perceived horizontal, vertical distance – preceding trial	1,0 2,1	2,2	1,1	0,1 1,2	0,2

Perceived horizontal, vertical distance – target trial	2,0	2,0	2,0	2,0	2,0
Difference	1,0 0,1	0,2	1,1	2,1 1,2	2,2
Simulated error rate	.170	.184	.220	.229	.240
Observed error rate	.176	.203	.188	.196	.164

Note. For each hypothesis, the table summarizes idealized predictions for target trials on which the distance combination was {2,0}. Complementary predictions held for target trials on which the distance combination was {0,2}. In the first line under each hypothesis, the table shows the predicted ordering of subsets of trial-pair types, in terms of error rate, where the trial-pair types are identified in terms of the distance combination on the preceding trial. In the second and third lines for each hypothesis the table shows, for each subset of trial-pair types, the idealized critical values on which the responses to the preceding trial and the target trial would be based under the hypothesis. In the fourth line the table shows, for each subset of trial-pair types, the difference between those critical values. In the fifth line the table shows the mean error rate that was simulated across the trial-pair types in each subset of trial-pair types and the complementary subset. In the sixth line the table shows the mean error rate that was observed across the trial-pair types in each subset of trial-pair types and the complementary subset. See text for details.

Table 2. Experiment 1: Performance as Function of Distance between Test Locations on Horizontal and Vertical Dimensions

Horizontal distance (position units)	Vertical distance (position units)		
	0	1	2
	Error rate		
0		.13 (.12)	.20 (.14)
1	.13 (.13)	.17 (.13)	.31 (.17)
2	.20 (.12)	.33 (.16)	.54 (.18)
3	.37 (.16)	.31 (.16)	.21 (.14)
4	.17 (.16)	.16 (.13)	.15 (.14)
5	.14 (.15)	.13 (.15)	.11 (.14)
6	.11 (.14)	.11 (.15)	.12 (.14)
	Response time		
0		558 (158)	559 (65)
1	554 (58)	542 (64)	579 (82)
2	562 (67)	583 (100)	595 (108)
3	599 (104)	598 (156)	576 (161)
4	563 (55)	557 (67)	556 (95)
5	557 (94)	540 (56)	538 (53)
6	548 (67)	540 (63)	541 (62)

Note. Standard deviations are given in parentheses. Response time means include the 400 ms that elapsed before the response signal appeared.

Table 3. Experiment 1: Error rate predicted under IE Hypothesis

Horizontal distance (position units)	Vertical distance (position units)		
	0	1	2
0		.14	.29
1	.13	.18	.33
2	.27	.32	.46
3	.52	.48	.37
4	.30	.27	.19
5	.14	.12	.09
6	.05	.04	.03

Table 4. Experiment 1: Error rate predicted under IP Hypothesis

Horizontal distance (position units)	Vertical distance (position units)		
	0	1	2
0		.15	.25
1	.15	.25	.40
2	.25	.40	.58
3	.57	.42	.25
4	.42	.25	.14
5	.25	.14	.06
6	.14	.06	.02

Table 5. Experiment 1: Error rate predicted under ID Hypothesis

Horizontal distance (position units)	Vertical distance (position units)		
	0	1	2
0		.13	.25
1	.05	.16	.28
2	.21	.42	.62
3	.58	.33	.17
4	.43	.20	.09
5	.23	.09	.03
6	.37	.16	.07

Table 6. Experiment 2: IP Hypothesis - Distances between Means of Distributions for Test Locations from 2 Trials on Dimensions of Perceived Focal and Non-focal Position as Function of Actual Focal and Non-focal Distance between Locations

Actual focal distance	Actual non-focal distance					
	0		1		2	
	Psychological dimension					
	Focal	Non-focal	Focal	Non-focal	Focal	Non-focal
0			.3825	.9240	.7650	1.8479
1	.9240	.3825			1.6893	2.2301
2	1.8478	.7650	2.2301	1.6893		

Note. The table reports the average distances between the means of the distributions for all pairs of test locations associated with each combination of focal and non-focal distance. The distances are measured on the dimensions of perceived focal and non-focal position with MSI partial ($\sigma = .5$; see Appendix for explanation of σ). Similarly symmetrical patterns were predicted under different degrees of MSI.

Table 7. Experiment 2: Performance on Absolute Distance Judgments as Function of Distance between Test Locations on Focal and Non-focal Dimensions

Focal distance (position units)	Non-focal distance (position units)		
	0	1	2
	Error rate		
0		.16 (.18)	.22 (.17)
1	.19 (.20)	.19 (.21)	.27 (.19)
2	.33 (.23)	.34 (.20)	.52 (.22)
3	.30 (.19)	.29 (.18)	.23 (.17)
4	.19 (.17)	.17 (.17)	.16 (.16)
5	.15 (.13)	.15 (.15)	.13 (.14)
6	.15 (.17)	.14 (.16)	.13 (.17)
	Response time		
0		661 (355)	705 (307)
1	665 (273)	643 (245)	694 (293)
2	691 (297)	705 (296)	722 (303)
3	683 (240)	696 (289)	684 (357)
4	665 (264)	665 (301)	647 (282)
5	616 (217)	623 (195)	638 (262)
6	637 (228)	626 (272)	615 (214)

Note. Standard deviations are given in parentheses. Response time means include the 400 ms that elapsed before the response signal appeared.

