A Quartet of Asset Pricing Models in Nominal and Real Economies

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Abstract

This paper studies the equity premium implications of a canonical New Keynesian model with investment. We find that the presence of a time-varying marginal cost dampens the expansionary impact of a positive technology shock. With a given fraction of firms standing ready to satisfy demand at predetermined prices, the variations in the marginal utility of consumption attributed to technology shocks can easily be smoothed. Thus, technology shocks contribute little to the equity premium. Under a standard monetary policy rule, the real effect of monetary policy shocks is too weak and short-lived to generate a reasonable equity premium.

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Key Words: New Keynesian Model, equity premium, nominal rigidities, sticky prices, capital adjustment costs

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1 Introduction

Despite the success of New Keynesian models in matching key real business cycle features, the asset pricing implications of New Keynesian models have not been fully studied. This is especially true with regards to the equity premium. Compared with real business cycle models, New Keynesian models possess distinctly different features which may lead to different asset pricing implications. The most notable of these features is the inability of firms to set prices optimally every period. The paper addresses the following intriguing issues regarding asset pricing in New Keynesian models. First, we examine if it is possible to explain the equity premium in a standard New Keynesian framework. Second, we examine if the three features required to match the observed equity premium in real business cycle models also help to deliver a larger equity premium in New Keynesian models. These three features include (1) high curvature of the utility function, (2) high capital adjustment costs, and (3) inelastic labor supply. Lastly, we examine the role of monetary policy in achieving a sizable equity premium.

In this paper, we study a quartet of asset pricing models in both nominal and real economies. By turning on and off a few parameters of a canonical New Keynesian model, we are able to identify the impact of nominal rigidities and investment on the equity premium. The table below categorizes these four models:

	Flexible Prices	Sticky Prices
Without Investment	Model I (incl. exchange economy)	Model II
With Investment	Model III	Model IV

Most research on the financial aspects of New Keynesian models has focused on term structure, but not on equity premium issues[1]. Sangiorgi and Santoro (2005) study how the equity premium depends upon whether it is prices or wages that are sticky in New Keynesian models without internal persistence. Their study differs from ours in one key aspect: there is no investment or capital stock in their model. Including investment is important for three reasons. First, in reality, the value of a firm's securities measures the value of the firm's productive assets in general, and the capital stock in particular. Second, both consumption and investment are endogenously determined in our models. Investment demand, being an important component of the demand side of the economy, affects the marginal value of contemporary goods. The marginal value of contemporary goods is, in turn, an important determinant of the stochastic discount factor used to price all financial assets. Third, capital accumulation is an important process through which technology and monetary shocks are propagated over time. Since asset prices are forward-looking variables, it is important to capture the propagation of these shocks by incorporating investment in the model.

We also incorporate internal persistence mechanisms such as habit formation, inflation inertia and interest rate smoothing in Models III and IV. In addition, we study a model with both sticky prices and sticky wages to examine the robustness of our results.

These are the three main findings of the paper:

First, technology shocks contribute little to the equity premium. The equity premium generated by technology shocks diminishes as nominal rigidities in price-setting increase. There are two explanations for this behavior: first, the time-varying

[1] For example, Rudebusch and Wu (2007). The paper also relates to an earlier literature, including Giovannini and Labadie (1991), as well as Bansal and Coleman (1996), which study asset prices in a general equilibrium monetary endowment economy. Alternative models have also been developed to explore equity premium using the prospect theory, including Barberis et al. (2001) and Gruene and Semmler (2005).

marginal cost in a model with nominal rigidities dampens the expansionary impact of a positive technology shock. Second, the sticky-price feature in the model implies that a fraction of firms must have the flexibility to adjust labor input in order to satisfy demand at pre-set prices. That flexibility in varying labor input, however, can moderate most variations in the marginal utility of consumption caused by technology shocks.

Second, when the economy moves toward more nominal rigidities in price-setting, monetary policy shocks become the major contributor to the equity premium. However, under a standard monetary policy rule, the real effect of monetary policy shocks seems too weak and short-lived to generate a reasonable equity premium. Only when the exogenous monetary policy shock is assumed to be highly persistent (which is unusual in the literature) are there enough variations in the marginal utility of consumption and the real stock return to explain a substantial equity premium. Without this added exogenous persistence, even a sticky-wage sticky-price model cannot explain a large equity premium.

Third, variations in investment spending allow agents to alter their production plans to reduce fluctuations in consumption. In both nominal and real economies with production, costly capital adjustment leaves consumption susceptible to fluctuations caused by both monetary and technology shocks. The result is more variation in the marginal utility of consumption. However, the presence of capital adjustment costs is a necessary, but not sufficient, condition for a large equity premium in both economies.

The paper is organized as follows: Section 2 presents a canonical New Keynesian model which can be specialized into the four sub-models of interest. Sections 3 describes the asset pricing implications of each sub-model. Section 4 concludes.

2 A Canonical New Keynesian Model

2.1 Model Environment

In this section, we describe a generalized New Keynesian model, which can be further specialized into four sub-models when we turn on or off particular features.

