MATHEMATICAL MYSTICISM AND THE GREAT PYRAMID

by

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I remember vividly my first view of the Great Pyramid. I had seen many pictures of it, but nothing prepared me for the way its grey bulk looms over the restaurants and tourist hotels of Giza. It could first be seen from our taxi as we crossed the Nile. Then it seemed in no way remarkable: just a large grey triangle behind the skyline of Giza. But as we continued to drive the realization crept over me that we were as yet miles away and this object must be inconceivably huge. At that moment I finally understood why the Great Pyramid has come to be surrounded by such clouds of mysticism.

A standard claim of Great Pyramid mysticism is that its structure encodes a number of physical and mathematical constants. For example, Hunter Havelin Adams III writes in the Portland *African-American Baseline Essays* that the dimensions of the Great Pyramid contain

> the value of pi, the principle of the golden section [**sic**], the number of days in the tropical year, the relative diameters of the earth at the equator and the poles, and ratiometric distances of the planets from the sun, the approximate mean length of the earth's orbit around the sun, the 26,000-year cycle of the equinoxes, and the acceleration of gravity.(Adams, 1990)

More specifically, it is commonly asserted that the perimeter of the base of the Great Pyramid divided by twice its height gives a remarkably accurate estimate of pi. Indeed if one performs this computation a good estimate of the value of pi (3.150685) does emerge. This leads immediately to a conundrum: If the Egyptians had such a good approximation of pi when the Great Pyramid was erected in the 4th Dynasty why does the famous Rhind mathematical papyrus (written during the 13th Dynasty) imply a less accurate value (3.160494)?

There is more serious problem with the notion that the builders of the Great Pyramid encoded the value of pi in its fabric. There is reason to doubt that the ancient Egyptians even had the concept of pi. The Rhind mathematical papyrus describes the Egyptian method for calculating the area of a circle from its diameter:

> To get the area, one ninth of the diameter is first calculated; this fraction is subtracted from the value of the diameter and the result squared. This is equivalent to using a value of pi equal to 256/81.

This procedure for calculating the area of a circle appears to have been arrived at empirically. (Gilling, 1972) Note that the notion of squaring the diameter or radius and then multiplying by a constant does not appear.

Another frequently heard claim is that the slant height (or apothegm) of a face of the pyramid divided by one half of the length of the base gives phi, the so-called Golden Ratio (1.61803....). Some of the interesting properties of the Golden Ratio are presented in the mathematical excursus at the end of this article. In the 19th Century psychologist Gustav Fechner and others amassed considerable experimental evidence that rectangular objects whose sides are in the Golden Ratio are preferred by human test subjects.(Huntley, 1970) The 20th Century architect Le Corbusier incorporated the Golden Ratio into his designs.(Wells, 1986) However, Markowsky (1992) has recently pointed out that the concept of the Golden Ratio as the basis of harmonious proportions actually originated during the Renaissance. According to Markowsky, there is no evidence that the ancient Greeks or Egyptians made any intentional use of phi in their art or architecture. Even the Pythagoreans, to whom the discovery of the Golden Ratio is attributed, seemed to have been much more interested in other numbers such as $\sqrt{2}$, $\sqrt{3}$ and pi. (Fideler, 1993) Wells (1986) claims that a reference to a "sacred ratio" appears in the Rhine mathematical papyrus, but despite a careful search of a facsimile edition I have been unable to verify this.

Pyramidologists characteristically restrict their attention to the Great Pyramid and all but ignore other Egyptian pyramids. It is far from clear that ancient Egyptians regarded the Great Pyramid with the same awe as modern mystics. The hieroglyphic name for the Great Pyramid translates as "the pyramid which is the place of sunrise and sunset;" on the other hand, the hieroglyphic name for the smaller nearby Pyramid of Khephren translates as "the great pyramid." (Baines and Malek, 1980) At present forty-seven royal pyramids are known to exist or to have existed. From these I selected the twenty-two true pyramids whose heights and base dimensions can be determined with a reasonable degree of accuracy; I then used these dimensions to calculate values of both pi and phi. As can be seen in Table 1 the calculated values of pi and phi do not cluster closely around the true values. In the absence of any documentary evidence of an Egyptian interest in pi or phi, it is reasonable to conclude that any particular agreement between the calculated and true values is purely coincidental.

The values of pi and phi calculated in Table 1 are critically dependent on the slopes of the pyramids. It is a simple matter to show that if α is the angle of the slope or batter angle of the side of the pyramid then

$$\pi = \frac{4}{\tan(\alpha)} = 4\cot(\alpha)$$

and

$$N \text{ ' } \frac{1}{\cos(")} \text{ ' } \sec(")$$

For both pi and phi to fall within ±1% of their true values " must lie between 51º35' and 52º8'. Of the twenty-two royal pyramids in Table 1 only three have batter angles lying within this range.

Extant Egyptian mathematical papyri have problems in which four different values of the slopes of pyramids are used. (Gillings, 1972) These are presented in Table 2, along with the values of pi and phi which result from each. As can be seen, none of the values of pi or phi is strikingly close to the true values.

The mathematical sophistication of the ancient Egyptians can be gauged by comparing their value of pi with that determined by other ancient cultures. Table 3 summarizes all explicit and implicit values of pi known from writings dating from before 1000 CE As can be seen, the Egyptian value is the second worst used by any ancient people.