Table 8. Experiment 2: Performance on Relative Distance Judgments as Function of Distance between Test Locations on Focal and Non-focal Dimensions

Focal distance (position units)	Non-focal distance (position units)		
	0	1	2
	Error rate		
0		.40 (.27)	.38 (.27)
1	.38 (.26)		.45 (.24)
2	.31 (.22)	.36 (.19)	.
	Response time		
0		1239 (750)	1170 (621)
1	1283 (821)		1191 (586)
2	1056 (548)	1077 (571)	

Note. Standard deviations are given in parentheses.

Table 9

Descriptions of Parameters for Three Accounts of Experiment 1 Results

IE Hypothesis

d_{h12} distance, on dimension of perceived horizontal position, between means of distributions for locations with horizontal positions 1 and 2

d_{h23} distance, on dimension of perceived horizontal position, between means of distributions for locations with horizontal positions 2 and 3

d_{h34} distance, on dimension of perceived horizontal position, between means of distributions for locations with horizontal positions 3 and 4

d_{h45} distance, on dimension of perceived horizontal position, between means of distributions for locations with horizontal positions 4 and 5

d_{h56} distance, on dimension of perceived horizontal position, between means of distributions for locations with horizontal positions 5 and 6

d_{h67} distance, on dimension of perceived horizontal position, between means of distributions for locations with horizontal positions 6 and 7

d_{v12} distance, on dimension of perceived vertical position, between means of distributions for locations with vertical positions 1 and 2

d_{v23} distance, on dimension of perceived vertical position, between means of distributions for locations with vertical positions 2 and 3

sd_h standard deviation of distributions for locations on dimension of perceived horizontal position

ρ weighting given vertical dimension in calculation of overall distance

c criterion value of distance

IP Hypothesis

- α value of angle determined by intersection of ℓ and m ; reflects relative degree of anisotropy in the estimation of distances on the horizontal and vertical dimensions
- d_{AB} distance between points A and B on composite axis
- d_{BC} distance between points B and C on composite axis
- d_{CD} distance between points C and D on composite axis
- d_{DE} distance between points D and E on composite axis
- d_{EF} distance between points E and F on composite axis
- d_{FG} distance between points F and G on composite axis
- d_{GH} distance between points G and H on composite axis
- d_{HI} distance between points H and I on composite axis
- sd standard deviation of distributions for all locations on both dimensions of psychological space
- σ determines degree of MSI in representation of task array
- c criterion value of distance

ID Hypothesis

- d_{h01} distance, on dimension of perceived horizontal distance, between means of distributions for distance combinations with horizontal distances 0 and 1
- d_{h12} distance, on dimension of perceived horizontal distance, between means of distributions for distance combinations with horizontal distances 1 and 2
- d_{h23} distance, on dimension of perceived horizontal distance, between means of distributions for distance combinations with horizontal distances 2 and 3
- d_{h34} distance, on dimension of perceived horizontal distance, between means of distributions for distance combinations with horizontal distances 3 and 4

d_{h45} distance, on dimension of perceived horizontal distance, between means of distributions for distance combinations with horizontal distances 4 and 5

d_{h56} distance, on dimension of perceived horizontal distance, between means of distributions for distance combinations with horizontal distances 5 and 6

d_{v01} distance, on dimension of perceived vertical distance, between means of distributions for distance combinations with vertical distances 0 and 1

d_{v12} distance, on dimension of perceived vertical distance, between means of distributions for distance combinations with vertical distances 1 and 2

sd_h standard deviation of distributions for all distance combinations on dimension of perceived horizontal distance

$c1, c2, c3$ coefficients of decision bound

Table 10

Mean Values of Parameters for Three Accounts of Experiment 1 Results

IE Hypothesis

d_{h12}	d_{h23}	d_{h34}	d_{h45}	d_{h56}	d_{h67}	d_{v12}	d_{v23}	sd_h	ρ	c
1.00	1.01	1.04	.95	.90	1.03	.93	.96	.82	.99	3.89

IP Hypothesis

α	d_{AB}	d_{BC}	d_{CD}	d_{DE}	d_{EF}	d_{FG}	d_{GH}	d_{HI}	sd	σ	c
1.57	1.000	.999	1.001	1.001	1.000	1.001	.999	.999	1.13	.998	2.51

ID Hypothesis

d_{h01}	d_{h12}	d_{h23}	d_{h34}	d_{h45}	d_{h56}	d_{v01}	d_{v12}	sd_h	$c1$	$c2$	$c3$
.35	.67	.71	1.93	.05	1.16	3.47	2.73	1.41	.93	3.77	-2.09

Figure captions

Figure 1. Hypothetical layout. Solid circles A - F are objects. Open circle X is a hypothetical point. Let us assume that the layout of objects A – F is understood in terms of a reference frame consisting of orthogonal axes, one of which intersects A, C, D, and F (Mou et al., 2009; Mou, et al., 2007). Understanding the distance BC in terms of this reference frame involves extracting the distances BX and CX, respectively, from that distance.

Figure 2. Task array used in the complex distance task of Dopkins (2005). Positions are indicated on the horizontal and vertical dimensions (participants did not see these numbers). Participants were to indicate on each trial whether the test locations were two or fewer horizontal positions apart.

