

Can CDO Equity Be Short on Correlation?*

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Abstract

By examining the impact of an increase in correlation among underlying assets on the value of a collateralized debt obligation (CDO) equity tranche, we show that, contrary to general perception, CDO equity can be *short on correlation*. Specifically, when the underlying reference portfolio comprises high quality assets (assets with low probability of default) or diverse assets (assets with low correlations), the upfront price of a CDO equity tranche can increase with correlation. The implication of our findings is that not all senior and equity tranche trade combinations provide effective correlation hedging. In fact, in some cases, this type of ‘hedge’ might actually increase the correlation risk.

JEL Classification: G11, G12, G13

Keywords: Credit risk, Collateralized debt obligations, Monte-Carlo simulation, Gaussian copula model

1. Introduction

Collateralized debt obligations (CDO) became very popular investment vehicles in recent years until the financial crisis that started in summer 2007. As evidenced by the recent crisis, there was a failure by market participants to understand the complex relationships between various risk drivers such as correlations and their impact on the credit worthiness of different tranches across a CDO capital structure. As a step in the direction of understanding these risks, we examine the impact of correlation on the value of a CDO equity tranche.

CDO equity tranches became particularly popular in the marketplace in an era of tight credit spreads as they offered reasonably high returns. Since equity tranches are the riskiest first loss tranches, they earn high spreads or upfront payments to compensate for the higher default risk. For instance, for the CDX North America Investment Grade and iTraxx Europe Investment Grade index tranches, the equity tranche is denoted as a 0-3 tranche¹ and is responsible for covering the first three percent of losses in the underlying index that typically comprises an equally weighted portfolio of 125 reference obligations.

Equity tranches are perceived to be *long on correlation*, i.e. an increase in correlation across underlying credits is supposed to reduce the risk to the equity tranche investor. For example, Bond Market Association's Synthetic CDO Primer [2005] states that the seller of equity tranche protection wants correlation to increase, and equity tranche spreads compress as correlation rises.² The common explanation for this is that any increase in correlation increases the likelihood of extreme events in the underlying portfolio's loss distribution. Since losses on equity tranches are capped at the detachment point of the tranche, an increase in the likelihood of positive extreme events

(corresponding to zero or very few defaults) should be beneficial to an equity tranche i.e. the value of equity tranche should increase, while a corresponding increase in the likelihood of negative events (corresponding to a very high number of joint defaults) should not hurt it since losses are capped. (See Hull and White [2004] for a detailed discussion).

In this paper, we show that when correlations across assets increase, a CDO equity tranche does not necessarily become less risky. On the contrary, the value of an equity tranche may decrease (i.e. the fair spread or upfront price can increase) as correlations increase. The reason is that even though an increase in correlations across assets does not increase the physical likelihood of default for the underlying reference obligations, it increases their systematic risk and thereby their risk neutral probability of default if increases in systematic risk get priced in the market. In certain cases, this increase in risk neutral default probabilities can outweigh the beneficial impact of high correlation on the equity tranche, thereby reducing its value. Moreover, correlations typically increase in periods of market declines along with a concurrent increase in the market price of risk during market collapses. This further exacerbates the negative effect of increased correlation on the value of CDO equity.³ Thus, contrary to general belief in the market, CDO equity is likely not to be long on correlation in the real world.

To examine how CDO equity value is affected by an increase in correlation across assets, we conduct a Monte-Carlo simulation analysis. The motivation for using a Monte-Carlo analysis is that it allows us to control for the impact of various risk factors that drive CDO equity values and thus look at the effect of correlations in isolation⁴. We do however provide empirical support for our results as well.

Our model is the industry standard one-factor Gaussian copula model of CDO pricing introduced first by Li [2000].⁵ We consider a reference portfolio of 125 credit default swaps (similar to the CDX.NA.IG index), each with a tenor of 5 years. We explore the pricing of the 0-3 tranche (equity tranche) on this portfolio.⁶ Following industry practice, we assume a certain initial identical correlation across all assets and obtain the risk neutral probability of default from the parameters of the model. Then, using the Merton [1974] model, we transform risk neutral default probabilities to physical default probabilities. Keeping the physical default probabilities constant, we increase the correlation across all assets. Since correlation is directly tied to the systematic risk of an asset in our model, this leads to an increase in the risk neutral probability of default even though there is no change in the physical default probability. We then examine the impact of an increase in correlation on the value of standardized CDO equity tranches.

Our results suggest that when correlation across assets increases, the detrimental effect of an increase in risk neutral default probabilities on the value of a CDO equity tranche can very often be substantial. Specifically, when underlying assets are of high quality or when the initial correlation across assets is low i.e. when the portfolio is well diversified, CDO equity tranche value decreases when correlations rise. Thus CDO equity can be *short on correlation*.

Since the general perception in credit markets is that CDO equity is always long on correlation while more senior tranches are not, equity tranches are used not only for investment purposes but also in combination with more senior tranches for hedging against correlation movements. The implication of our findings is that not all equity

tranche trades provide effective correlation hedging. In fact, in some cases, the ‘hedge’ might actually increase the correlation risk.

The rest of the paper is organized as follows: Section 2 explains the underlying model and the simulation methodology. Section 3 presents and discusses the impact of an increase in correlation across assets on CDO equity tranche value . Section 4 concludes.

2. Model and Simulation Methodology

2.1. Model

The industry standard default time model that is used to price CDOs captures default dependency through a one-factor Gaussian copula function. The objective of the standard default time model is to determine the timing of losses on a collateral portfolio by taking into account the default probabilities of underlying assets, their correlations and assumptions about recovery in the instance of default. The one-factor Gaussian copula model introduced by Li [2000] became popular in the industry since it is easy to implement and computationally efficient.

