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The Performance of Alternative Interest Rate Risk Measures and Immunization Strategies under a Heath-Jarrow-Morton Framework

Senay Agca*

Abstract

Using a Monte Carlo simulation, this study addresses the question of how traditional risk measures and immunization strategies perform when the term structure evolves in a Heath-Jarrow-Morton (1992) manner. The results suggest that, for immunization purposes, immunization strategies and portfolio formation strategies are more important than interest rate risk measures. The performance of immunization strategies depends more on the transaction costs and the holding period than on the risk measures. Moreover, the immunization performance of bullet and barbell portfolios is not very sensitive to interest rate risk measures.

I. Introduction

Over the past two decades, development of a complicated set of models for describing the evolution of the term structure of interest rates has occurred. Research in this area has been driven primarily by the fact that interest rate risk management is increasingly an important component of the risk management practices of financial institutions.[1] At the same time, even the most sophisticated financial institutions continue to use traditional risk measures, such as the duration measures developed by Macaulay (1938) and Fisher and Weil (1971), as well as convexity measures. While one can model the term structure and then use the corresponding interest rate risk measures as an alternative to traditional

*Agca, sagca@gwu.edu, George Washington University, 2023 G St NW, Lisner Hall 540G, Washington, DC 20052. This paper is part of my dissertation work at Virginia Tech, and has benefited from comments by Isabelle Bajeux-Besnainou, Randall Billingsley, Stephen Brown (the editor), Martin Day, Narasimhan Jegadeesh, Raman Kumar, Abon Mozumdar, and seminar participants at the 2003 Euro/Informs Conference, 2001 FMA doctoral student seminar, Freddie Mac, George Washington University, Rutgers University, SUNY-Binghamton, and Virginia Tech. I am especially grateful to Don M. Chance, Saiyid Islam, and Pierluigi Balduzzi (the referee) for detailed and insightful comments. I thank Okan Kolak for technical assistance and the National Crash Analysis Center at George Washington University for providing additional computing resources. I acknowledge a summer research grant from the School of Business, George Washington University. Any remaining errors are my responsibility.

[1]The effect of interest rates on the banking industry is rigorously analyzed in the literature. Hasan and Sarkar (2002) review the literature in this area, and provide an analysis of the effect of interest rates on bank portfolios.

risk measures, doing so is more costly in terms of both human capital and computing technology. Thus, the effort is useful only if there are substantial benefits to be gained in using these more complex risk measures instead of their simpler traditional counterparts.

This paper compares the immunization performance of traditional risk measures with that derived from specific term structure models. Previous studies in a similar direction provide inconclusive evidence. For instance, Cox, Ingersoll, and Ross (1979), Brennan and Schwartz (1983), and Nelson and Schaefer (1983) develop term structure models and their corresponding interest rate risk measures. They then compare the immunization performance of Macaulay and Fisher-Weil duration measures with the immunization performance of their model-specific risk measures. The evidence suggests that the risk measures of term structure models do not offer better immunization performance than their traditional counterparts. Gultekin and Rogalski (1984) test the implications of alternative duration measures in terms of explaining the return variance of U.S. government bonds. They find that none of the duration measures analyzed are superior to the others. Babbel (1983) and Carcano and Foresi (1997) compare the immunization strategies that control for differing volatility with those that depend on simple duration matching strategies. While Babbel finds that simple duration matching strategies perform as well as the proposed alternatives, Carcano and Foresi provide counterevidence by considering the correlations between interest rates. Ilmanen (1992) finds that simple duration measures explain 80% to 90% of the return variance of U.S. government bonds. Finally, Ho, Cadle, and Theobald (2001), using Sterling futures data, show that the hedging performance of a specific Heath, Jarrow, and Morton (HJM hereafter) model-based risk measure is no better than that of modified duration.

These results are not comparable since they are based on different data sets, portfolio formation strategies, and analysis periods. Additionally, if the model or stochastic process conjectured to develop the risk measures does not represent the true stochastic process underlying the data at certain times, it would be hard to interpret the immunization performance of the risk measures. A comparison of the immunization performance of risk measures based solely on term structure models with empirical data amounts to an analysis of which term structure best fits the data. Moreover, when the performance of certain risk measures is poor, it is hard to infer the reasons for this poor performance only using empirical data. To address these problems, I use a selected set of one-factor HJM (1992) models as a benchmark.[2] Then I carry out a Monte Carlo simulation-based study to compare the immunization performance of traditional risk measures with that of benchmark risk measures. I estimate the parameters of the benchmark models using U.S. government bond data, and I use these parameters throughout the simulation. I examine the immunization performance both with and without trans-

[2]I focus on one-factor HJM models for several reasons. First, traditional risk measures assume one random underlying factor, and therefore it is consistent to compare risk measures that depend on only one random factor. Also Litterman and Scheinkman (1991) and Chapman and Pearson (2001) document that more than 85% of the variation in the U.S. Treasury yield curve can be explained by one factor. These authors refer to this factor as the level of interest rates. Additionally, all short rate models can be represented as specific cases of one-factor HJM models, and short rate models still receive considerable attention in the literature.

action costs. Additionally, to consider the impact of sampling variability on the immunization performance of HJM risk measures, I calculate these measures using parameters estimated in the simulation rather than those used to simulate the data.[3] I use bullet, barbell, and random portfolios for each scenario so that the immunization performance of the risk measures is not overly dependent on the way the immunization portfolios are formed.[4] I consider two widely used immunization strategies in evaluating the performance of each risk measure—one that matches only the duration of the assets and liabilities, and one that matches both their duration and convexity.

The evidence of this paper suggests that major performance differences among immunization strategies depend more on the immunization strategy than on the risk measures. When a duration matching strategy is used, immunization performance with traditional interest rate risk measures is very similar to that with HJM risk measures. When there are no transaction costs, a duration and convexity matching strategy substantially improves immunization performance. When the parameters of the HJM risk measures are estimated in the simulation, sampling error affects the immunization performance of the HJM risk measures. Since traditional risk measures do not require any estimation—and hence are free of this sampling error—under most scenarios, they provide similar or better performance than HJM risk measures that are based on estimated parameters.

When there are transaction costs, a duration and convexity matching strategy performs better for short holding periods, whereas a duration matching strategy performs better for medium and long holding periods. The transaction costs incurred increase with the length of the holding period. Since a duration matching strategy incurs relatively fewer transaction costs than a duration and convexity matching strategy, the former strategy performs better over medium to long holding periods. Therefore, when transaction costs are present, immunization strategies should be selected with respect to holding periods.

Among portfolio formation strategies, both bullet and barbell portfolios offer very good immunization performance over short holding periods. Since bullet portfolios generally have a yield advantage over barbell portfolios, their returns over short holding periods are closer to the respective target returns than the corresponding returns of barbell portfolios. However, as the holding period increases, the convexity advantage of barbell portfolios overcomes the yield advantage of bullet portfolios. Therefore, over medium and long holding periods, barbell portfolios provide better immunization than bullet portfolios. When there are no transaction costs, a duration and convexity matching strategy improves the immunization performance of both bullet and barbell portfolios. When transaction costs are present, however, duration and convexity matched bullet portfolios offer better performance over short holding periods, and duration matched barbell

[3] I assume no transaction costs for this set of simulations. Since the aim is to examine the impact of sampling variability on the performance of HJM risk measures, introduction of transaction costs would make the inference less clear. It would not be possible to associate the differences in performance solely with the sampling variability.

[4] A barbell portfolio consists of securities that have the lowest and highest durations. A bullet portfolio, on the other hand, consists of securities with durations closest to the target holding period.

portfolios offer better performance over long holding periods. In both cases, the portfolio performance is not very sensitive to interest rate risk measures.

The overall evidence suggests that immunization and portfolio formation strategies affect immunization performance more than risk measures. Additionally, the length of holding period and transaction costs has a considerable impact on the immunization performance of these strategies. HJM risk measures provide better immunization only when used with the appropriate immunization strategy. When the parameters of the HJM risk measures are not known but are estimated, under most scenarios, traditional risk measures that do not require estimation provide a similar or better immunization performance than HJM risk measures.

The paper proceeds as follows. Section II gives a brief description of traditional duration and convexity measures, outlines the HJM framework, and presents the risk measures developed for specific one-factor HJM models. Section III describes the simulation framework. Section IV compares the immunization performance of interest rate risk measures and portfolio formation strategies. Section V presents the conclusions.