Figure 3. Mental representation underlying performance in the complex distance task according to the IP Hypothesis. Figure 3A shows the task array when MSI is absent. Each point corresponds to one of the locations in the array. Figure 3B shows the task array under MSI as it has typically been studied in past work. A location with a given horizontal position has a greater value on the dimension of perceived horizontal position the greater its vertical position. However, an increase on the dimension of horizontal position produces a greater increase on the dimension of perceived horizontal position than does an increase on the dimension of vertical position. Figure 3C shows the array under absolute MSI. The perceptual effects of increases on the dimension of horizontal and vertical position are indistinguishable. The mental representation of the task array collapses the horizontal and vertical dimensions of the array onto the minor diagonal of the psychological space. Most of the points in the resulting representation correspond to multiple locations. The dotted lines show the trajectories according to which the two-dimensional representation collapses into a one-dimensional representation under MSI.

Figure 4. Mental representation underlying performance in the complex distance task according to the ID Hypothesis. Each point corresponds to the center of the distribution for one of the distance combinations in the task. The dashed line shows a decision bound under which the response depends on perceived horizontal and vertical distance equivalently.

Figure 5. Mental representation assumed in fitting the data of Experiment 1 under the IP Hypothesis. Each point corresponds to the center of a distribution. The location(s) represented by each distribution is/are listed in brackets next to the point for the distribution. Each location is identified in terms of a pair of integers that give positions on the horizontal and vertical dimensions of the task array, according to the system of Figure 2. Figure 5A: The representation under absolute MSI. Lines ℓ and m intersect at L , forming angles of sizes α and β , reflecting the relative degree of anisotropy in the estimation of distances on the horizontal and vertical dimensions. Only nine distributions are needed to represent the 21 locations because most distributions represent multiple locations. The points are labeled A-H for comparison with Figures 5B and 5C. Figure 5B: The representation under the complete absence of MSI. Angles γ and δ are given by angles α and β , respectively. Each point is labeled A – H to indicate the nature of its derivation relative to the representation for the case of absolute MSI in Figure 5A. Some of the points are also labeled **J-Z** for comparison with the representation for the degree of MSI that actually held for the dataset in Figure 5C. Notice that each location is represented by a separate distribution. Figure 5C: The representation under the degree of MSI that actually held for the dataset. The angles ϵ and ζ give the degree of MSI that actually held. In other respects the representation follows that of Figure 5B. See text for details.

Figure 6. Demonstrating that the representation of the task array under the IP Hypothesis must collapse on a given trial toward the diagonal most nearly parallel to the

hypothetical line connecting the points for the test locations. The filled circles indicate the task array when MSI is absent. The dashed line indicates the diagonal onto which the task array collapses under MSI. The open circles indicate the task array when MSI is complete. The solid lines indicate the trajectory according to which the array collapses.

In Figure 6A, the squares indicate points representing two locations when MSI is absent. Notice that the points are separated by 3 units of horizontal distance and 2 units of vertical distance. A hypothetical line connecting the points is more nearly parallel to the minor diagonal than the main diagonal. Triangles indicate points representing the locations when the array collapses onto the minor diagonal and MSI is complete. Now the points are separated by 5 units of horizontal/vertical distance (notice that the density of points increases when MSI is complete). Thus, the horizontal/vertical distance separating the points when MSI is complete is the sum of the horizontal and vertical distances separating the points when MSI is absent. This relationship allows the explanation of effects of non-focal distance.

In Figure 6B, the squares again indicate points representing two locations when MSI is absent. Notice that the points are separated by 4 units of horizontal distance and 2 units of vertical distance. A hypothetical line connecting the points is more nearly parallel to the main diagonal than the minor diagonal. Triangles indicate points representing the locations when MSI is complete and the array collapses onto the minor diagonal. Notice that the points are separated by 2 units of horizontal/vertical distance. Thus, the horizontal/vertical distance separating the points when MSI is complete is not the sum of the horizontal and vertical distances separating the points when MSI is absent. Effects of non-focal distance cannot be explained in terms of this representation.

Figure 6C shows the points of 6B with the array collapsing onto the main diagonal. Now, when MSI is complete, the points are separated by 6 units of horizontal/vertical distance. The horizontal/vertical distance separating the points when MSI is complete is the sum of the

horizontal and vertical distances separating the points when MSI is absent. This relationship allows the explanation of effects of non-focal distance.