2.1.1 The Representative Household

The representative household obtains utility from consumption, leisure, and real money balances. The household's maximization problem is standard and specified as in Appendix A. Solving the household's problem yields first-order conditions for the nominal bond, real equity, and labor supply[2]:

$$E_t\left[\beta\frac{\Psi_{t+1}}{\Psi_t}\frac{R_t}{P_{t+1}/P_t}\right] = 1, \tag{1}$$

$$E_t\left[\beta\frac{\Psi_{t+1}}{\Psi_t}\left(V_{t+1}+D_{t+1}\right)\right] = V_t, \tag{2}$$

$$\frac{W_t}{P_t} = \tau\frac{L_t^{\theta-1}}{\Psi_t}, \tag{3}$$

$$\text{where } \Psi_t = \left(C_t-bC_{t-1}\right)^{-\sigma}-b\beta E_t\left[\left(C_{t+1}-bC_t\right)^{-\sigma}\right]$$

Here β is the subjective discount factor, and Ψ_t is the marginal utility of consumption at date t for the habit formation preference specification. R_t and P_t are the nominal interest rate and aggregate price level respectively. V_t denotes the real value of the firm, and D_t represents dividends in real terms. $\frac{W_t}{P_t}$ represents the real wage, L_t denotes the labor supply, τ reflects the degree of disutility from working, and θ, which indexes the degree of labor supply elasticity, is greater than or equal to 1. C_t

[2]The money demand equation serves only to determine how much money the central bank needs to supply to clear markets given its interest rate target. This equation can be dropped when a monetary policy rule is present.

represents consumption. The coefficient σ represents the degree of the relative risk aversion. When $b > 0$, there is habit formation in consumption preferences.

2.1.2 The Production Technology and Pricing Decisions

We follow Bernanke, Gertler and Gilchrist (1998) in assuming a wholesale sector for production and a retail sector for pricing. Competitive firms produce the wholesale good, make decisions on how much output to produce and how much to invest. There is a continuum of retail firms in the retail sector. The retail firms purchase the wholesale good from the wholesale production sector, differentiate these goods at no cost and sell them to an output aggregator. The aggregator combines heterogeneous products into a Dixit-Stiglitz type final output, and then sells to households and the wholesale production sector for consumption and investment.

Production Technology of the Wholesale Sector The wholesale good is produced using the following technology:

$$Y_{w,t} = Z_t K_t^{1-\alpha} L_t^{\alpha}, \tag{4}$$

where z_t, the logarithm of the technology shock Z_t, follows an $AR(1)$ process

$$z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z,t}. \tag{5}$$

Here $0 < \rho_z < 1$ and the zero-mean, serially uncorrelated innovation $\varepsilon_{z,t}$ is drawn from a standard normal distribution. For the remainder of the paper, all lower case letters will represent log-linearized deviations of corresponding variables from their steady state values.

Wholesale firms take the relative price of the wholesale good, $\frac{P_{w,t}}{P_t}$, as given, and choose the labor input optimally to maximize their profits. The maximization condition is given by:

$$\frac{P_{w,t}}{P_t} = \frac{W_t/P_t}{MPL_t}, \tag{6}$$

where the right hand side is the real wage over the marginal product of labor.

Investment Decisions of the Wholesale Sector We assume convex capital adjustment costs[3] so that,

$$K_{t+1} = (1-\delta)\, K_t + \phi\left(\frac{I_t}{K_t}\right) K_t, \tag{7}$$

where

$$\phi\left(\frac{I_t}{K_t}\right) = \frac{\delta^\eta}{1-\eta}\left(\frac{I_t}{K_t}\right)^{1-\eta} + \frac{\eta\delta}{\eta-1}. \tag{8}$$

The parameter δ represents the depreciation rate, and η measures the adjustment cost. As η approaches 0, adjustment costs go to zero. Likewise, as η approaches infinity, capital adjustment costs go to infinity.

The maximization problem facing wholesale firms is given by:

$$\max_{I_t} \sum_{n=0}^{\infty} E_t \left\{ \beta^n \frac{\Psi_{t+n}}{\Psi_t} \left[\frac{P_{w,t+n}}{P_{t+n}} Y_{w,t+n} - \frac{W_{t+n}}{P_{t+n}} L_{t+n} - I_{t+n} \right] \right\}. \tag{9}$$

The first-order condition for investment is given by:

$$\begin{aligned} \frac{1}{\phi'\left(\frac{I_t}{K_t}\right)} &= E_t \left\{ \beta \frac{\Psi_{t+1}}{\Psi_t} \left[\frac{P_{w,t+1}}{P_{t+1}} (1-\alpha) \frac{Y_{w,t+1}}{K_{t+1}} \right] \right. \\ &\quad \left. + \beta \frac{\Psi_{t+1}}{\Psi_t} \left[1 - \delta - \phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\left(\frac{I_{t+1}}{K_{t+1}}\right) + \phi\left(\frac{I_{t+1}}{K_{t+1}}\right) \right] \frac{1}{\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)} \right\} \end{aligned} \tag{10}$$

where the left-hand side is the relative price of a unit of installed capital versus output today, and the right hand side is the present value of its benefits.

Pricing Decisions of The Retail Sector There is a continuum of retail firms in the retail sector. An output aggregator combines heterogeneous retail goods into

[3] This specification of convex adjustment costs is similar to Jermann (1998) and Boldrin, Christiano and Fisher (2001).