II. Interest Rate Risk Measures

A. Traditional Interest Rate Risk Measures

The development of interest rate risk measures dates back to Macaulay (1938), Redington (1952), Hicks (1939), and Samuelson (1945). These authors independently developed an interest rate risk measure that assumes a constant yield and, therefore, theoretically provides accurate immunization under a flat yield curve and parallel shifts in the yield curve. This interest rate risk measure is referred to as Macaulay's duration. Fisher and Weil (1971) relaxed the assumption of a constant yield in Macaulay's duration and developed a new duration measure, henceforth referred to as the Fisher-Weil duration. The discount factors of the cash flows in the Fisher-Weil duration are derived from the current term structure. Theoretically, Fisher-Weil duration provides accurate immunization under parallel shifts in the yield curve. These duration measures at time t are summarized for a coupon bond maturing at $T = \tau_n$, with coupon payments at time periods τ_i and principal payment at τ_n, and with price $B(t, T)$,[5,6]

$$(1) \qquad D_{\mathrm{FW}}(t) = \frac{1}{B(t,T)} \sum_{i=1}^{n} C_i(\tau_i - t) e^{-y(t,\tau_i)(\tau_i - t)}.$$

Here, C_i is the coupon payment at date τ_i, where $t \leq \tau_1 < \tau_2 < \ldots < \tau_n \leq T$, for a trading interval of $[t, T]$; $P(t, \tau_i)$ is the price of a zero-coupon bond maturing at time τ_i, and $y(t, \tau_i)$ is the yield to maturity of $P(t, \tau_i)$. In Macaulay's duration, the yield to maturity is constant and, therefore, $y(t, \tau_i) = y(t)$.

Duration models are based on the assumption of a linear relation between bond prices and interest rates. Such an assumption is valid for infinitesimal

[5]None of the bonds considered in this study contain options.

[6]Since HJM models assume continuously compounded forward rates, the yield to maturity is considered continuously compounded.

changes in the interest rate. For non-infinitesimal changes, the nonlinear relation between bond prices and interest rates must be taken into account in order to obtain an accurate immunization. The second derivative of the bond price with respect to the interest rate provides the expression, often called convexity, to be considered in addition to duration,

$$\text{(2)} \qquad \text{Conv}_{\text{FW}}(t) = \frac{1}{B(t,T)} \sum_{i=1}^{n} C_i(\tau_i - t)^2 e^{-y(t,\tau_i)(\tau_i - t)}.$$

The convexity measure corresponding to Macaulay's duration has a constant yield, $y(t,\tau_i) = y(t)$, while that corresponding to the Fisher-Weil duration has a varying yield.

B. HJM Interest Rate Risk Measures

HJM (1992) develop an arbitrage-free pricing framework for capturing the evolution of forward rates and therefore modeling the term structure of interest rates. The specification of different volatility functions together with the input of the initial forward curve results in different term structure models. The general HJM model is a multi-factor model. An n-factor HJM model specifies the evolution of the instantaneous forward rate according to the stochastic process,

$$\text{(3)} \qquad df(t,T) = \alpha(t,T,w)dt + \sum_{i=1}^{n} \sigma_i(t,T,w)dW_i(t).$$

Here, $f(t,T)$ is the instantaneous forward rate at time t for date T, $T > t$; dW_t is Brownian motion at time t;[7] $\alpha(t,T,w)$ is a random drift term of the forward rate curve, and $\sigma_i(t,T,w)$ is the volatility function i for $i = 1,\ldots,n$, where $T > t$ and w is a sample point defined on Ω.

The no-arbitrage condition puts a restriction on the relation of drift to volatilities. Under the equivalent martingale measure, the drift restriction and corresponding forward rate process are, respectively,

$$\text{(4)} \qquad \alpha(t,T,w) = \sum_{i=1}^{n} \sigma_i(t,T,w) \int_t^T \sigma_i(t,v,w)dv$$

and

$$\text{(5)} \qquad df(t,T) = \left(\sum_{i=1}^{n} \sigma_i(t,T,w) \int_t^T \sigma_i(t,v,w)dv \right) dt + \sum_{i=1}^{n} \sigma_i(t,T,w)d\tilde{W}_i(t).$$

In equation (5), $\tilde{W}(t)$ corresponds to the Wiener process under the equivalent martingale measure. In HJM models, different volatility functions lead to dif-

[7] A stochastic process W_t defined for $t \geq 0$ on (Ω, F, P) is a Brownian motion if $W_0 = 0$, W_t is continuous with respect to t; the increment $W_{s+t} - W_s$ is distributed as normal $N(0,t)$, under P and is independent of F_s.

ferent term structure models.[8] The price at time t for a zero-coupon bond maturing at time T, $P(t,T)$, is determined by the forward rate dynamics, $P(t,T) = \exp(-\int_t^T f(t,u)du)$.

This study focuses on one-factor HJM models that have only one random factor driving the forward rate dynamics. After specification of the volatility functions, the resulting term structure model provides the bond pricing framework. This bond pricing framework is used to derive the duration and convexity measures corresponding to the selected HJM models.

Au and Thurston (1995) derive the duration measures for certain continuous-time one-factor HJM term structure models. Munk (1999) and Jeffrey (2000) use an alternative method to determine duration measures for HJM models.[9] The duration measure for one-factor HJM models, $D_{\text{HJM}}(t)$, with deterministic volatility is

$$D_{\text{HJM}}(t) = \frac{\sum_{i=1}^{n} C_i \left(\int_t^{\tau_i} \sigma(t,v)dv \right) P(t,\tau_i)}{\sigma(t,t) \sum_{i=1}^{n} C_i P(t,\tau_i)}. \tag{6}$$

For constant volatility, $\sigma(t,T) = \sigma$, $D_{\text{HJM}}(t)$ reduces to the Fisher-Weil duration measure. Following Amin and Morton (1994), I only consider one-factor HJM models that have the time-invariance property. The volatility functions of these HJM models depend only on the time to maturity, not the calendar date.[10] In this respect, I consider the constant and exponentially decaying volatility functions analyzed by Au and Thurston (1995). Additionally, I analyze the volatility function Mercurio and Moraleda (2000) develop. Mercurio and Moraleda develop a volatility function to represent the humped term structure of volatility sometimes observed in the market. The duration measures that correspond to these one-factor HJM models are as follows.

Constant Volatility $(\sigma(t,T) = \sigma)$:

$$D_{\text{HJM}}(t) = \frac{\sum_{i=1}^{n} C_i(\tau_i - t)P(t,\tau_i)}{\sum_{i=1}^{n} C_i P(t,\tau_i)}. \tag{7}$$

Exponentially Decaying Volatility $(\sigma(t,T) = \sigma e^{-\lambda(T-t)})$:

$$D_{\text{HJM}}(t) = \frac{\sum_{i=1}^{n} C_i P(t,\tau_i)(1 - e^{-\lambda(\tau_i - t)}) \Big/ \lambda}{\sum_{i=1}^{n} C_i P(t,\tau_i)}. \tag{8}$$

[8]For example, in one-factor HJM models, if volatility is constant, i.e., if $\sigma(t,T) = \sigma$, the model reduces to the Ho and Lee (1986) model. If volatility is exponentially decaying, i.e., if $\sigma(t,T) = \sigma \exp(-\lambda(T-t))$, the model reduces to the Hull and White (1993) extended Vasicek model. Other short rate models can also be represented as special cases of one-factor HJM models.

[9]Bierwag (2000) also derives the duration measure of the Ho and Lee (1986) model, i.e., the constant volatility HJM model. Bierwag's duration measure is different than the one stated in this paper since Bierwag derives the measure for a binomial interest rate stochastic process.

[10]Since I only consider a restricted number of one-factor HJM models, it seems appropriate to select the models that have common properties.

Humped Volatility $(\sigma(t,T) = \sigma(1+\gamma(T-t))e^{-\lambda(T-t)})$:[11]

$$(9)\quad D_{\text{HJM}}(t) = \frac{1}{\sum_{i=1}^{n} C_i P(t,\tau_i)} \sum_{i=1}^{n} C_i P(t,\tau_i) \times \frac{\gamma}{\lambda}\left[\left(\frac{1}{\gamma}+\frac{1}{\lambda}\right)\left(1-e^{-\lambda(\tau_i-t)}\right) - (\tau_i - t)e^{-\lambda(\tau_i-t)}\right].$$

Following the framework used by Au and Thurston (1995), I derive the convexity measures of one-factor HJM models.[12] Under the HJM framework with deterministic volatilities, the convexity of a coupon bond, Conv_{HJM}, is[13]

$$(10)\quad \text{Conv}_{\text{HJM}}(t) = -\frac{1}{\sum_{i=1}^{n} C_i P(t,\tau_i)} \sum_{i=1}^{n} C_i \frac{\partial P(t,\tau_i)}{\partial f(t,t)} \frac{1}{\sigma(t,t)} \int_t^{\tau_i} \sigma(t,v)dv$$

$$= \frac{1}{B(t,T)} \sum_{i=1}^{N} C_i \frac{P(t,\tau_i)}{\sigma(t,t)^2} \left(\int_t^{\tau_i} \sigma(t,v)dv\right)^2.$$

For constant volatility, $\sigma(t,T) = \sigma$, $\text{Conv}_{\text{HJM}}(t)$ reduces to the varying yield convexity given in equation (2). The convexity measures for the selected one-factor HJM models are summarized below.