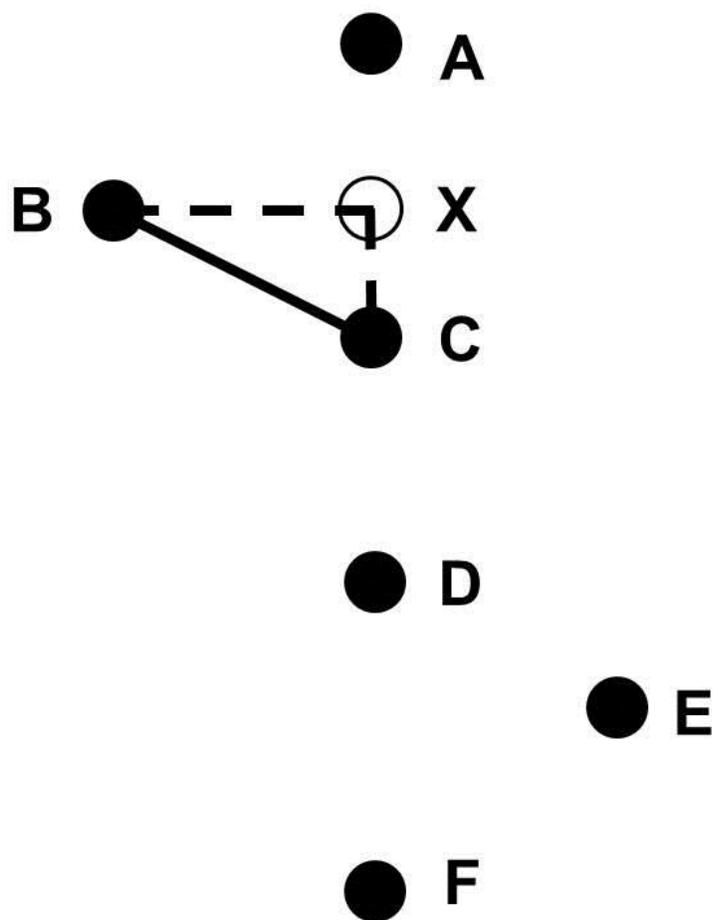


Figure 1

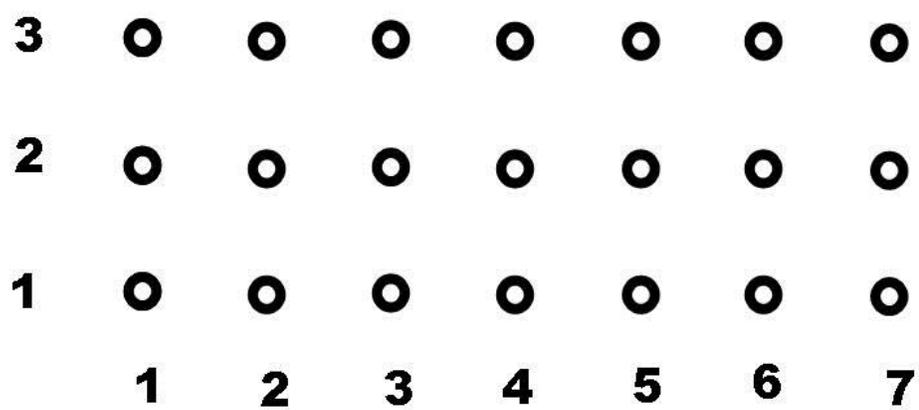


Figure 2