Dixit-Stiglitz type final output, Y_t :

$$Y_t = \left\{ \int_0^1 Y_{j,t}^{\gamma} dj \right\}^{\frac{1}{\gamma}}, 0 < \gamma < 1 \tag{11}$$

where γ measures the elasticity of demand for each retail good. Since retail goods are heterogeneous, retail firms set prices taking their respective demand curves as given. As in Calvo (1983), we assume that at each period a fraction φ of randomly chosen retail firms are free to set prices. A firm that cannot reoptimize its price adopts the following updating scheme[4]:

$$P_{j,t} = \exp(\pi_{t-1}) P_{j,t-1}. \tag{12}$$

Following standard procedures outlined in Christiano, Eichenbaum and Evans (2005), we obtain the Phillips curve

$$\pi_t = \frac{1}{1+\beta}\pi_{t-1} + \frac{\beta}{1+\beta}E_t\pi_{t+1} + \frac{\varphi[1-\beta(1-\varphi)]}{(1+\beta)(1-\varphi)}mc_t, \tag{13}$$

where $\pi_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$ and mc_t is the logarithm of the marginal cost of retail goods. Since retailers differentiate the wholesale good at no extra cost, the marginal cost of retail goods is the relative price of the wholesale good, as defined in equation (6).

2.1.3 Monetary Policy Rule

We consider a simple monetary policy rule, according to which the central bank adjusts the current nominal interest rate in response to the inflation rate and the lagged interest rate:

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(1+\rho_\pi)\pi_t + \xi_t. \tag{14}$$

[4] According to Christiano, Eichenbaum and Evans (2005), this specification generates sufficient inflation inertia to match that observed in the data. In model II, we use the specification $P_{j,t} = \exp(\overline{\pi}) P_{j,t-1}$, where $\overline{\pi}$ is the steady-state inflation rate. This specification generates no inflation inertia, but the resulting model can be solved analytically for further insight.

where $0 < \rho_r < 1$, and $\rho_\pi > 0$. A positive ρ_π guarantees that the central bank adjusts the short-term nominal interest rate so that the targeted *ex post* real interest rate rises when inflation exceeds its target value, which is assumed to be the steady-state rate of inflation. Rules of this form do a reasonably good job of describing the variation of short term interest rates (see Clarida, Gali and Gertler (2000)). We also considered variants that allow for responses to output as well as inflation, in the spirit of the Taylor (1993) rule[5]. The equity premium is smaller when the central bank is able to dampen output fluctuations by adjusting the interest rate in response to the output gap.

We assume that ξ_t follows an $AR(1)$ process:

$$\xi_t = \rho_\xi \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t}, \tag{15}$$

where $\varepsilon_{\xi,t}$ is drawn from a standard normal distribution and uncorrelated with the technology shock ε_z at all leads and lags. The autoregressive parameter ρ_ξ is set to zero in our benchmark calibration, as is standard in the literature. We then vary the size of the parameter to examine how persistent exogenous monetary policy shocks should be to generate a substantial equity premium.

2.1.4 Goods Market Equilibrium

In a model with investment, the goods market is in equilibrium when

$$Y_t = C_t + I_t. \tag{16}$$

2.2 Asset Pricing Implications

In this section, we derive generalized formulae for the equity premium in the canonical model. In the remainder of the paper, whenever we discuss asset pricing implications,

[5] The details of these variants of the models are available from the author upon request.

we shall focus on the value of a conglomerate consisting of retail and wholesale firms. This makes sense since both sectors are owned by households in the model.

We use the loglinear-lognormal approximations as first proposed by Jermann (1998). We first represent the solutions of the model by a loglinear state-space system, with the vector of state variables, $\mathbf{S}_t$, following a first order autoregressive process with multivariate normal i.i.d impulses[6]:

$$\mathbf{S}_t = \mathbf{\Lambda}\mathbf{S}_{t-1} + \mathbf{\Gamma}\boldsymbol{\varsigma}_t, \tag{17}$$

where the square matrix $\mathbf{\Lambda}$ governs the dynamics of the system, $\boldsymbol{\varsigma}_t$ is a column vector of $\{\varepsilon_{z,t}, \varepsilon_{\xi,t}\}$, and $\mathbf{\Gamma}$ is the covariance matrix.

The level of dividends, D_t, can be written as

$$D_t = C_t - \frac{W_t}{P_t}L_t \tag{18}$$

For asset pricing, this system provides us with the log of dividends d, and the log of the stochastic discount factor, $\psi_{t+1} - \psi_t$, as linear combinations of the state vector. The log-linear asset pricing framework of Campbell and Shiller (1988) allows us to express the real stock return, rr_t^s, as a function of state variables. We can subsequently approximate the conditional equity premium as

$$EP = -cov_t\left(\psi_{t+1} - \psi_t, rr_{t+1}^s\right). \tag{19}$$

Due to the presence of homoskedasticity in the model, the conditional equity premium is constant for all t.

2.3 Calibration

We adhere closely to the general equilibrium business cycle literature in choosing values for the parameters to be calibrated. The model parameters can be categorized

[6]For the economy considered in the paper, $\mathbf{S}_t$ contains the capital stock, the habit level, the interest rate, the inflation rate, and the monetary and technology shocks.

into the following three groups:

2.3.1 Preferences

The subjective time discount rate, β, is set to 0.987. Each model period is considered to be one quarter. We set σ to 1 to be consistent with balanced growth. The parameter b, which indexes the degree of habit persistence, is set to 0.9, similar to the value used in Jermann (1998). The parameter θ, which describes the labor supply elasticity, is set to 2, as in Christiano, Eichenbaum and Evans (2005). The parameter τ is set so that the fraction of labor used for production is 0.25.