Constant Volatility $(\sigma(t,T) = \sigma)$:

$$(11)\quad \text{Conv}_{\text{HJM}}(t) = \frac{1}{\sum_{i=1}^{n} C_i P(t,\tau_i)} \sum_{i=1}^{n} C_i P(t,\tau_i)(\tau_i - t)^2.$$

Exponentially Decaying Volatility $(\sigma(t,T) = \sigma e^{-\lambda(T-t)})$:

$$(12)\quad \text{Conv}_{\text{HJM}}(t) = \frac{1}{\lambda^2 \sum_{i=1}^{n} C_i P(t,\tau_i)} \sum_{i=1}^{n} C_i P(t,\tau_i)(1-e^{-\lambda(\tau_i-t)})^2.$$

Humped Volatility $(\sigma(t,T) = \sigma(1+\gamma(T-t))e^{-\lambda(T-t)})$:[14]

$$(13)\quad \text{Conv}_{\text{HJM}}(t) = \frac{1}{\sum_{i=1}^{n} C_i P(t,\tau_i)} \sum_{i=1}^{n} C_i P(t,\tau_i) \times \frac{\gamma^2}{\lambda^2}\left[\left(\frac{1}{\gamma}+\frac{1}{\lambda}\right)\left(1-e^{-\lambda(\tau_i-t)}\right) - (\tau_i - t)e^{-\lambda(\tau_i-t)}\right]^2.$$

[11] The derivation of the humped volatility HJM duration measure is given in the Appendix.

[12] Full derivations are given in the Appendix.

[13] Fruhwirth (2002) follows a similar approach in deriving the convexity measures of one-factor HJM models. My results agree with his.

[14] The Appendix gives the derivation of the humped volatility HJM convexity measure.

III. The Simulation Framework

Forward rates are based on the selected HJM models; therefore, the risk measures of the HJM models provide the benchmarks. The inputs of the simulations are estimated using U.S. government bond data. Forward rate data up to 30 years of maturity and the corresponding zero-coupon yield curve data for the period January 1947 to February 1991 are obtained from McCulloch and Kwon (1993).[15] For the period January 1947 to February 1991, I form yield curve charts for each month that I categorize by shape: flat, declining, increasing, and humped. Then, I select one representative yield curve from each shape. The selected yield curves are: for flat, August 1989; for increasing, January 1985; for decreasing, May 1981; and for humped, October 1986. Figure 1 shows these yield curves and the corresponding forward rate curves. I use the cubic spline method to smooth the forward rates and to interpolate missing monthly forward rates.[16,17] Forward rates corresponding to these selected dates are then used as the initial forward rate curves in the simulations.

Another necessary input for the simulations is the forward rate volatilities. To determine the volatilities, I use four years of historical forward rate data corresponding to each of the selected four yield curves.[18] Corresponding forward rates are smoothed and the missing monthly forward rates are interpolated using the cubic spline method. Then, I calculate changes in the forward rates. The standard deviation vector of different maturity forward rates gives the forward rate volatilities.[19] Using regression methods, I fit forward rate volatilities into the volatility functions of certain one-factor HJM models to estimate the parameters of each volatility function. I then use these estimated parameters in the simulation of the forward rates. Table 1 presents the results. The exponential decay factor in both exponentially decaying volatility and humped volatility models is negative due to the increase in the respective forward rate volatilities after 20 years. The linear increase factor for the humped volatility is positive but very small. As a result, the humped and exponentially decaying volatility functions are determined mainly by the negative decay factor. In this respect, the forward rate volatilities increase

[15]For this data set, both the zero-coupon bond yields and the forward rates are derived by McCulloch (1975) using a tax-adjusted cubic spline method. All forward rates are given as percentages per annum on a continuously compounded basis. All data are from the afternoon of the last business day of the month indicated. For each month, 56 maturities are available. The data are monthly from zero to 18 months, then quarterly to two years, then semi-annually to three years, then annually to 35 years; the data then make a five-year jump to 40 years.

[16]Bliss (1997) compares different curve fitting procedures and finds that the McCulloch (1975) cubic spline procedure and the Fama and Bliss (1987) bootstrapping procedure outperform other existing methods.

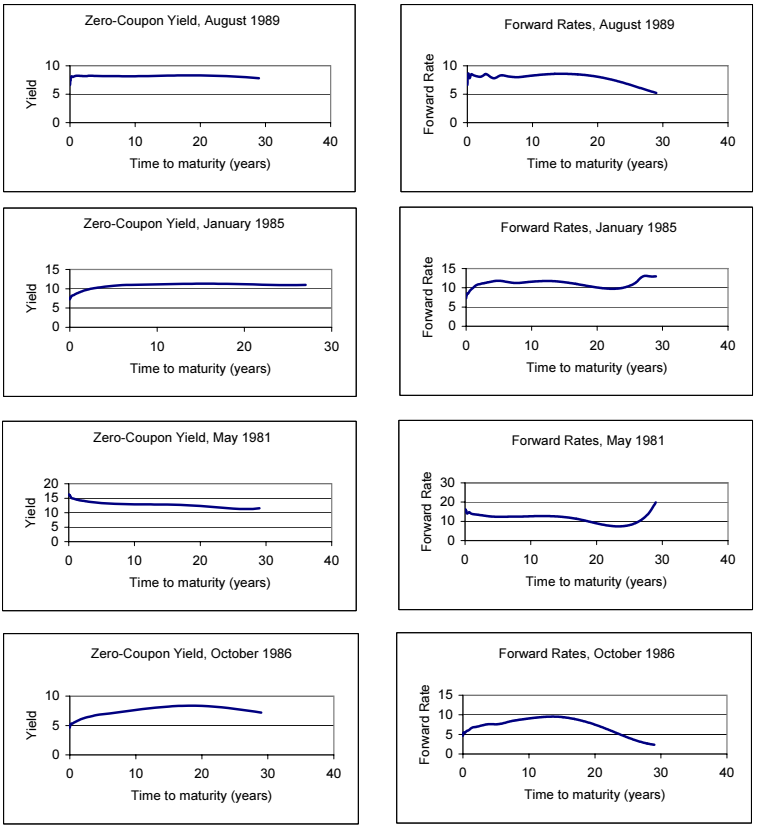
[17]Since the forward rate data of McCulloch and Kwon (1994) are not available at the monthly frequency after 18 months, I use the cubic spline method to interpolate the missing monthly forward rates.

[18]Rebonato ((1998), pp. 51–74) and Amin and Ng (1997) use four years of historical data in their volatility estimations.

[19]Using a principal component analysis, Litterman and Scheinkman (1991) and Chapman and Pearson (2001) show that all variation in the U.S. Treasury yield curve can be explained by three factors, and more than 85% of it by one factor alone. Since the focus of this paper is one-factor HJM models, I assume that all variation in the yield curve is explained by one factor. In an earlier version, I employ a three-factor principal component analysis to estimate volatilities. I then use the first factor in the simulations of the selected one-factor HJM models. The results are similar to the ones reported here.

FIGURE 1

Initial Yield Curves and Forward Rate Curves



with the term to maturity in both models. I plot these volatility functions, along with their estimated parameters, against the observed forward rate volatilities in Figure 2. While the constant volatility model fit is poor, the best fit is for humped and exponentially decaying volatilities. The fits of these two volatility models are indistinguishable from each other. Hence, in the rest of the study I focus mainly on the exponentially decaying volatility HJM model.

