Department of Mathematics of the George Washington University

Math 6720 (CRN 53847): Topics in Logic
Topics in Computability Theory and Applications
Fall 2013; TuTh 3:45–5:00p.m.
Government Hall (2115 G Street), Room 325

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Description
Computability theory is a branch of mathematical logic, which explores the limits of computation. Classical computability theory formalized the intuitive notion of a decision procedure or an algorithm, and provided the theoretical basis for digital computers. It also demonstrated the limitations of algorithms and showed that most sets of natural numbers and problems they encode are not decidable (computable). Important results of modern computability theory include the classification of the computational difficulty of sets and the discovery of new classes of computational difficulty. Turing degrees provide an important measure of the level of such difficulty. There are uncountably many Turing degrees and they are partially ordered. We will study the structure of Turing degrees. They form an upper semilattice but not a lattice, and there are minimal degrees. An important countable substructure of this structure is the structure of computably enumerable degrees. These degrees also form an upper semilattice, but are dense (hence do not have minimal degrees). We will use Turing degrees and other computability-theoretic tools to investigate the complexity of problems in modern computable mathematics.
The main method of computability theory that we plan to use is the priority method. Although it was introduced in its simplest form in 1956–57, this powerful and intricate method is still not completely understood. We use the priority method when we want to satisfy several very complicated and mutually conflicting requirements by fitting together opposite strategies. We first decompose such requirements into infinitely many simpler subrequirements, assign priorities to the subrequirements, and then attempt to satisfy them in order of their priority. The execution of such a method is somewhat like an infinite game against a clever opponent. The construction succeeds when it becomes clear that the game, though infinite, has been won.

**Topics**


**Required Background**

Mathematical maturity and familiarity with the basic theory of algorithms. Math 6720 can be taken for credit repeatedly. Advanced undergraduate students may also take this course for credit.

**Grading**

Based on class participation and in-class presentation (30%), take-home assignments (50%), and the final project (20%).

**Learning Outcomes**

As a result of completing this course students should be able to:

1. Analyze computability theoretic properties of functions, sets, and countable algebraic structures;
2. Use the coding method to construct objects with various complexity properties;
3. Apply the extension and the priority methods to diagonalizing against infinitely many requirements;
4. Establish and apply correspondence between definability and computability;
5. Classify and compare mathematical objects within the arithmetical and the Turing degree hierarchies.
Course Material

All reading material will be provided in class. Here are some good references:


E. Fokina, V. Harizanov, and A. Melnikov, “Computable model theory,” accepted for publication in the centennial volume *Turing’s Legacy*, Cambridge University Press/ASL, 71 pages. (Survey chapter without proofs.)


D. Cenzer, V. Harizanov, and J. Remmel, “Computability-theoretic properties of injection structures,” accepted for publication in *Algebra and Logic*.


D. Cenzer, V. Harizanov, and J. Remmel, “$\Sigma^0_1$ and $\Pi^0_1$ equivalence structures,” *Annals of Pure and Applied Logic* 162 (2011), pp. 490–503.