2.3.2 Production and Investment

The capital depreciation rate, δ, is 0.025. The constant labor share in the Cobb-Douglas production function is $\frac{2}{3}$. The persistence parameter of the technology shock is 0.95, and σ_z is set to 0.007, as is standard in real business cycle models. The parameter η stands for the inverse of the elasticity of the investment-capital ratio with respect to Tobin's Q. We set η to $\frac{1}{0.23}$, which is the value used in Jermann (1998) and Boldrin, Christiano and Fisher (1999). The parameter γ measures the elasticity of demand for each retail good. As γ approaches 1, the economy approaches perfect competition. We set γ to 0.25 in the benchmark case. The parameter φ represents the fraction of firms that cannot set prices in a given period. We set its value to 0.5.

2.3.3 Monetary Policy Rule Parameters

The autoregressive coefficient in the monetary policy rule, ρ_r, is set to 0.86, close to various estimates (e.g. Clarida, Gali, and Gertler 2000) of the Federal Reserve's policy rule. The parameter ρ_π is set to 0.5 as suggested by Taylor (1993). The persistence parameter of exogenous monetary policy shocks, ρ_ξ, is set to 0 in the benchmark case, as is standard in the literature. The standard deviation of the monetary policy shocks, σ_ξ, is set to 0.005, consistent with Ireland's (2004) estimate.

3 A Quartet of Asset Pricing Models

By turning on and off a few key parameters, the canonical New Keynesian model can be specialized into four sub-models. Table 1 summarizes the respective equilibrium conditions of these four sub-models. In this section, we examine the asset pricing implications of each.

3.1 Models Without Internal Persistence

We assume no internal persistence in Models I and II by setting b and ρ_r to zero and by adjusting the Phillips curve accordingly. The relationship between the equity premium and macro fundamentals is made transparent by the resulting closed-form solutions for the equity premium.

3.1.1 Model I

Model I describes an economy with flexible prices ($\varphi = 1$) and no investment ($\widehat{c}$, the consumption-output ratio, equals 1). Since there is no investment in Models I and II, we simplify the model by assuming a linear production economy where α equals 1 in equation (4). In such an economy, the value of a firm essentially comes from the monopoly rent in the retail sector. Money has no real effects in this economy.

In a model with flexible prices, the prices of all retail goods (a markup over the real marginal cost) are equal to 1. Hence, the real marginal cost, as defined in equation (6), is also a constant. As a result, both the real labor cost and dividends are a constant fraction of total output, just like in an exchange economy.

It is well known that with assumptions like those in Model I, a sizable equity premium can be obtained when the representative agent is strongly averse to fluctuations in consumption (high σ), and yet the inelastic labor supply ($\theta \to \infty$) prevents him from adjusting his labor supply to smooth consumption. The equity premium is close to zero when labor supply is reasonably elastic.

3.1.2 Model II

As shown in Table 1, Model II has one key difference from Model I. The Phillips curve has an extra term, representing the degree of price stickiness. The lower is the value of φ (the fraction of firms free to set prices), the larger is the effect of this extra term on the model equilibrium. This opens the door for the monetary policy rule to have real effects.

Without internal persistence or investment, the aggregate state is summarized exclusively by two exogenous variables, $\{z_t, \xi_t\}$. The equilibrium of Model II can be obtained once we have the solutions to ψ_t and π_t.

Proposition 1 *The solutions to ψ_t and π_t can be written as*

$$\psi_t = \psi_{s,1} z_t + \psi_{s,2} \xi_t, \quad \pi_t = \pi_{s,1} z_t + \pi_{s,2} \xi_t, \tag{20}$$

where

$$\psi_{s,1} < 0, \psi_{s,2} > 0, \pi_{s,1} < 0, \pi_{s,2} < 0.$$

Proof. The proof is contained in Appendix B. ∎

The proposition indicates that a positive technology shock reduces both the marginal utility of consumption and inflation, while a positive monetary policy shock raises the former and reduces the latter.

Dividends are no longer a fixed fraction of output in sticky-price models. The marginal cost, which is proportional to the fraction of labor compensation out of output, fluctuates in response to shocks.

3.1.3 Magnitude of The Equity Premium with only Technology Shocks

Now we examine in detail how the equity premium resulting from technology shocks, EP_1, depends upon the new parameters introduced in Model II.

Proposition 2 *Under the assumption $\gamma(\theta+\sigma) < 1$, technology shocks contribute positively to the equity premium*[7]*. We can show that*

$$\frac{\partial EP_1}{\partial \varphi} > 0, \frac{\partial EP_1}{\partial \rho_\pi} > 0, \frac{\partial EP_1}{\partial \theta} > 0.$$

In particular, when there are only technology shocks, the equity premium in a sticky-price economy ($\varphi < 1$) is lower than its counterpart in a flexible-price economy ($\varphi = 1$) except when $\rho_\pi \to \infty$.

Proof. The results follow the proposition in Appendix B. ■

In Model I ($\varphi = 1$), a positive technology shock raises the marginal product of labor. As a result, an increase in labor input leads to increases in output and consumption. In Model II where a fraction of firms are not free to set prices, the relative price of the wholesale good, which is also the real marginal cost, has to decline to stimulate retailers' demand. The decline in the relative price dampens the incentive of the wholesale sector to boost production at the time of a positive technology shock. The result is a smaller increase in consumption compared to Model I. The time-varying marginal cost moderates the impact of technology shocks on real variables. In general the marginal utility of consumption responds very little to technology shocks when φ is smaller than 1. As a result, the contribution of technology shocks to the equity premium is very limited.

An alternative interpretation is that as inflation goes down, the relative prices of the fraction of the retail firms which cannot reset prices go up, thus dampening the

[7] For the remainder of the paper, we will focus on the case $(\sigma+\theta)\gamma < 1$, which is a sufficient condition for both EP_1 and EP_2 to be positive.

responses of aggregate consumption and dividends to positive technology shocks, and leading to a lower equity premium. The effect of technology shocks on real variables decreases as the response of inflation to these shocks increases. This observation demonstrates the importance of monetary policy rules.