For the initial time period, I derive the zero-coupon bond prices from the forward rates, and I construct the coupon bonds from the zero-coupon bond prices. To determine the coupon rates, I use CRSP monthly Treasury data. For each of the four dates of initial yield curves, I compute the average coupon rate from the

TABLE 1

Forward Rate Volatility Functions

	Flat (Aug. 1989)	Increasing (Jan. 1985)	Decreasing (May 1981)	Humped (Oct. 1986)
Panel A. Constant Volatility ($\sigma(t,T) \equiv \sigma$, a constant, and $\sigma(t,T) = \hat{\sigma} + \varepsilon$)				
$\hat{\sigma}$	0.01775	0.02974	0.02193	0.02811
	(4.57E−04)	(3.01E−04)	(8.53E−04)	(6.76E−04)
Panel B. Exponential Decay Volatility ($\sigma(t,T) \equiv \sigma e^{-\lambda(T-t)}$, σ, λ, constants, and $\ln(\sigma(t,T)) = \ln(\hat{\sigma}) - \hat{\lambda}(T-t) + \varepsilon$)				
$\ln(\hat{\sigma})$	−1.0769	−0.83962	−0.67006	−0.85898
	(2.49E−02)	(2.91E−02)	(4.61E−02)	(2.31E−02)
$\hat{\lambda}$	−0.0208	−0.03727	−0.01294	−0.04043
	(1.54E−03)	(1.80E−03)	(2.75E−03)	(1.43E−03)
$\hat{\sigma}$	0.01180	0.01496	0.01773	0.014675
Panel C. Humped Volatility ($\sigma(t,T) \equiv \sigma(1+\gamma(T-t))e^{-\lambda(T-t)}$, σ, λ, and γ, constants, and $\ln(\sigma(t,T)) = \ln(\hat{\sigma}) - \hat{\lambda}(T-t) + \ln(1+\hat{\gamma}(T-t)) + \varepsilon$)				
$\ln(\hat{\sigma})$	−1.0528	−0.8042	−0.6519	−0.8061
	(3.73E−02)	(4.37E−02)	(6.90E−02)	(3.49E−02)
$\hat{\lambda}$	−0.0195	−0.0353	−0.0120	−0.0377
	(1.54E−03)	(2.90E−03)	(2.75E−03)	(6.56E−03)
$\hat{\gamma}$	0.000021	0.000086	0.000014	0.000032
	(0.00000)	(0.00000)	(0.00000)	(0.00000)
$\hat{\sigma}$	0.012088	0.015500	0.018050	0.015471

The one-factor Heath-Jarrow-Morton (HJM) model specifies the instantaneous forward rate as

$$df(t,T) = \left(\sigma(t,T)\int_t^T \sigma(t,s)\,ds\right)dt + \sigma(t,T)\,d\tilde{W}_t,$$

where $f(t,T)$ is the instantaneous forward rate at time t for date T, $T > t$, $\tilde{W}_t$ is a Brownian motion under an equivalent Martingale measure at time t, and $\sigma(t,T)$ is the stochastic volatility function. Three different volatility functions are considered in Panels A, B, and C: constant, exponentially decaying, and humped volatilities, respectively. For each volatility function, the parameters are estimated using four years of historical monthly forward rate data from McCulloch and Kwon (1993). These estimated volatilities are fitted into the above-mentioned three volatility functions in order to estimate the parameters. To estimate the parameters of humped volatility, the Marquardt nonlinear least squares method is used. Other volatility parameters are estimated using the linear regression method. Volatility parameter estimates are for August 1989 (flat yield curve), January 1985 (increasing yield curve), May 1981 (decreasing yield curve), and October 1986 (humped yield curve). The volatility estimates presented are the annualized volatilities. In each panel, the estimates of volatility parameters are presented. Standard errors are given in parentheses.

coupon bonds trading on that specific date. I then use these average coupon rates to generate the coupon bonds.[20]

The immunization performance of the interest rate risk measures are analyzed over one-, five-, and 10-year holding periods.[21] The target yield is the yield on a zero-coupon bond maturing at the end of the holding period. Table 2 gives the target yields for the selected initial yield curves and holding periods. A portfolio is considered to be successfully immunized if the holding period return of the portfolio is close to the yield of the target zero-coupon bond.[22] Using these criteria, I analyze portfolios that have holding period returns within one, five, and 10 basis points of the target yield.

[20]In Fisher and Weil (1971), the coupon rate is fixed at 4%, whereas in Bierwag, Kaufman, Schweitzer, and Toevs (1981), the coupon rate is fixed at 5%.

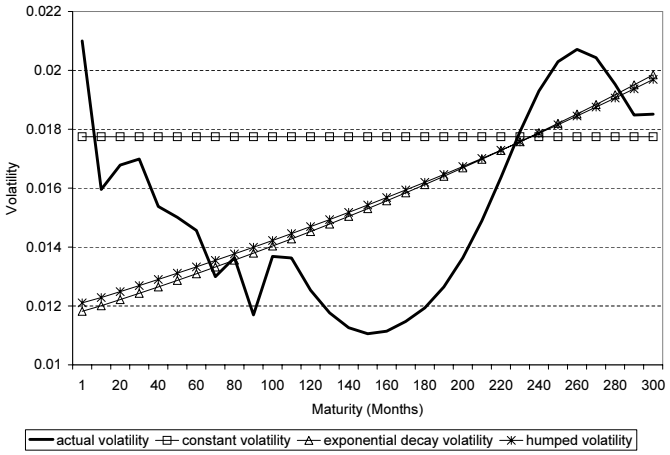
[21]Fisher and Weil (1971) use five-, 10-, and 20-year holding periods. The holding periods in Cox, Ingersoll, and Ross (1979) are five, 10, 15, 20, 35, and 50 years. Brennan and Schwartz (1983) and Gultekin and Rogalski (1984) use five- and 10-year holding periods. Nelson and Schaefer (1983) use five-, 10-, and 15-year holding periods.

[22]Brennan and Schwartz (1983) and Nelson and Schaefer (1983) use this standard.

FIGURE 2

Forward Rate Volatility Functions

Graph A. Forward Rate Volatility Curve, August 1989



Graph B. Forward Rate Volatility Curve, January 1985

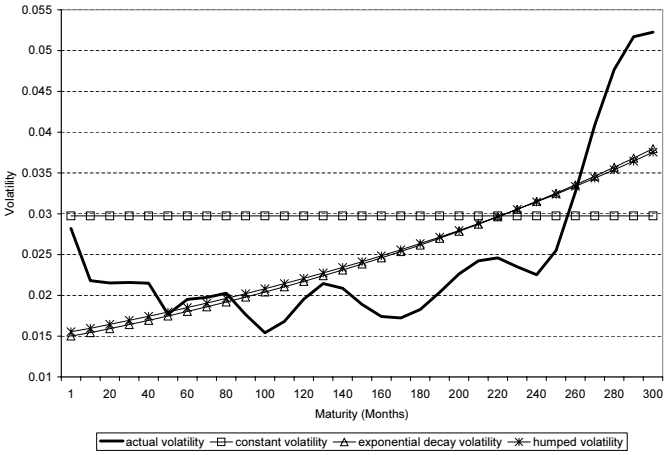
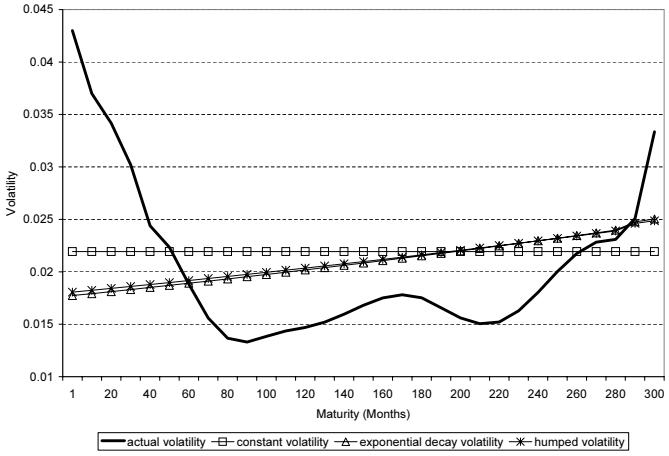


FIGURE 2 (continued)

Forward Rate Volatility Functions

Graph C. Forward Rate Volatility Curve, May 1981



Graph D. Forward Rate Volatility Curve, August 1986

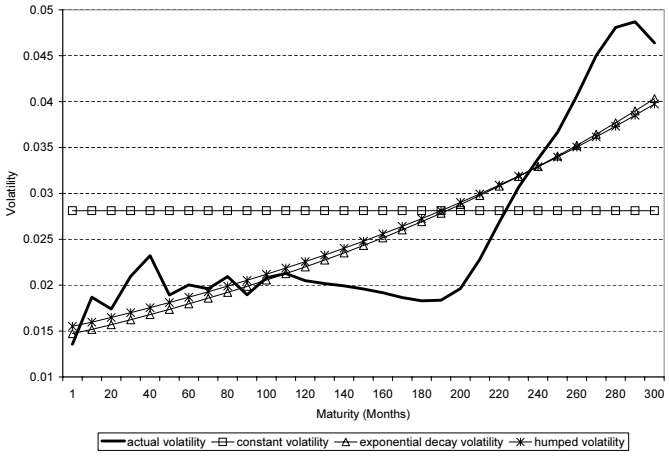


TABLE 2

Yields of Target Zero-Coupon Bond Prices

Yield	Holding Period		
	One-Year	Five-Year	Ten-Year
Flat (Aug. 1989)	8.1523%	8.1546%	8.1465%
Increasing (Jan. 1985)	8.8667%	10.7388%	11.0990%
Decreasing (May 1981)	14.7832%	13.3495%	12.9036%
Humped (Oct. 1986)	5.6248%	6.8994%	7.6217%

Table 2 gives the yields of the target zero-coupon bonds for each of the initial yield curves and holding periods analyzed. Four initial yield curve shapes are considered: flat (August 1989), increasing (January 1985), decreasing (May 1981), and humped (October 1986). Zero-coupon bond prices are from McCulloch and Kwon (1993). Three different holding periods are considered: short-term (one-year), medium-term (five-year), and long-term (10-year). The target bond is the zero-coupon bond that has maturity equal to the holding period. The yields of these zero-coupon bonds are the target yields used to test the immunization performance of alternative interest rate risk measures.