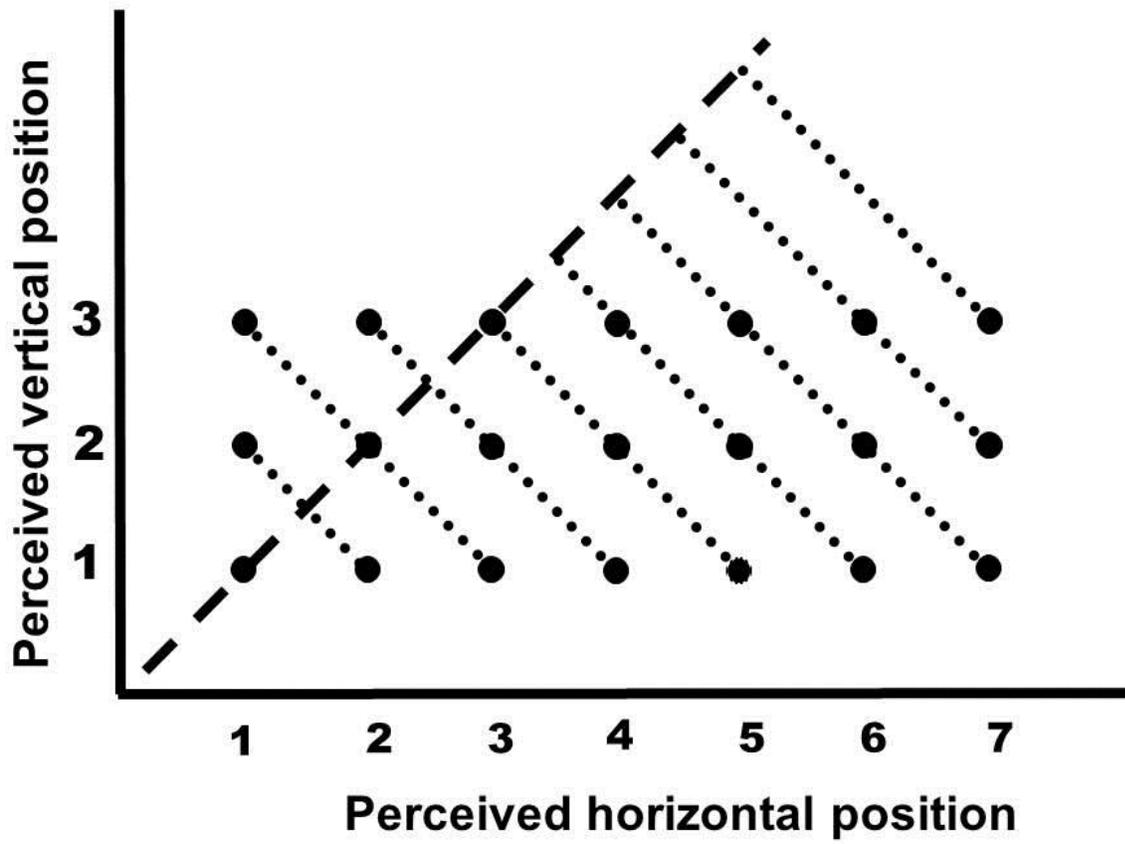


Figure 3A

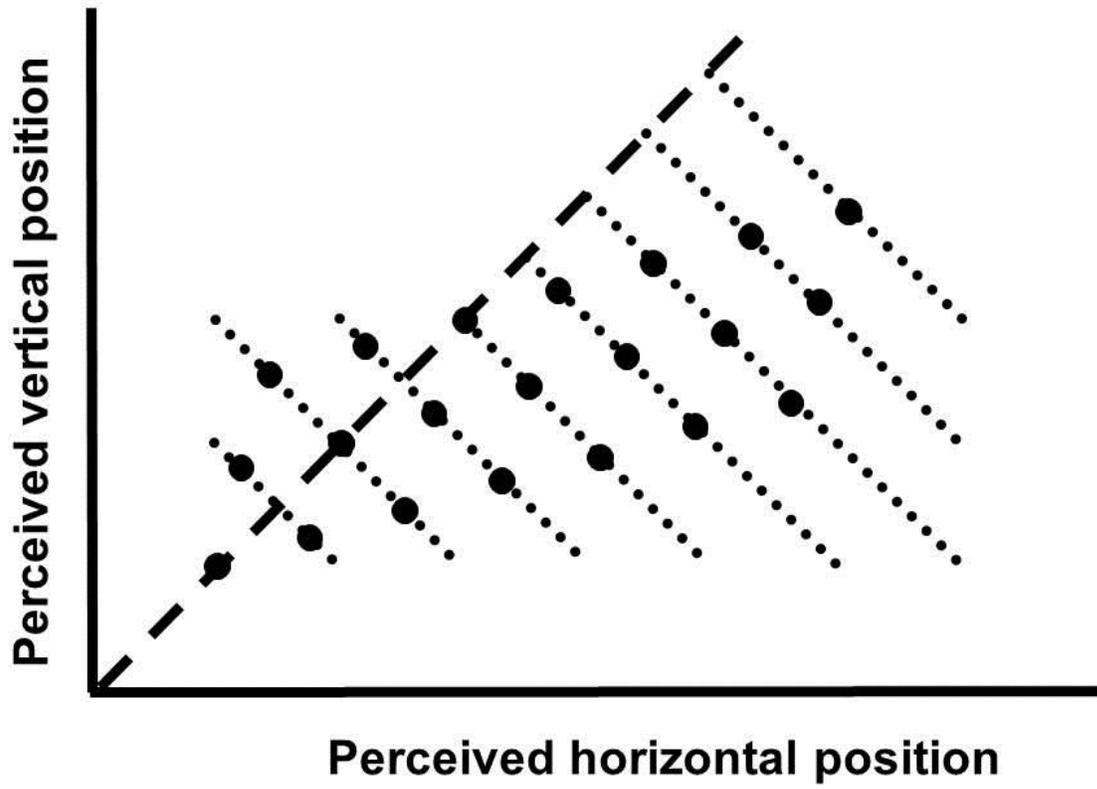


Figure 3B