A higher ρ_π discourages deviations from the inflation target, naturally leading to a smaller response of inflation to technology shocks. As ρ_π goes to infinity, inflation barely responds to technology shocks, and the equity premium approaches its counterpart in Model I.

In an economy with only technology shocks, theoretically we can raise θ sufficiently high to compensate for the decline in the equity premium due to a smaller φ. However, even in an endowment economy ($\theta \to \infty$) with flexible prices ($\varphi = 1$), the coefficient of relative risk aversion σ has to be unreasonably high to generate a sufficiently large equity premium. Technology shocks are not the answer to a sizable equity premium in Model II.

3.1.4 Magnitude of Equity Premium With Only Monetary Shocks

When only monetary policy shocks are present, we have the following proposition for the equity premium:

Proposition 3 *Under the assumption $\gamma(\theta+\sigma) < 1$, we can show that*

$$\begin{aligned} \frac{\partial EP_2}{\partial \varphi} &< 0, \frac{\partial EP_2}{\partial \rho_\pi} < 0, \frac{\partial EP_2}{\partial \theta} < 0, \frac{\partial EP_2}{\partial \gamma} < 0, \\ \frac{\partial EP_2}{\partial \rho_\xi} &> 0 \textit{ iff } \rho_\pi < \frac{1}{\beta} - 1. \end{aligned}$$

Proof. The results follow the proposition in Appendix B. ■

Intuitively, a positive monetary shock dampens the demand for the wholesale good, and reduces the real marginal cost, which is also the relative price of the wholesale good. The wholesale firms respond by contracting their production, which results in lower consumption and dividends.

An alternative interpretation is that as inflation goes down, the relative prices of the retail firms which cannot reset prices go up, thus aggravating the adverse effect of a positive monetary shock on aggregate consumption and dividends, and leading to a higher equity premium. The equity premium increases with the responsiveness of inflation to monetary shocks. This observation again points to the importance of monetary policy rules.

When ρ_π is high, the monetary authorities are more aggressive with regard to fighting inflation. As a result, the adverse effect of positive monetary shocks is contained, and the equity premium is smaller.

The responsiveness of labor supply to monetary policy shocks increases as θ decreases. Since the equity premium in such an economy is mostly driven by monetary shocks, a lower θ leads to a larger equity premium.

It is interesting to observe that for both technology and monetary shocks, the derivative of the equity premium with respect to θ has the same sign as with respect to φ. This is understandable since both a higher θ and higher φ indicate that relative prices respond more to shocks. When θ is high, the labor supply is more inelastic and the real wage is more responsive to shocks. When φ is high, the relative prices of goods are more strongly linked to real wage changes.

As γ increases, the responsiveness of the relative price of the wholesale good to a positive monetary policy shock declines. Thus as γ increases, the equity premium declines.

3.2 Models with Internal Persistence

We now allow for internal persistence by assuming habit formation, inflation inertia, and interest rate smoothing in the monetary policy rule. The model is thus closer to what has become standard in the literature. Numerical methods are used to solve the model.

3.2.1 Model III

Model III is a model with investment and flexible prices. An intertemporal investment equation and capital accumulation equation need to be incorporated into the system. As in Model I, a classical dichotomy applies in this model setting. All real variables can be determined in the real sector of the economy.

When both labor supply and investment are inelastic (as θ and η approach ∞), Model III becomes similar to that of Jermann (1998). In such a model, the representative agent is strongly averse to consumption fluctuations, but inelastic labor supply and high capital adjustment costs prevent him from smoothing consumption. The result is a substantial equity premium. Panel A Table 2 reports selective numerical results of Model III. When we set θ to 100 and η to 8 while keeping other parameters at their benchmark values, the resulting equity premium is as high as 6.2%. However, when we assume zero capital adjustment costs, the equity premium decreases to zero even when the labor supply is close to inelastic. These results indicate that when investment is elastic, consumption can be greatly smoothed, and the equity premium can be substantially diminished. These findings are consistent with Jermann (1998).

3.2.2 Model IV

Model IV is the canonical model described in Section 2. The core system of Model IV is the same as that of Model III except for the sticky-price feature in the Phillips curve.

Role of Sticky Prices Panel B of Table 2 documents the equity premium from technology and monetary shocks when φ takes the values of $1, 0.5$ and 0 respectively.

Other parameters are calibrated at their benchmark values. Several findings are evident. First, the equity premium turns out to be close to zero. Second, when φ approaches 1, only technology shocks contribute to the equity premium, and money has no real effect. By contrast, as φ approaches zero, monetary policy shocks are the major contributor to the equity premium. We have obtained similar results in Model II.

It is well known that when prices are flexible money has no real effect, thus generating no equity premium. However, the reason why technology shocks contribute so little to the equity premium in a sticky-price economy has not been explored. The intuition we have obtained in Model II applies here. As φ approaches zero, almost all the retail firms are not free to reset prices. As a result, increases in the output of the wholesale good in response to a positive technology shock only leads to a decline in their relative price. Thus the wholesalers' incentive to expand production is reduced.

Panel A Table 3 documents the responses of relevant variables to contemporary technology and monetary policy impulses. When φ decreases from 1 to 0, the real marginal cost declines by more in response to a positive technology shock. The effect of such a shock on the marginal utility of consumption and the real stock return is diminished.