I focus on the two most widely used immunization strategies, namely the duration matching and the duration and convexity matching strategies. The duration matching strategy is expected to perform well under small yield curve shifts. The duration and convexity matching strategy is expected to be successful for non-infinitesimal changes in the yield curve as well. The target zero-coupon bond can be considered as the liability portfolio and the coupon bond portfolio as the asset portfolio. The duration of a coupon bond portfolio is matched to the duration of the respective target zero-coupon bond to form a duration matched portfolio. To form a duration and convexity matched portfolio, both the duration and the convexity of a coupon bond portfolio are matched to those of the respective target zero-coupon bond. After a portfolio is formed, the weights of the securities are changed at every rebalancing period to match the duration, or the duration and convexity, of the target bond. The rebalancing period is one month, since monthly forward rate data are used. The immunization portfolios are self-financing.

Portfolio formation strategies also affect immunization performance. To consider the impact of portfolio formation strategies on immunization performance, a bullet, a barbell, and 100 random portfolios are formed for each scenario. The random portfolios are formed to imitate portfolio formation strategies that cannot be observed or strategies where the portfolio manager has to act in response to exogenous factors. Uniformly distributed random numbers are used to determine the maturity of the coupon bonds in the random portfolios. The maturities of the bonds in random portfolios vary between the holding period and 30 years.[23] Two sets of 100 random portfolios are formed; one is for a duration matching strategy that has two coupon bonds in each portfolio; and the other one is for a duration and convexity matching strategy that has three coupon bonds in each portfolio. I use the same random portfolio set for each HJM model considered in the simulations.

Bullet and barbell portfolios are widely used strategies. A barbell portfolio consists of bonds with the highest and lowest durations available. With a duration matching strategy, a barbell portfolio is formed using two coupon bonds; with one maturing at the end of the holding period, and the other maturing in 20 years. Rolling portfolios are not allowed. To match both duration and convex-

[23]The minimum maturity of a coupon bond in the portfolio is the holding period, since rolling over is not allowed.

ity, an additional coupon bond is necessary. This additional coupon bond is a medium-term bond. For a one-year holding period, I select a 10-year bond. For five- and 10-year holding periods, the selected coupon bond has a maturity of 12 years and 15 years, respectively. A bullet portfolio consists of bonds that have durations close to the target duration. As is the case with other portfolios, bullet portfolios are formed using two securities with a duration matching strategy, and three securities with a duration and convexity matching strategy.[24] To determine these bonds, an optimization program is run to minimize the absolute difference between the duration of a bond portfolio and the target duration. The optimization program considers bond portfolios consisting of all possible combinations of bonds with maturities ranging between the holding period and 360 months. The optimal portfolio has the closest duration to that of the target bond, and hence is considered the bullet portfolio. I form separate bullet portfolios for each one-, five-, and 10-year holding periods, and for each risk measure considered in the study—Macaulay, Fisher-Weil, and the benchmark HJM risk measures.

I examine the immunization performance of duration matched, and duration and convexity matched, portfolios both with and without transaction costs. Theoretically, duration and convexity matched portfolios should provide better immunization than duration matched portfolios, since the duration and convexity matching strategy takes the nonlinearity between bond price and yield into account. However, duration and convexity matched portfolios incur higher transaction costs than duration matched portfolios due to the greater amount of bond rebalancing in each period.[25] To analyze the impact of transaction costs on the respective immunization strategies, I use the median effective bid-ask spreads of different maturity on-the-run U.S. Treasury bonds reported by Fleming and Sarkar (1999) for the year 1993.[26] These median spread data are duplicated in Table 3. Missing maturity spreads are linearly interpolated. Simulated forward rates are assumed to generate the mid-price of the coupon bonds. Using the spread data, bid and ask prices of the bonds are then determined. Throughout the analysis of the immunization strategies with transaction costs, bonds are bought at ask and sold at bid at each rebalancing period.

HJM risk measures depend on volatility parameters, whereas traditional risk measures do not. While comparing the immunization performance of risk measures, the volatility parameters I use in the simulation of the forward rates are also used with the HJM risk measures. Using HJM risk measures that depend on the volatility parameters estimated in the simulation rather than those used in simulating the forward rates can affect the immunization performance of the respective HJM risk measures.[27] To examine the impact of sampling variability on the performance of HJM risk measures, I run another set of simulations under the

[24]In Gultekin and Rogalski (1984), random immunization portfolios are formed from two to 10 securities. Brennan and Schwartz (1983) use 20 securities in forming their immunization portfolio. In most studies, such as Fisher and Weil (1971), Bierwag, Kaufman, and Toevs (1983), and Nelson and Schaefer (1983), only two securities are used in forming the immunization portfolio.

[25]I thank the referee for this constructive comment.

[26]Fleming and Sarkar (1999) use GovPX, Inc. data to determine the effective spreads of on-the-run Treasuries. Fleming and Sarkar give a detailed description of the data.

[27]I thank the referee for this constructive comment.

TABLE 3

Median Bid-Ask Spreads in the U.S. Treasury Spot Market

Sector	Effective Spread
13-week bill	0.02
26-week bill	0.07
52-week bill	0.26
2-year note	0.44
3-year note	0.80
5-year note	1.00
7-year note	2.06
10-year note	1.67
30-year note	2.73

Table 3 reports the median effective spread of on-the-run U.S. Treasuries that Fleming and Sarkar (1999) document for the year 1993. Bid-ask spreads are measured in proportion to the bid-ask midpoint. All spreads are in basis points.

exponentially decaying volatility HJM model and with no transaction costs.[28,29] In this set of simulations, I estimate the volatility parameters of HJM risk measures using four years of forward rate data at each rebalancing period.[30]

For simulation purposes, the continuous-time HJM models are discretized. An Euler scheme provides a satisfactory discretization method for one-factor HJM models.[31,32] For a given discretization of trading interval $[t_0, T]$, $t_0 < t_1 < \ldots < t_N = T$, the forward rate equation for a one-factor HJM model can be discretized using the Euler scheme as

$$\tilde{f}(t_{i+1}, T) = \tilde{f}(t_i, T) + \left(\sigma(t, T) \int_t^T \sigma(t, v) dv \right) (t_{i+1} - t_i) + \sigma(t_i, T) \left(\tilde{W}_{t_{i+1}} - \tilde{W}_{t_i} \right), \quad i = 0, 1, \ldots, N. \tag{14}$$

Here, $\tilde{f}(t_i, T)$ is the value of the approximation of $f(t_i, T)$ at discretization time t_i.[33]

[28]The performance of HJM risk measures that depend on estimated volatility parameters are analyzed when there are no transaction costs. The introduction of transaction costs could alter the results, and it would not be possible to attribute the difference in the results solely to the sampling variability corresponding to the parameter estimation.

[29]The analysis is carried out only with the exponentially decaying volatility HJM model. Since the respective fits to the data of the humped volatility and exponentially decaying volatility HJM models are indistinguishable from one another, the paper focuses mainly on the exponentially decaying volatility HJM model. Furthermore, since the humped volatility HJM model has more parameters, the deterioration in the immunization performance of the corresponding HJM risk measures due to sampling variability should not be less than that of the exponentially decaying volatility HJM risk measures.

[30]The forward rate data used for parameter estimation is composed of real and simulated forward rates for the first 48 months. After that, the data consists of forward rates simulated in the previous time steps.

[31]For multi-factor HJM models, more complex alternatives should be considered for higher order weak convergence. One of these alternatives is the Milstein scheme, discussed in Kloeden and Platen (1992), pp. 345–351, and James and Weber (2000), pp. 362–363.

[32]For one-factor models, the Euler scheme has satisfactory convergence properties (see Duanmu (1994), pp. 83–89, and James and Weber (2000), pp. 362–363).

[33]Since I use monthly data in this study, the time increment $t_{i+1} - t_i$ is taken as one month.

I simulate forward rate evolutions over the holding period with each selected one-factor HJM model.[34,35] Following Duffie and Glynn (1995), I conduct 20,000 simulation trials.[36] I use three one-factor HJM models (constant, exponential decay volatility, and humped volatility), three holding periods (one-, five-, and 10-year), four initial yield curves (flat, increasing, decreasing, and humped), and two immunization strategies (duration and duration convexity matching) to form the portfolios. I analyze the performance of immunization strategies both with and without transaction costs. When no transaction costs are present, exponentially decaying volatility HJM risk measures are also formed using the volatility parameters estimated in the simulation from the forward rate data rather than those used in simulating the forward rates.[37] These simulations reveal the conditions under which the immunization performance of the HJM and traditional risk measures differ.