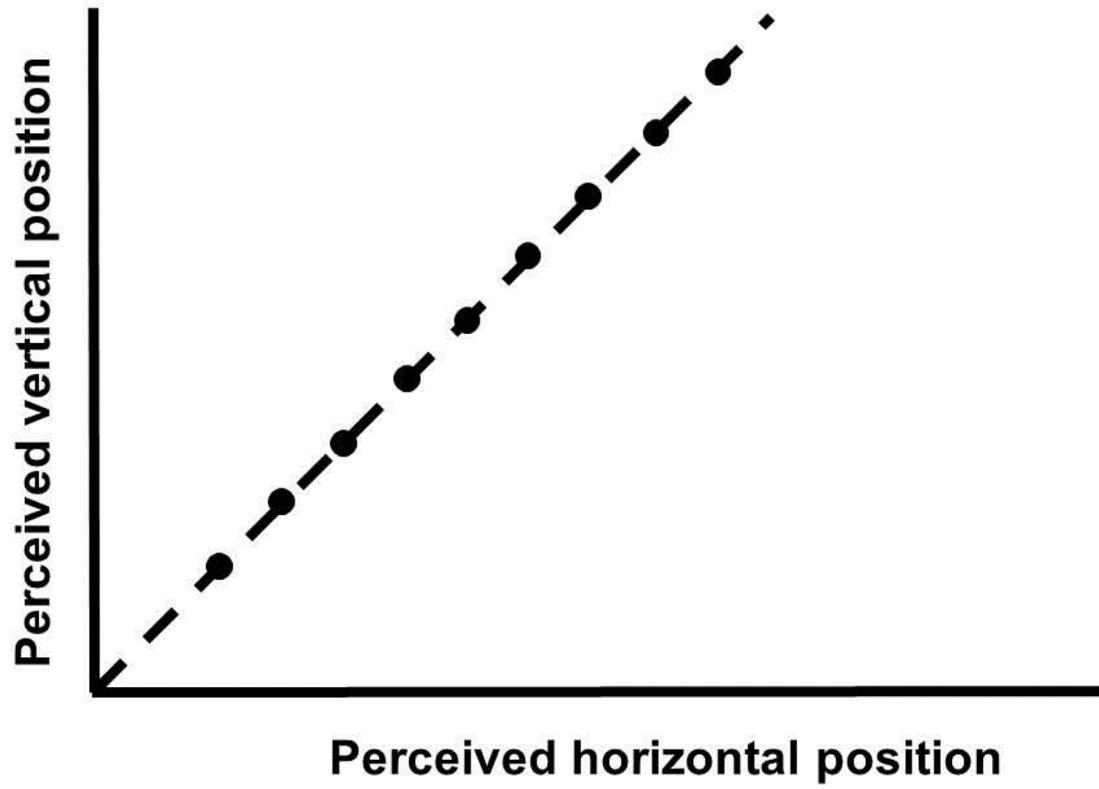


Figure 3C

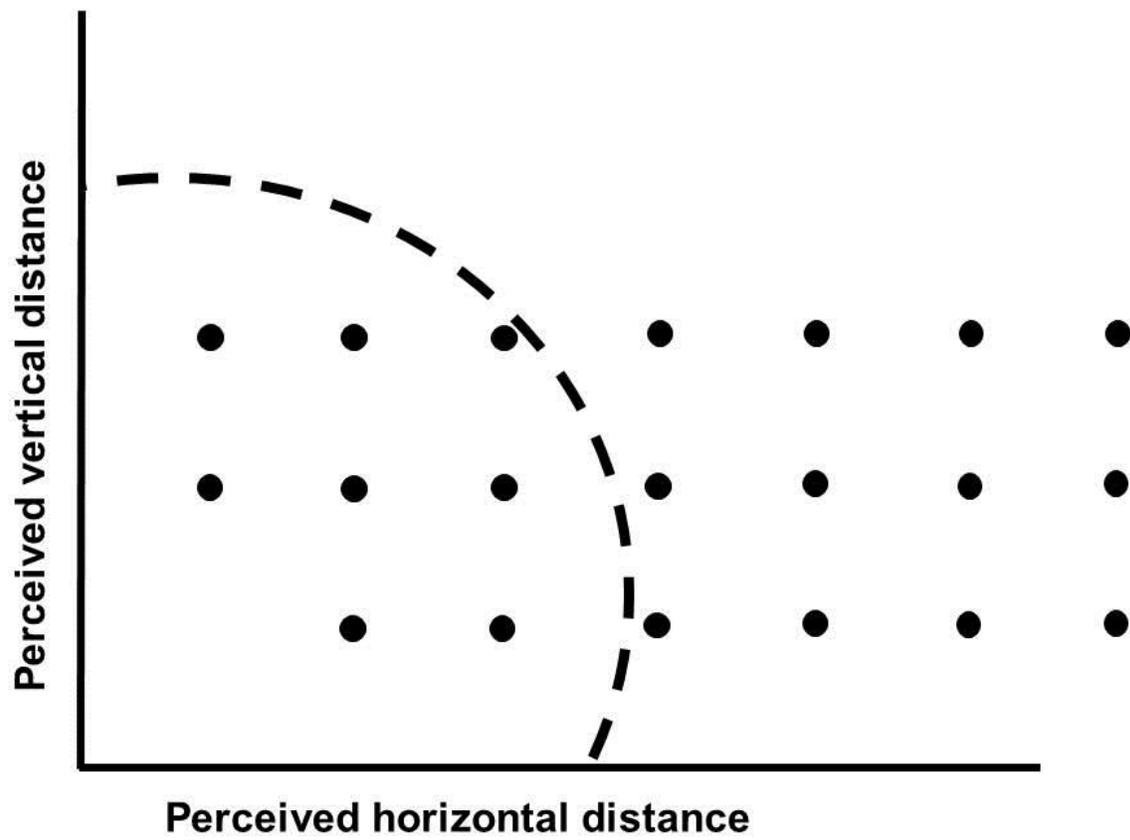


Figure 4