A positive monetary policy shock reduces inflation, reduces the demand for the wholesale good, and discourages the incentive of wholesale producers to increase production. Panel A Table 3 shows that real marginal costs decrease as φ decreases. Also the negative effect of a positive monetary shock on consumption and the real stock return increases as φ decreases. The result is a larger equity premium.

The equity premium is fairly low when φ is the only parameter deviating from its benchmark value. In Panel C Table 2, we follow the intuition in Proposition 3 by setting ρ_π to zero. The monetary authority responds little to inflation under this policy rule, thus allowing the positive monetary shock to exert its full negative impact. The equity premium is indeed higher compared to the benchmark case, but the increase is too small to make any difference.

It is interesting to observe that monetary shocks and technology shocks are "orthogonal" in the sense that the former contributes to the equity premium in a New Keynesian type sticky-price economy, while the latter in a flexible-price real economy. This is due to the time-varying marginal cost which weakens the impact of technology shocks, but strengthens the real effect of monetary shocks.

Persistence of Monetary Policy Shocks In order to generate a substantial equity premium, monetary policy shocks should be persistent enough to create large variations in the real stock return and in the marginal utility of consumption. In the standard literature the monetary policy shock itself is often assumed to be i.i.d, with interest rate smoothing and inflation inertia being the major propagation mechanisms. Here we allow monetary policy shocks ξ to be a persistent AR process, and examine whether such external persistence can help to generate a substantial equity premium.

As shown in Panel D of Table 2, when ρ_ξ is equal to 0.95, the equity premium is as high as 7.22%. The equity premium quickly diminishes as ρ_ξ becomes smaller. Panel B Table 3 compares the responses of key variables when $\rho_\xi = 0.95$ versus the benchmark case when $\rho_\xi = 0$. The long lasting effect of a persistent monetary policy shock is especially evident through its strong adverse impact on forward-looking variables, such as the real stock return and the marginal utility of consumption. The strong impact on these two variables leads to a substantial equity premium.

Role of Capital Adjustment Costs We work with two sets of alternative calibrations to examine the importance of capital adjustment costs. First we set η to 1000 instead of its benchmark value of 4.35, and examine whether inelastic investment can help to generate a larger equity premium in a sticky-price economy. Panel E of Table 2 shows that higher capital adjustment costs only lead to a very small increase in the equity premium compared to the benchmark case. These results show that variations in the marginal cost and the flexibility to vary labor inputs, by themselves, can smooth variations in consumption caused by technology and monetary policy shocks to a great extent, despite costly investment adjustment.

In the second set of calibrations, we start with the case of $\rho_\xi = 0.95$, and assume investment to be infinitely elastic by setting η close to zero. Surprisingly, the equity premium drops to only slightly above zero. As shown in Panel B Table 3, comparing to the case of $\rho_\xi = 0.95$ and $\eta = 4.35$, investment declines significantly in response to a positive monetary policy shock as η approaches 0. The decrease in investment moderates the decline in consumption caused by positive monetary policy shocks.

Inelastic investment by itself is not sufficient to generate a large equity premium. However, when investment is elastic, it shields consumption from exposure to outside shocks, resulting in smoothed consumption and a small equity premium.

The results of Model IV show that there are two indispensable ingredients for a substantial equity premium in the sticky-price model. They are persistent exogenous monetary policy shocks and high capital adjustment costs. The former generates persistent impact on real variables, while the latter exposes consumption to exogenous shocks.

In order to obtain a substantial equity premium in a sticky-price economy, we need considerably persistent real effects of monetary policy shocks, much more persistent than a standard monetary policy rule would permit. A natural question to ask is whether additional internal persistence stemming from wage rigidity can help to deliver a large equity premium. We examine this issue by studying a New Keynesian model with both sticky-price and sticky-wage features[8]. As shown in Table

[8]The details of the model are contained in Appendix C, which is available upon request.

4, the equity premium is now 0.36%, which is slightly larger than in our benchmark model without sticky wages. The marginal utility of consumption and the real stock return indeed respond more strongly to monetary policy shocks in the presence of extra persistence. However it is still not strong enough to explain a substantial equity premium. A persistent exogenous monetary policy shock is still required to explain a large equity premium in such a model.

4 Conclusion

In this paper, we study the equity premium implications of a canonical New Keynesian model with investment. We start with a simple flexible-price economy without investment, and then introduce sticky prices and investment sequentially into the simple setup. We find that technology shocks contribute little to the equity premium as the model moves toward more nominal rigidity in price-setting. Capital adjustment costs only have marginal effects on the equity premium once nominal rigidities are present. We attempt to build in more internal persistence by incorporating habit formation, inflation inertia, interest rate smoothing, and sticky wages into the model. However, under a standard monetary policy rule, monetary policy shocks are still too weak and short-lived to generate a substantial equity premium. It is a tougher challenge to generate a substantial equity premium in New Keynesian models as compared to their real business cycle counterparts.