IV. The Immunization Performance of Interest Rate Risk Measures and Portfolio Formation Strategies

The immunization performance of traditional and HJM risk measures is analyzed under the selected HJM models.[38] First, I discuss the performance of random portfolios. Then, I examine the bullet and barbell portfolios.

A. The Immunization Performance of Random Portfolios

Under a constant volatility HJM model, I use the Fisher-Weil risk measures as benchmarks because the related HJM risk measures reduce to them. The results under a constant volatility HJM model show no substantial difference between the immunization performance of Macaulay and Fisher-Weil risk measures.[39] Therefore, the rest of this section focuses on the performance of risk measures and immunization strategies under an exponentially decaying volatility HJM model.

[34]The random number generator of L'Ecuyer (1999) is implemented for the generation of uniform random numbers since it is more efficient than its similar counterparts. The polar method of Marsaglia and Bray (1964) is used to generate normal random variates from uniform random numbers. To increase the speed of convergence, I use antithetic variates. Variance reduction techniques are discussed in detail in Nelson and Schmeiser (1986), Duffie and Glynn (1995), and Boyle, Broadie, and Glasserman (1997).

[35]If a simulated forward rate is negative, it is dropped and I simulate another forward rate.

[36]Duffie and Glynn (1995) provide an algorithm based on the trade-off between the discretization and the Monte Carlo averaging errors. According to the proposed algorithm, for a given number of time steps N, the optimal number of simulations is N^2 with an Euler scheme. In this paper, an Euler scheme is used for discretization, and monthly forward rates are simulated for up to a 10-year holding period. This results in a maximum time step of 120 for any portfolio simulation. Therefore, the optimal number of simulations is 14,450. I use 20,000 simulations for each portfolio irrespective of the holding period. This criterion satisfies the number of simulations suggested by Duffie and Gylnn (1995).

[37]These combinations lead to a total of 124,848 portfolios.

[38]I report only the results corresponding to the exponentially decaying volatility HJM model. The humped volatility HJM model produces similar results.

[39]These results are available from the author. As expected, when there are no transaction costs, a duration and convexity matching strategy improves the immunization performance of both risk measures. When there are transaction costs, the immunization performance deteriorates regardless of the risk measure and immunization strategy.

Table 4 shows the performance of the Fisher-Weil and HJM risk measures under the exponentially decaying volatility HJM model.[40] Panel A in Table 4 shows the immunization performance of duration matched, and duration and convexity matched, portfolios with transaction costs. Panel B in Table 4 shows the immunization performance of random portfolios when there are no transaction costs and when the HJM risk measures depend on the estimated volatility parameters rather than the volatility parameters used to simulate the forward rate data.[41]

TABLE 4

Performance of Random Portfolios

	Fisher-Weil			HJM		
	One-Year	Five-Year	Ten-Year	One-Year	Five-Year	Ten-Year
Panel A. (with transaction costs)						
Duration Matched Portfolios						
Within 1 basis point	9.00%	17.00%	8.25%	10.50%	19.25%	17.25%
Within 5 basis points	33.50%	41.00%	54.75%	37.25%	46.75%	69.75%
Within 10 basis points	44.50%	55.50%	76.50%	52.25%	62.00%	79.25%
MaxAD	6.57E−03	6.54E−03	5.10E−03	6.49E−03	6.52E−03	5.73E−03
MaxRD	7.41E−02	6.40E−02	4.60E−02	4.39E−02	4.88E−02	5.16E−02
Duration and Convexity Matched Portfolios						
Within 1 basis point	29.00%	12.50%	11.50%	44.00%	27.25%	22.25%
Within 5 basis points	41.50%	24.50%	17.75%	58.00%	40.25%	35.75%
Within 10 basis points	46.75%	37.50%	29.00%	63.00%	54.75%	47.00%
MaxAD	6.31E−03	6.41E−03	5.91E−03	4.79E−03	5.32E−03	4.41E−03
MaxRD	5.72E−02	5.38E−02	5.17E−02	4.21E−02	4.24E−02	3.97E−02
Panel B. (with parameter estimation and no transaction cost)						
Duration Matched Portfolios						
Within 1 basis point	9.50%	22.75%	19.00%	8.25%	18.75%	16.50%
Within 5 basis points	35.75%	45.25%	62.25%	29.70%	40.00%	63.00%
Within 10 basis points	49.00%	62.25%	84.25%	44.25%	58.25%	85.00%
MaxAD	5.27E−03	5.53E−03	3.42E−03	6.35E−03	6.87E−03	3.62E−03
MaxRD	5.66E−02	5.57E−02	4.01E−02	5.83E−02	7.48E−02	4.02E−02
Duration and Convexity Matched Portfolios						
Within 1 basis point	56.50%	38.50%	40.25%	61.75%	51.25%	49.25%
Within 5 basis points	70.75%	65.50%	77.00%	75.75%	65.00%	72.75%
Within 10 basis points	76.00%	78.50%	92.50%	84.75%	80.50%	90.25%
MaxAD	5.78E−03	5.66E−03	3.24E−03	7.12E−03	6.08E−03	3.01E−03
MaxRD	6.20E−02	5.56E−02	3.65E−02	7.47E−02	6.38E−02	3.81E−02

Table 4 summarizes the immunization performance of random portfolios under the exponentially decaying volatility HJM model. The benchmark risk measures are the exponentially decaying volatility HJM risk measures. Panel A presents the immunization performance with transaction costs. In Panel B, the parameters of the HJM risk measures depend on the volatility parameters estimated in the simulation at each rebalancing period rather than the parameters used in simulating the forward rate data. Two immunization strategies are considered: a duration matching strategy, and a duration and convexity matching strategy. With the duration matching strategy, portfolio durations are matched with the duration of the target zero-coupon bond using the HJM and traditional duration measures. With the duration and convexity matching strategy, both duration and convexity of portfolios are matched with those of the target zero-coupon bond by HJM and traditional risk measures. Three holding periods are considered: one-, five-, and 10-year. The target zero-coupon bonds mature at the end of the respective holding period. Immunization performance is assessed as the absolute difference between the holding period returns of portfolios and the yield on the respective target zero-coupon bond. The percentages of portfolios that have holding period returns within one, five, and 10 basis points of the target yield are reported. Also, the maximum absolute deviation (MaxAD) and maximum relative deviation (MaxRD) of portfolio holding period returns from the target yield are reported.

In the exponentially decaying volatility HJM model, the estimated decay factors are negative for the analyzed periods. Thus, forward rate volatilities are

[40] Since Macaulay and Fisher-Weil risk measures yield similar immunization performance, only the results of the Fisher-Weil risk measures are reported here.

[41] The performance of random portfolios without transaction costs and with no parameter estimation are comparable to the results reported in Table 4, Panels A and B. Specifically, the performance of risk measures is comparable to that of with transaction costs are with no parameter estimation that are reported in Table 4, Panel A. The performance of immunization strategies is comparable to that without transaction costs and with parameter estimation reported in Table 4, Panel B.

higher for longer holding periods. Panel A in Table 4 shows that, as expected, there is some deterioration in the immunization performance of all risk measures for five- to 10-year holding periods due to the higher volatilities over longer holding periods. The impact of transaction costs on immunization performance is more prominent over longer holding periods. For short holding periods, a duration and convexity matching strategy provides better immunization than a duration matching strategy, though the difference is not large. For medium and long holding periods, a duration matching strategy produces better immunization performance. Even though HJM risk measures perform better than Fisher-Weil risk measures when used with the same immunization strategy, this result does not hold when they are used with different immunization strategies.[42] Therefore, a risk manager's main concern should be to choose the appropriate immunization strategy. Notably, immunization strategies should also be chosen with respect to the holding period.

The aforementioned results compare traditional risk measures with benchmark HJM risk measures where the latter are based on the exponential decay parameter, λ, used in the simulation of the forward rates. If the HJM risk measures depend on the estimated exponential decay parameter rather than the one used in the simulation, the resultant sampling variability could affect the performance of this risk measure. To examine the impact of the sampling variability on the performance of HJM risk measures, the exponential decay parameter, λ, is estimated using four years of historical forward rate data at each rebalancing period. The HJM risk measures are determined according to the estimated exponential decay parameter. No transaction cost is assumed.

Panel B, Table 4 gives the results and shows that with a duration matching strategy, the Fisher-Weil risk measure provides slightly better immunization than the HJM risk measure for short and medium holding periods. For long holding periods, the immunization performance of HJM and Fisher-Weil risk measures is similar. The duration and convexity matching strategy improves immunization performance of all risk measures when there are no transaction costs. With a duration and convexity matching strategy, HJM risk measures provide slightly better immunization than Fisher-Weil risk measures, though only for short holding periods. For medium and long holding periods, Fisher-Weil and HJM risk measures exhibit similar immunization performance. These results suggest that when one accounts for sampling variability, HJM risk measures are not superior to traditional risk measures. In fact, in most cases the immunization performance of the traditional risk measures is similar to or better than that of the HJM risk measures.