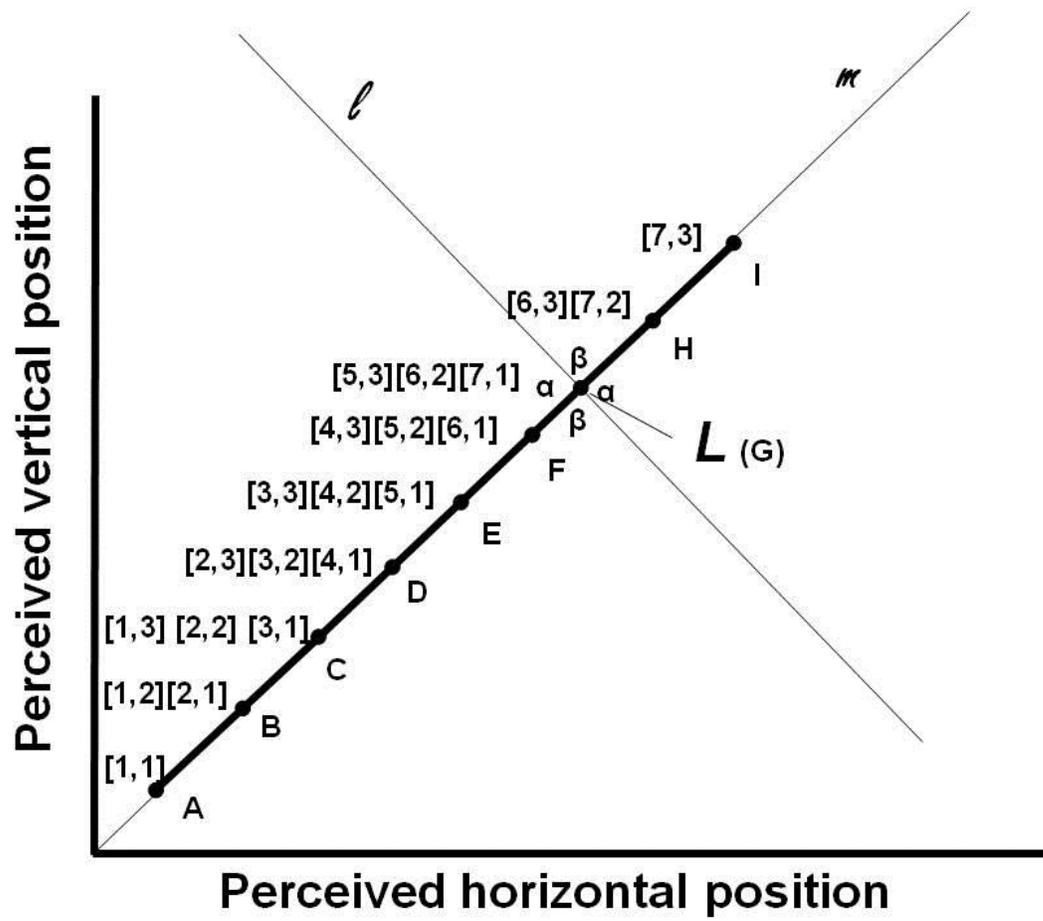


Figure 5A

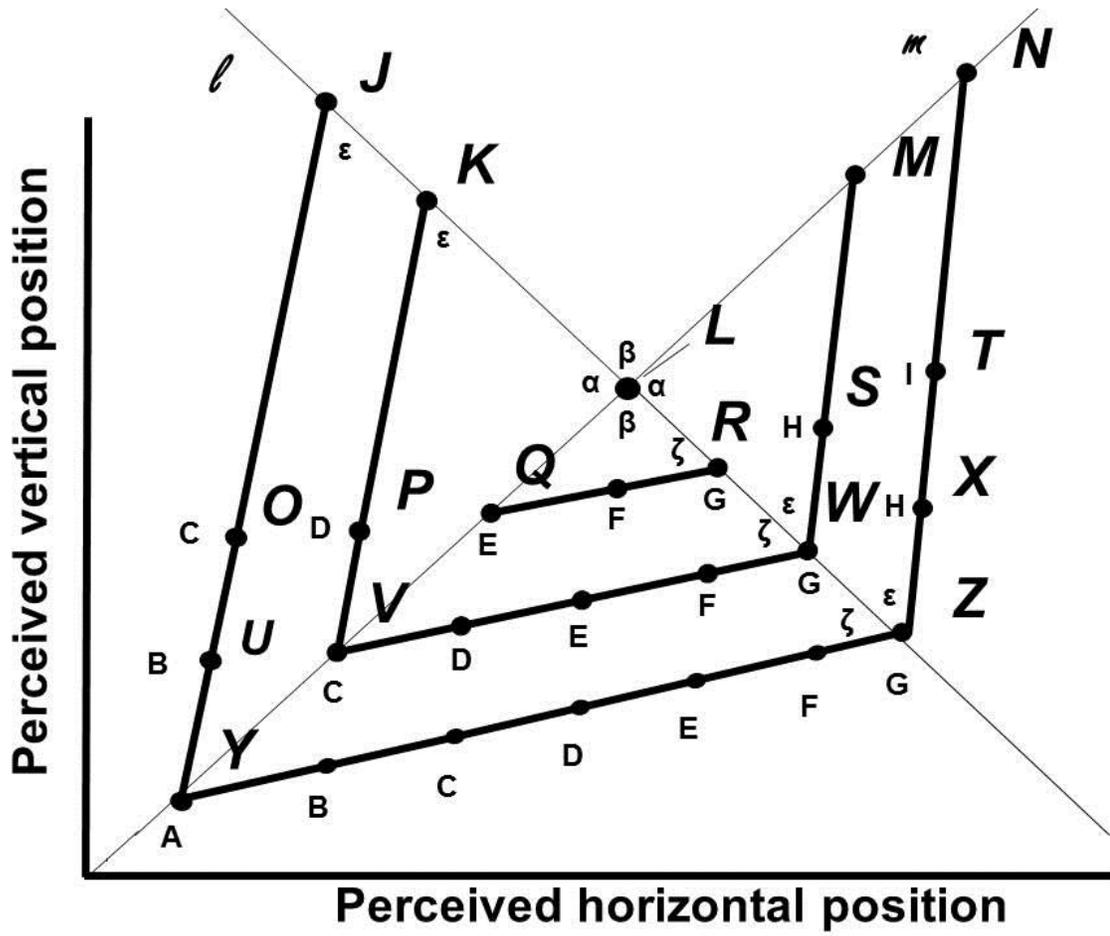


Figure 5C