References

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MATHEMATICAL EXCURSUS

Most people have heard of **B**; the concept of the Golden Ratio or Divine Proportion **N** is probably less familiar. The value of **N** (from Phidias, the Greek sculptor) is

$$\frac{\sqrt{5} \,\%\, 1}{2}$$

The Golden Ratio or Divine Proportion results if a straight line is divided in the following way. Let the line be cut into segments of length **N** and 1 (**N** >1) so that the ratio of the length of the whole line to the length of the longer of the two segments is the same as the ratio of the length of the longer segment to the length of the shorter. That is,

$$\frac{N \,\%\, 1}{N} \;,\; \frac{N}{1}$$

From which one obtains

$$N^2 \,\&\, N \,\&\, 1 \;'\; 0$$

The quadratic formula then yields as a solution for **N**

$$N \;'\; \frac{1 \pm \sqrt{5}}{2}$$

In antiquity only the positive root was accepted as real.

A rectangle having sides equal to **N** + 1 and **N** can be divided into a **N** X **N** square and a geometrically similar **N** X 1 rectangle; this process can be continued <u>ad infinitum</u>. An equiangular spiral can be drawn through the points of intersection of the sides of the sequence of rectangles. Equiangular spirals occur frequently in nature (sunflower heads, nautilus shells and so forth).

N is also equal to the simplest continued fraction, viz.

$$\mathbf{N} \;'\; 1 \,\% \cfrac{1}{1 \,\% \cfrac{1}{1 \,\% \cfrac{1}{1 \,\% \cfrac{1}{\cdots}}}}$$

Further interesting properties of **N** may be found in Huntley (1970).

Table 1 -- Values of Pi and Phi Calculated From Dimensions of Twenty-Two Egyptian Royal Pyramids

Location	Pyramid	Base (m)	Height (m)	Pi	Phi
Giza	Khufu	230	146	3.150685	1.616105
	Khephren	214.5	143.5	2.989547	1.760399
	Menkaure	105	65.5	3.206107	1.598923
Abusir	Sahure	78.5	47	3.340426	1.560094
	Neuserre	81	51.5	3.145631	1.617708
	Neferirkare	105	70	3.000000	1.666667
Saqqara	Teti	78.5	52.5	2.990476	1.670066
	Userkaf	73.5	49	3.000000	1.666667
	Wenis	57.5	43	2.674419	1.799160
	Pepy I	78.5	52.5	2.990476	1.670066
	Izezi	78.5	52.5	2.990476	1.670066
	Meremre	78.5	52.5	2.990476	1.670066
	Pepy II	78.5	52.5	2.990476	1.670066
	Khendjer	52.5	37	2.837838	1.728224
Dahshur	Senwosret	105	78.5	2.675159	1.798815
	Snofru	220	104	4.230769	1.376185
	Amenemhet	105	81.5	2.576687	1.846588
el-Lisht	Amenemhet	78.5	55	2.854545	1.721502
	Senwosret	105	61	3.442623	1.532978
Maidum	Huni	147	93.5	3.144385	1.618104
Hawara	Amenemhet	100	58	3.448276	1.531535
el-Lahun	Senwosret	106	48	4.416667	1.349156

Source: Baines and Malek, 1980.

Table 2 -- Slopes of Pyramids Used in Problems in Egyptian Mathematical Papyri

Source	**Height (cubits)**	**Base (cubits)**	**Slope "**	**Pi**	**Phi**
RMP, Problem 56	250	360	54°14'	2.88	1.71
RMP, Problems 57 & 58	93 1/3	140	53°8'	3.00	1.67
RMP, Problem 59	8	12	53°8'	3.00	1.67
RMP, Problem 60	30	15	75°58'	1.00	4.12
MMP, Problem 14	6[1]	4	80°34'	0.66	6.10

RMP = Rhine mathematical papyrus

MMP = Moscow mathematical papyrus

Source: Gillings, 1972.

[1]This problem involves the volume of a truncated pyramid.

Table 3 -- Values of Pi Determined in Antiquity

Value	Date	Source
3	ca. 550 BCE	1 Kings 7:23/ 2 Chronicles 4:2
3.160493827 ($^{256}/_{81}$)	2000-1800 BCE	Rhind mathematical papyrus
3.125	uncertain	Babylonian cuneiform tablet
3.1373 3.1447	uncertain	ancient Greek gematria[2]
3.140845<B<3.142857 (3 $^{1}/_{7}$ <B< 3 $^{10}/_{71}$)	3rd Century BCE	Archimedes
3.14167	1st Century CE	Claudius Ptolemy
3.1622	130 CE	Hou Hau Shu
3.14159	265 CE	Liu Hui
3.1416	499 CE	Aryabhata
3.1415926<B<3.145927	5th Century CE	Tsu Chung-Chih/ Tsu Keng-Chih

Sources: Beckmann, 1971 and Fideler, 1993

[2]In ancient Greece the letters of the alphabet did double duty as numbers, with " = 1 $ = 2 and so forth. Consequently, each Greek word could be represented by the sum of the number values of its letters. The most familiar example of gematria is Revelations 13:18: "This calls for wisdom: let him who has understanding reckon the number of the beast for it is a human number, its number being six hundred and sixty-six." These two values of pi come from two Pythagorean (Fideler, 1993) diagrams. In the first, the circumference of a circle is assigned a length of 1000 units and the diameter is designated "Helios" (=318); in the second, the circumference is designated "Ouranos" (=891) and the diameter "Theos" (=284). Ancient gnostic Christians used similar diagrams to teach initiates about the cosmic order.