Overall, the evidence suggests that the performance of immunization strategies depends more on the holding period and the transaction costs than the risk measures. When the volatility parameter of the HJM risk measure is estimated, in most cases the traditional risk measures are preferable to the HJM risk measures due to the sampling variability in the HJM risk measures.

[42]For example, according to Panel A in Table 4, duration matching strategy is the appropriate strategy for long holding periods. If one uses a duration matching strategy with the Fisher-Weil risk measure for a 10-year holding period, 76% of the portfolios are within 10 basis points of the target yield. On the other hand, if one uses the benchmark HJM risk measure with a duration and convexity matching strategy for a 10-year holding period, only 47% of the portfolios are within 10 basis points of the target.

B. The Immunization Performance of Bullet and Barbell Portfolios

The performance of bullet and barbell portfolios under the constant volatility HJM model is similar to that under the exponentially decaying volatility HJM models with the Fisher-Weil risk measures.[43] In this respect, the rest of this section examines the performance of bullet and barbell portfolios under the exponentially decaying volatility HJM model.

Table 5 gives evidence concerning the immunization performance of bullet and barbell portfolios under an exponentially decaying volatility HJM model. Panel A of Table 5 shows the results with transaction costs. Panel B of Table 5 shows the results without transaction costs when the HJM risk measures depend on the estimated volatility parameters rather than the volatility parameters used to simulate the data.[44]

Panel A in Table 5 shows that when transaction costs are present for both bullet and barbell portfolios, a duration and convexity matching strategy performs worse than a duration matching strategy over medium and long holding periods. Duration and convexity matched bullet portfolios are preferable for short holding periods and duration matched barbell portfolios are preferable for long holding periods. The immunization performance of Fisher-Weil and HJM risk measures is similar.

As Panel B in Table 5 shows, when there are no transaction costs and when the exponential decay parameter of HJM risk measures is estimated rather than taken as the one used in the simulation of the forward rate data, the immunization performance of the Fisher-Weil risk measure is comparable to that of the HJM risk measure. In fact, the maximum errors corresponding to the Fisher-Weil risk measure are slightly less than those corresponding to the HJM risk measure. When there are no transaction costs, bullet and barbell portfolios exhibit very good immunization performance for short holding periods, though the holding period returns of bullet portfolios are closer to the target yield. For long holding periods, however, barbell portfolios exhibit a better immunization performance. Furthermore, the duration and convexity matching strategy improves immunization performance of both bullet and barbell strategies.

These results support evidence concerning the immunization performance of bullet and barbell portfolios.[45] Bullet portfolios generally exhibit higher initial yields than corresponding barbell portfolios. Barbell portfolios offer higher convexities than bullet portfolios. The higher convexity of barbell portfolios is desirable when there are large shifts in the yield curve. If the yield curve shifts are small, due to the yield advantage, bullet portfolios provide better immunization than barbell portfolios. In this study, I simulate forward rates according to the selected one-factor HJM models that depend on a normal random shock. For

[43]These results are available from the author.

[44]The performance of bullet and barbell portfolios without transaction costs and with no parameter estimation are comparable to the results reported in Table 5, Panels A and B. Specifically, the performance of risk measures is comparable to that with transaction costs and with no parameter estimation that Table 5, Panel A reports. The performance of immunization strategies is comparable to that without transaction costs and with parameter estimation that Table 5, Panel B reports.

[45]Ilmanen (2000) provides a detailed analysis of the respective immunization performance of bullet and barbell portfolios in terms of the yield advantage of the former and the convexity advantage of the latter.

TABLE 5

Performance of Bullet and Barbell Portfolios

	Fisher-Weil			HJM		
	One-Year	Five-Year	Ten-Year	One-Year	Five-Year	Ten-Year
Panel A. (with transaction costs)						
Bullet Portfolios						
Within 1 basis point	4,4	1,0	1,0	4,4	2,1	1,0
Within 5 basis points	4,4	4,2	2,2	4,4	4,2	3,2
Within 10 basis points	4,4	4,4	4,4	4,4	4,4	4,4
MaxAD	2.65E−06	4.96E−04	7.51E−04	1.58E−06	4.96E−04	5.73E−04
	1.11E−06	7.77E−04	8.63E−04	1.09E−06	6.33E−04	6.39E−04
MaxRD	1.86E−05	5.41E−03	6.46E−03	1.78E−05	4.71E−03	6.16E−03
	2.21E−05	7.80E−03	8.04E−03	1.23E−05	8.67E−03	5.56E−03
Barbell Portfolios						
Within 1 basis point	4,4	2,2	0,0	4,4	2,1	1,1
Within 5 basis points	4,4	4,4	3,2	4,4	4,4	3,3
Within 10 basis points	4,4	4,4	4,4	4,4	4,4	4,4
MaxAD	5.82E−05	4.90E−04	5.45E−04	5.93E−06	4.63E−04	4.38E−04
	2.81E−06	4.69E−04	5.93E−04	2.79E−06	4.17E−04	6.92E−04
MaxRD	6.78E−04	5.01E−03	5.95E−03	5.31E−05	3.45E−03	5.04E−03
	1.23E−05	2.07E−03	5.15E−03	1.97E−05	2.38E−04	4.48E−03
Panel B. (with parameter estimation and no transaction cost)						
Bullet Portfolios						
Within 1 basis point	4,4	2,3	1,2	4,4	2,3	1,2
Within 5 basis points	4,4	4,4	3,4	4,4	4,4	3,4
Within 10 basis points	4,4	4,4	4,4	4,4	4,4	4,4
MaxAD	1.35E−07	2.42E−04	5.04E−04	1.78E−07	3.72E−04	6.20E−04
	4.61E−08	1.73E−04	2.88E−04	5.60E−08	2.22E−04	3.80E−04
MaxRD	1.28E−06	3.95E−03	5.17E−03	1.38E−06	5.06E−03	6.21E−03
	3.79E−07	1.89E−03	2.93E−03	4.48E−07	2.50E−03	3.62E−03
Barbell Portfolios						
Within 1 basis point	4,4	2,4	1,3	4,4	3,4	1,3
Within 5 basis points	4,4	4,4	4,4	4,4	4,4	4,4
Within 10 basis points	4,4	4,4	4,4	4,4	4,4	4,4
MaxAD	3.81E−06	1.32E−04	4.74E−04	4.22E−06	2.72E−04	4.84E−04
	8.73E−07	4.24E−05	1.85E−04	9.85E−07	5.92E−05	2.62E−04
MaxRD	3.42E−05	1.66E−03	4.35E−03	3.62E−05	2.18E−03	4.73E−03
	8.94E−07	5.20E−04	1.37E−03	9.10E−06	5.82E−04	2.20E−03

Table 5 summarizes the performance of bullet and barbell portfolios under the exponentially decaying volatility HJM model with the duration matching and duration-convexity matching strategies. With the duration matching strategy, bullet and barbell portfolio durations are matched with the duration of the target zero-coupon bond using the Fisher-Weil and HJM duration measures. With the duration and convexity matching strategy, both the duration and convexity of bullet and barbell portfolios are matched with those of the target zero-coupon bond using Fisher-Weil and HJM risk measures. The immunization performance of bullet and barbell portfolios with transaction costs is reported in Panel A. In Panel B, no transaction costs are assumed and the parameters of the HJM risk measures depend on the volatility parameters estimated in the simulation at each rebalancing period rather than the parameters used in simulating the forward rate data. Two immunization strategies are considered: a duration matching strategy, and a duration and convexity matching strategy. Three holding periods are considered: one-, five-, and 10-year. The respective target zero-coupon bonds mature at the end of the holding period. Immunization performance is assessed as the absolute difference between the holding period returns of portfolios and the yield on the target zero-coupon bond. For each holding period, four bullet and four barbell portfolios are formed corresponding to the four initial yield curve shapes being considered. The number of portfolios with holding period returns within one, five, and 10 basis points of the target yield is reported. The first number in the cells gives the number of portfolios matched with duration, and the second one gives the number of portfolios matched with both duration and convexity. Also, the maximum absolute deviation (MaxAD) and maximum relative deviation (MaxRD) of portfolio holding period returns from the target yield are reported. Again the first reported MaxAD (MaxRD) corresponds to the one with duration matching strategy and the one reported below it is for the duration and convexity matching strategy.

a given probability, the number of large shocks increases with the length of the holding period, since the number of simulated time steps increases.[46] Therefore, it is more likely to observe large yield curve shifts for a 10-year holding period. I expect barbell portfolios to perform better than bullet portfolios for a 10-year holding period. For a one-year holding period, it is less likely to observe large shifts in the yield curve. Hence, the yield advantage of bullet portfolios overcomes the convexity advantage of barbell portfolios for a one-year holding period.