A Household's Maximization Problem

The representative household carries M_{t-1} units of money, B_{t-1} units of nominal bonds, and a portfolio(vector) of financial assets $\mathbf{f}_{t-1}$ into period t. The asset vector $\mathbf{f}$ contains real bonds, shares of the representative firm and possibly other assets. $\mathbf{V}_t^f$ are vectors of asset prices in real terms. At the beginning of the period, the household receives a lump-sum monetary transfer T_t from the central bank. During period t, the representative household supplies L_t units of labor to the wholesale firms at the nominal wage rate W_t, and receives real payouts $\mathbf{D}_t^f$. The household purchases C_t units of consumption goods at the nominal price P_t from an output aggregator of heterogeneous retail goods. The household also purchases new bonds of real value $\frac{B_t}{P_t R_t}$, where R_t denotes the gross nominal interest rate from t to $t+1$. Finally, the household carries M_t, B_t and the new asset portfolio f_t into the next period. Thus, the flow budget constraint that the representative household faces is

$$C_t + \frac{M_t}{P_t} + \frac{B_t}{P_t R_t} + \mathbf{f}_t^{\prime}\mathbf{V}_t^f = \frac{W_t}{P_t}L_t + \mathbf{f}_{t-1}^{\prime}\left(\mathbf{V}_t^f + \mathbf{D}_t^f\right) + \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} + \frac{T_t}{P_t}. \tag{21}$$

Faced with these budget constraints, the household acts to maximize the expected lifetime utility of consumption

$$E_t \sum_{n=0}^{\infty} \beta^n \left[\frac{\left(C_{t+n} - bC_{t+n-1}\right)^{1-\sigma} - 1}{1-\sigma} - \tau \frac{L_{t+n}^{\theta}}{\theta} + \ln\left(\frac{M_{t+n}}{P_{t+n}}\right)\right] \tag{22}$$

with $0 < \beta < 1$ and $\theta \geqslant 1$.

B Equilibrium Solutions and Equity Premium of Model II

Proposition 4 *Table* 1 *summarizes the log-linearized Euler equations of Model II. Solving the model yields the following coefficients as defined in Proposition* 1*:*

$$\psi_{s,1} = \frac{1}{N_1}\frac{(1+\rho_\pi-\rho_z)\,\sigma\theta}{(\sigma+\theta-1)} \tag{23}$$

$$\psi_{s,2} = -\frac{1}{N_2}\frac{(1-\varphi)\left(1-\beta\rho_\xi\right)\sigma}{\varphi\left[1-\beta(1-\varphi)\right](\sigma+\theta-1)} \tag{24}$$

$$\pi_{s,1} = \frac{1}{N_1}\frac{(1-\rho_z)\,\sigma\theta}{(\sigma+\theta-1)} \tag{25}$$

$$\pi_{s,2} = \frac{1}{N_2} \tag{26}$$

and

$$N_i = -(1+\rho_\pi-\lambda_i)-(1-\lambda_i)(1-\beta\lambda_i)\frac{(1-\varphi)\,\sigma}{\varphi\left[1-\beta(1-\varphi)\right](\sigma+\theta-1)} \tag{27}$$

$$\boldsymbol{\lambda}_1 = \rho_z, \lambda_2=\rho_\xi. \tag{28}$$

Proposition 5 *We use EP_1 and EP_2 to denote the fraction of the equity premium contributed by technology shocks and monetary policy shocks respectively:*

$$EP_1 = \left\{g_1-(1-g_1)\frac{[(\sigma+\theta)\gamma-1]}{(1-\gamma)\,\sigma}\right\}\psi_{s,1}^2\sigma_1^2-(1-g_1)\frac{\gamma\theta}{(1-\gamma)}\psi_{s,1}\sigma_1^2, \tag{29}$$

$$EP_2 = \left\{g_2-(1-g_2)\frac{[(\sigma+\theta)\gamma-1]}{(1-\gamma)\,\sigma}\right\}\psi_{s,2}^2\sigma_2^2, \tag{30}$$

where $g_i=\frac{\beta(1-\lambda_i)}{1-\beta\lambda_i}$.

Proof. The proof is contained in Appendix C, which is available upon request. ■

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Table 1: Equilibrium in the Quartet of Models

	Flexible Prices	Sticky Prices
w/o Inv.	Model I (1) $\psi_t = -\sigma c_t$, (2) $\psi_t = (\theta - 1) y_t - \theta z_t$, (3) $y_t = c_t$. (4) $r_t = (1 + \rho_\pi)\pi_t + \xi_t$, (5) $\psi_t = E_t\psi_{t+1} + R_t - E_t\pi_{t+1}$	Model II (1) $\psi_t = -\sigma c_t$, (2) $\psi_t = (\theta - 1) y_t - \theta z_t + \frac{(1-\varphi)}{\varphi[1-\beta(1-\varphi)]} (\beta E_t\pi_{t+1} - \pi_t)$, (3) $y_t = c_t$ (4) $r_t = (1 + \rho_\pi)\pi_t + \xi_t$, (5) $\psi_t = E_t\psi_{t+1} + R_t - E_t\pi_{t+1}$
w/ Inv.	Model III (1) $\psi_t = \frac{\sigma b\beta}{(1-b\beta)(1-b)} E_t c_{t+1} - \frac{\sigma\left(1+b^2\beta\right)}{(1-b\beta)(1-b)} c_t + \frac{\sigma b}{(1-b\beta)(1-b)} c_{t-1}$, (2) $\psi_t = \frac{\theta-\alpha}{\alpha} y_t - \frac{\theta}{\alpha} z_t - \frac{\theta(1-\alpha)}{\alpha} k_t$, (3) $y_t = \widehat{c} c_t + (1 - \widehat{c})\, i_t$, (4) $\eta\,(i_t - k_t) = E_t \{\psi_{t+1} - \psi_t + [1 - \beta\,(1 - \delta)]\,(y_{t+1} - k_{t+1}) + \beta\eta\,(i_{t+1} - k_{t+1})\}$, (5) $k_{t+1} = (1 - \delta)\, k_t + \delta i_t$, (6) $r_t = \rho_r r_{t-1} + (1 - \rho_r)\,(1 + \rho_\pi)\pi_t + \xi_t$, (7) $\psi_t = E_t\psi_{t+1} + r_t - E_t\pi_{t+1}$	Model IV (1) $\psi_t = \frac{\sigma b\beta}{(1-b\beta)(1-b)} E_t c_{t+1} - \frac{\sigma\left(1+b^2\beta\right)}{(1-b\beta)(1-b)} c_t + \frac{\sigma b}{(1-b\beta)(1-b)} c_{t-1}$, (2) $\psi_t = \frac{\theta-\alpha}{\alpha} y_t - \frac{\theta}{\alpha} z_t - \frac{\theta(1-\alpha)}{\alpha} k_t + \frac{(1+\beta)(1-\varphi)}{\varphi[1-\beta(1-\varphi)]} \left(\frac{\beta}{1+\beta} E_t\pi_{t+1} + \frac{1}{1+\beta}\pi_{t-1} - \pi_t\right)$, (3) $y_t = \widehat{c} c_t + (1 - \widehat{c})\, i_t$, (4) $\eta\,(i_t - k_t) = E_t \{\psi_{t+1} - \psi_t + [1 - \beta\,(1 - \delta)]\,(p_{w,t+1} - p_{t+1} + y_{t+1} - k_{t+1}) + \beta\eta\,(i_{t+1} - k_{t+1})\}$, (5) $k_{t+1} = (1 - \delta)\, k_t + \delta i_t$, (6) $r_t = \rho_r r_{t-1} + (1 - \rho_r)\,(1 + \rho_\pi)\pi_t + \xi_t$, (7) $\psi_t = E_t\psi_{t+1} + R_t - E_t\pi_{t+1}$