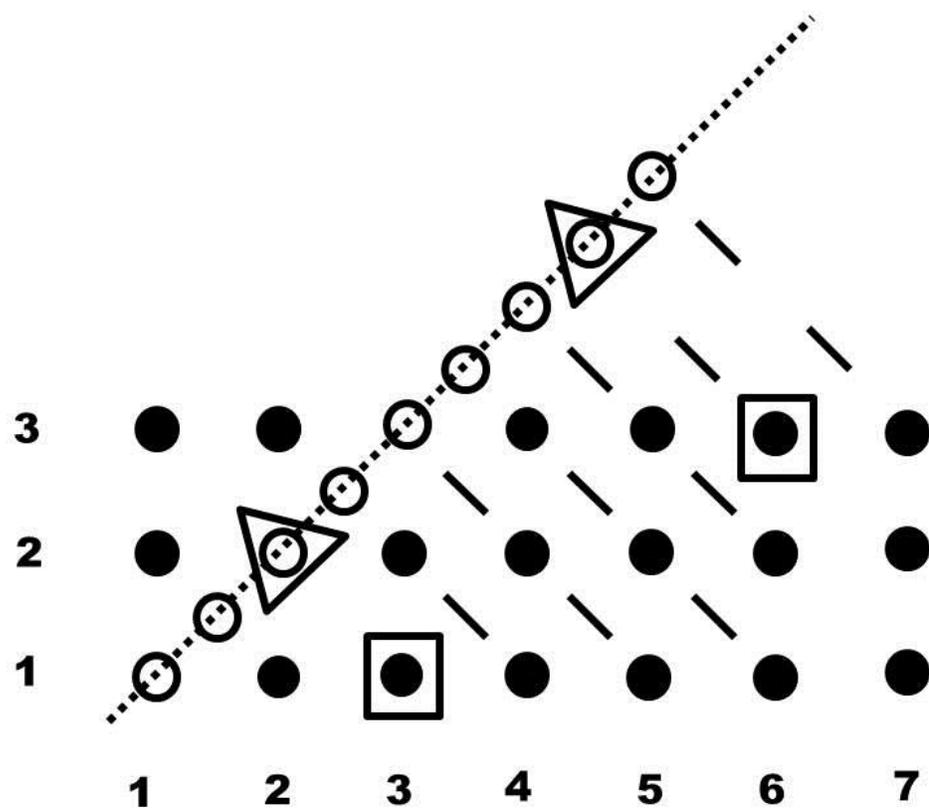


Figure 6A

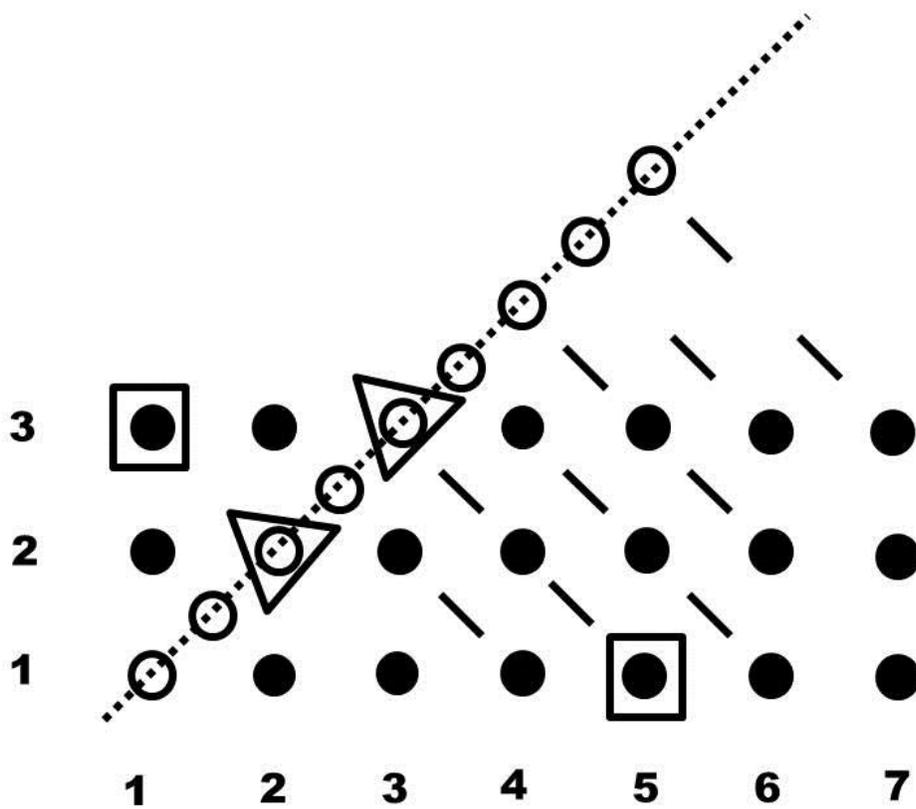


Figure 6B