Overall, the evidence suggests that one should decide jointly on the portfolio formation and immunization strategy, given the holding period and transaction costs. Also, the immunization performance of bullet and barbell portfolios is not sensitive to the interest rate risk measure used.

V. Conclusion

The results of this paper suggest that, for immunization purposes, determining and using the correct term structure model and the corresponding interest rate risk measure is not as effective in immunization as using the correct immunization strategy. When there are no transaction costs, a duration and convexity matching strategy exhibits better immunization performance than a duration matching strategy. When HJM risk measures depend on estimated volatility parameters rather than the parameters used in the simulation of the forward rate data, traditional risk measures provide, in most cases, similar or better immunization performance than HJM risk measures.

When transaction costs are present, the immunization performance of a duration and convexity matching strategy deteriorates substantially, especially for long holding periods, since this strategy requires a larger amount of rebalancing than a duration matching strategy. For short holding periods, a duration and convexity matching strategy still provides better immunization. For medium and long holding periods, however, a duration matching strategy immunizes better than a duration and convexity matching strategy.

The immunization performance of bullet and barbell portfolios is not very sensitive to the interest rate risk measures, but is, however, sensitive to immunization strategies and holding periods. Bullet portfolios perform better for short holding periods, and barbell portfolios perform better for long holding periods. When transaction costs are present, duration and convexity matched bullet portfolios immunize more effectively for short holding periods, while duration matched barbell portfolios are more effective for medium and long holding periods.

The evidence I present suggests that immunization strategies affect immunization performance more than risk measures. Immunization and portfolio formation strategies should be chosen jointly according to holding period and transaction costs. Additionally, in most cases, traditional risk measures are preferable to HJM risk measures when the parameters of the latter have to be estimated.

[46] As an example, consider that the upper and lower 2.5% of the distribution gives the large shifts. Then, the probability of observing a large shift is 5%. For a one-year holding period, there are 12 simulated monthly forward rate paths. Therefore, for this holding period, the expected number of large shifts is 0.6; i.e., either zero or one. For a 10-year holding period, there are 120 simulated monthly forward rate paths. Thus, the expected number of large shifts is six.

Since traditional risk measures do not depend on any parameter estimation, they are free of any sampling error that might cause the immunization performance of the HJM risk measures to deteriorate.

Appendix

A. Convexity Measures of One-Factor HJM Models

The basis risk of a zero-coupon bond is the change in the bond price when the short rate changes. As Au and Thurston (1995) show, under a one-factor deterministic volatility HJM framework, the basis risks of a zero-coupon bond $P(t, T)$ and a coupon bond $B(t, T)$ with N coupon payments of C_i at time periods τ_i are

$$\text{(A-1)} \qquad \frac{\partial P(t,T)}{\partial f(t,t)} = -\frac{P(t,T)}{\sigma(t,t)} \int_t^T \sigma(t,v)dv$$

and

$$\text{(A-2)} \qquad \frac{\partial B(t,T)}{\partial f(t,t)} = -\sum_{i=1}^{N} C_i \frac{P(t,\tau_i)}{\sigma(t,t)} \int_t^{\tau_i} \sigma(t,v)dv.$$

The convexity of a coupon bond is the change in the basis risk of the coupon bond when the interest rate changes divided by the bond price. Therefore, the convexity of a coupon bond $B(t, T)$ is

$$\text{(A-3)} \qquad \begin{aligned} \text{Conv} &= \frac{1}{B(t,T)} \frac{\partial^2 B(t,T)}{\partial f(t,t)^2} \\ &= -\frac{1}{B(t,T)} \sum_{i=1}^{N} C_i \frac{1}{\sigma(t,t)} \frac{\partial P(t,\tau_i)}{\partial f(t,t)} \int_t^{\tau_i} \sigma(t,v)dv. \end{aligned}$$

Substituting (A-1) into (A-3) gives the convexity of a coupon bond under a one-factor HJM framework,

$$\text{(A-4)} \qquad \text{Conv} = \frac{1}{B(t,T)} \sum_{i=1}^{N} C_i \frac{P(t,\tau_i)}{\sigma(t,t)^2} \left(\int_t^{\tau_i} \sigma(t,v)dv \right)^2 .$$

For selected volatility functions, equation (A-4) gives the convexity of the coupon bonds. For volatility functions of the form $\sigma(t, T) = \sigma g(t, T, \Omega)$, where $g(.)$ is a function and Ω is a set of non-stochastic parameters, the convexity of a coupon bond is not a function of σ. The volatility functions considered in this paper satisfy this condition, and hence the convexity measures corresponding to these selected models are independent of σ.

B. The Duration and Convexity Measures of a Humped Volatility HJM Model

The duration and convexity of one-factor HJM models are, respectively,

$$\text{(A-5)} \qquad D_{\text{HJM}}(t) = \frac{\sum_{i=1}^{n} C_i \left(\int_t^{\tau_i} \sigma(t,v)dv \right) P(t,\tau_i)}{\sigma(t,t) \sum_{i=1}^{n} C_i P(t,\tau_i)}$$

$$\text{(A-6)} \qquad \text{Conv}_{\text{HJM}}(t) = -\frac{1}{\sum_{i=1}^{n} C_i P(t,\tau_i)} \sum_{i=1}^{n} C_i \frac{\partial P(t,\tau_i)}{\partial f(t,t)} \frac{1}{\sigma(t,t)} \int_t^{\tau_i} \sigma(t,v)dv.$$

The humped volatility function, proposed by Mercurio and Moraleda (2000), is

$$\text{(A-7)} \qquad \sigma(t,T) = \sigma(1+\gamma(T-t))e^{-\lambda(T-t)}.$$

Therefore,

$$\text{(A-8)} \qquad \sigma(t,t) = \sigma.$$

The duration and convexity of a humped volatility HJM model requires the evaluation of the integral,

$$\text{(A-9)} \qquad \int_t^{\tau_i} \sigma(t,v)dv = \int_t^{\tau_i} \sigma(1+\gamma(v-t))e^{-\lambda(v-t)}dv.$$

The evaluation of the integral is

$$\text{(A-10)} \qquad \int_t^{\tau_i} \sigma(1+\gamma(v-t))e^{-\lambda(v-t)}dv =$$

$$\sigma(1-\gamma t)e^{\lambda t} \int_t^{\tau_i} e^{-\lambda v}dv + \sigma\gamma e^{\lambda t} \int_t^{\tau_i} ve^{-\lambda v}dv,$$

$$\text{(A-11)} \qquad \int_t^{\tau_i} e^{-\lambda v}dv = \frac{1}{\lambda}\left(e^{-\lambda t} - e^{-\lambda \tau_i}\right),$$

$$\text{(A-12)} \qquad \int_t^{\tau_i} ve^{-\lambda v}dv = \frac{e^{-\lambda t}}{\lambda}\left(\frac{1}{\lambda}+t\right) - \frac{e^{-\lambda \tau_i}}{\lambda}\left(\frac{1}{\lambda}+\tau_i\right).$$

Substituting (A-11) and (A-12) into (A-10) leads to

$$\text{(A-13)} \qquad \int_t^{\tau_i} \sigma(1+\gamma(v-t))e^{-\lambda(v-t)}dv =$$

$$\frac{\sigma\gamma}{\lambda}\left[\left(\frac{1}{\gamma}+\frac{1}{\lambda}\right)\left(1-e^{-\lambda(\tau_i-t)}\right) - (\tau_i-t)e^{-\lambda(\tau_i-t)}\right].$$

The duration and convexity measures of a humped volatility HJM model are obtained by substituting (A-8) and (A-13) into equations (A-5) and (A-6), respectively,

$$\text{(A-14)} \qquad D_{\text{HJM}}(t) = \frac{1}{\sum_{i=1}^{n} C_i P(t, \tau_i)} \sum_{i=1}^{n} C_i P(t, \tau_i) \times \frac{\gamma}{\lambda} \left[\left(\frac{1}{\gamma} + \frac{1}{\lambda} \right) \left(1 - e^{-\lambda(\tau_i - t)} \right) - (\tau_i - t) e^{-\lambda(\tau_i - t)} \right]$$

and

$$\text{(A-15)} \qquad \text{Conv}_{\text{HJM}}(t) = \frac{1}{\sum_{i=1}^{n} C_i P(t, \tau_i)} \sum_{i=1}^{n} C_i P(t, \tau_i) \times \frac{\gamma^2}{\lambda^2} \left[\left(\frac{1}{\gamma} + \frac{1}{\lambda} \right) \left(1 - e^{-\lambda(\tau_i - t)} \right) - (\tau_i - t) e^{-\lambda(\tau_i - t)} \right]^2 .$$

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