When $\varphi = 1$, Model II collapses to Model I. When $\widehat{c} = 1$ and $\alpha = 1$, equations (4) and (5) in Model III become irrelevant. When we assume away internal persistence by setting ρ_r and b to zero, Model III collapses to Model I. When $\varphi = 1$, Model IV collapses to Model III. In models I and III, we list the last two equations apart from the first equations to stress the latter as a self-contained system for the equilibrium solutions of real variables. We rewrite the Phillips curve to summarize the sticky-price component in the last term, and substitute the log-linearized form of equation (6) for the real marginal cost. Equation (4) in Model III and IV are the log-linearized form of equation (10).

Table 2: Sensitivity Analysis of Alternative Calibrations

#	Calibration	EP	$\frac{EP_1}{EP}$ (%)	$\frac{EP_2}{EP}$ (%)
	Data	6.18		
	Benchmark	0.148	30	70
	Panel A (Model III, $\varphi = 1$)			
1	$\eta = 0, \theta = 100$	0.00	100	0
2	$\eta = 4.35, \theta = 100$	2.77	100	0
3	$\eta = 8, \theta = 100$	6.21	100	0
	Panel B (Model IV, role of φ)			
4	$\varphi \to 1$	0.125	100	0
5	$\varphi = 0.5$	0.148	30	70
6	$\varphi \to 0$	0.465	0.04	99.95
	Panel C (Role of ρ_π)			
7	$\rho_\pi = 0$	0.22	7	93
	Panel D (Role of ρ_ξ)			
9	$\rho_\xi = 0.95$	7.22	0.6	99.4
	Panel E (Role of η)			
10	$\eta = 1000$	0.16	32	68
11	$\rho_\xi = 0.95, \eta = 0$	0.32	2	98

The second column lists the parameters which are assigned values different from their benchmark values.

Table 3: Responses of Key Variables to Shocks

Panel A						
	$\varphi = 1$		$\varphi = 0.5$		$\varphi = 0$	
	ε_z	ε_ξ	ε_z	ε_ξ	ε_z	ε_ξ
ψ_t	−0.018	0.000	−0.011	0.016	−0.000	0.036
rr_t^s	0.017	0.000	0.010	−0.016	0.001	−0.033
π_t	−0.011	−0.024	−0.004	−0.009	−0.000	−0.000
mc_t	0.000	0.000	−0.008	−0.018	−0.023	−0.044
Panel B						
	$\rho_\xi = 0$		$\rho_\xi = 0.95$		$\rho_\xi = 0.95$ $\eta \to 0$	
	ε_z	ε_ξ	ε_z	ε_ξ	ε_z	ε_ξ
ψ_t	−0.011	0.016	−0.011	0.135	−0.004	0.030
rr_t^s	0.010	−0.016	0.010	−0.133	0.004	−0.027
π_t	−0.004	−0.009	−0.004	−0.087	0.001	−0.192
mc_t	−0.008	−0.018	−0.008	−0.154	0.003	−0.657
i_t	0.002	−0.004	0.002	−0.031	0.039	−1.248

The entries report the contemporary responses of the variables in the first column to technology shock ε_z and monetary policy shock ε_ξ under various calibrations. We only list the values of parameters which are different from those assigned in the benchmark calibration.

Table 4: Results of the Sticky-Price Sticky-Wage Model

Panel A: Equity Premium		
EP	$\frac{EP_1}{EP}$ (%)	$\frac{EP_2}{EP}$ (%)
0.36	27	73
Panel B: Responses of Key Variables		
	ε_z	ε_ξ
ψ_t	−0.017	0.026
rr_t^s	0.014	−0.025
π_t	−0.004	−0.002
mc_t	0.045	−0.021
i_t	0.004	−0.006