DEAN'S SEMINAR
Fall 2018
MATH 1000 (G-PAC course)
Language and Logic

TR 11:10am – 12:25pm
Corcoran Hall, Room 207
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Course Description

Using the exact methods of mathematics, modern mathematical logic studies reasoning and the degree to which reasoning can be formalized and mechanized. We will cover the fundamental principles of formal logic and how it relates to informal logic. In addition to mathematical problems, we will analyze the real world problems and those arising in other academic disciplines where logic adds formal method, meaning, and mathematical rigor.

We will first consider propositional logic where formulas are built from symbols for atomic sentences using Boolean sentential operator symbols: unary for negation, and binary for "and," "or," "if … then." This logic is used in digital computer circuit design. Universally true formulas are called *tautologies*, and those that are not contradictions are called *satisfiable*. While tautologies represent laws of logic, satisfiable formulas formalize consistent arguments. It is not hard to show that there are algorithms to decide whether a given propositional formula is satisfiable. Hence the satisfiability problem is *decidable*. It is not known whether there is a fast algorithm for the satisfiability problem, one that can be executed in time polynomial relative to the size of the input formula. Establishing whether such an algorithm exists is among the seven millenium prize problems (each worth million dollars) posed in 2000 by Clay Mathematics Institute.

We will expand the propositional language to predicate language by allowing the use of predicate symbols, and names and variables for individuals in building formulas, as well as quantifiers "for all," "there is." Such formal languages are sufficient to express a great deal of current mathematical practice. Predicate formulas that are true in every possible world are called *universally valid*. The problem of finding an algorithm to decide whether a given formula is valid was posed by David Hilbert, one of the most influential mathematicians of the 20th century, and became known as *Entscheidungsproblem*. Alan Turing was among the first to show that it is impossible to devise such an algorithm. Hence the validity problem is *undecidable*.

Textbook

Sweet Reason (*A Field Guide to Modern Logic*) by Jim Henle, Jay Garfild, and Tom Tymoszko, 2nd edition, Willey Blackwell, 2011.

Required background

High school algebra.