Assume that there are n firms or assets. Let t_i be the default time of i^{th} firm and $P_i(t)$ be the cumulative probability of firm i defaulting before time t , i.e. probability that $t_i < t$.⁷ Define the asset return for firm i , X_i , $i = 1, 2, \dots, n$, as follows:

$$X_i = a_i S + \sqrt{1 - a_i^2} Z_i, \quad (1)$$

Here, S (systematic factor) and Z_i (idiosyncratic factor) are independent and have standard normal distributions. Note that all firms are subjected to the same systematic factor S and thus by construction, the correlation between X_i and X_j is $a_i a_j$ and X_i has a standard normal distribution. The normal variates X_i can be transformed to uniform

variates U_i such that $U_i = \Phi(X_i)$, where Φ is the standard cumulative normal distribution function. In this model, if U_i is greater than $P_i(t)$, then firm i does not default by time t . If $P_i(t_1) < U_i < P_i(t_2)$, then firm i defaults between time t_1 and t_2 .

The simplifying assumptions of the industry standard one-factor Gaussian copula model described above can be summarized as follows: (i) systematic and idiosyncratic factors have standard normal distributions; (ii) all assets have the same pair-wise correlation, thus $a_i = a_j = a$, for all i, j ($a \geq 0$); (iii) all assets have a fixed recovery rate; (iv) all assets have the same spread, s , where this spread is equal to the average spread on the portfolio (or the index spread in the case of an index); and (v) the default intensity, λ_i , is the spread⁸ per unit of LGD (loss given default, i.e. $(1 - Recovery)$)⁹. Accordingly, the default intensity, λ_i , and risk neutral probability of default of firm i , P_i^Q , are as follows:

$$\lambda_i = \lambda = s / LGD, \quad \text{for all } i = 1, 2, \dots, n \quad (2)$$

$$P_i^Q = P^Q = 1 - \exp(-\lambda t), \quad \text{for all } i = 1, 2, \dots, n \quad (3)$$

Since s and LGD are assumed to be the same for all assets, default intensity and risk neutral probability of default is identical for all assets in the portfolio.

In this paper, our objective is to examine how an increase in correlation across assets affects the value of CDO equity tranches even when physical default probabilities do not change. Therefore, we first transform risk neutral default probabilities derived from spreads to physical or ‘real-world’ default probabilities using the results of the Merton [1974] model¹⁰. In Merton’s model, the value of a firm, V , follows a geometric Brownian motion with mean μ and volatility σ , where $W(t)$ is a Weiner process.

$$dV/V = \mu dt + \sigma dW(t) \quad (4)$$

Assume that D is the outstanding debt of the firm. The firm defaults if $V < D$. Accordingly, assuming r is the risk-free rate, the physical and risk neutral cumulative probabilities of default between time 0 and t , P_t and P_t^Q , respectively, are as follows:

$$P_t = \Phi\left(\frac{-\ln(V/D) + (\mu - 0.5\sigma^2)t}{\sigma\sqrt{t}}\right) \quad (5)$$

$$P_t^Q = \Phi\left(\frac{-\ln(V/D) + (r - 0.5\sigma^2)t}{\sigma\sqrt{t}}\right) \quad (6)$$

Using the above equations, we can express physical and risk neutral default probabilities as follows:

$$P_t = \Phi\left(\Phi^{-1}(P_t^Q) - \left(\frac{\mu - r}{\sigma}\right)\sqrt{t}\right) \quad (7)$$

$$P_t^Q = \Phi\left(\Phi^{-1}(P_t) + \left(\frac{\mu - r}{\sigma}\right)\sqrt{t}\right) \quad (8)$$

We next consider a CAPM (Sharpe [1964], Lintner [1965]) type model where the excess return on asset i is a linear function of the systematic (or market) factor S^{II} . Since the correlation of asset i with S is a in the above model, we have:

$$\frac{\mu - r}{\sigma} = a\Lambda \quad (9)$$

where Λ is the market price of risk defined as the standardized excess market return. Using equations (7), (8) and (9), we can thus transform risk neutral and physical default probabilities to each other as follows:

$$P_t = \Phi\left(\Phi^{-1}(P_t^Q) - a\Lambda\sqrt{t}\right) \quad (10)$$

$$P_t^Q = \Phi\left(\Phi^{-1}(P_t) + a\Lambda\sqrt{t}\right) \quad (11)$$

As evident from equation (11), the risk neutral probability of default depends not only on the physical probability of default but also on the asset correlation. This is because the systematic component of an asset's return is tied to its correlation. As long as systematic risk is priced in the market i.e. the market price of risk is positive, an increase in correlation should lead to a corresponding increase in the asset's spread as compensation for bearing the additional systematic risk, which in turn leads to an increase in the asset's risk neutral default probability. This characteristic is detrimental to the value of the CDO equity but typically ignored in standard CDO analysis. Hence it is important to examine to what extent this may impact CDO pricing.

To accomplish this objective, we first determine the risk neutral default probability term structures, P_t^Q from a given (market observed) index spread as per equations (2) and (3), in conjunction with certain LGD assumptions. We then convert these to physical default probabilities P_t using equation (10). Keeping the physical probabilities constant, we increase correlations by increasing a i.e. the loading on the systematic latent factor. Accordingly, we find 'new' corresponding risk neutral probabilities from equation (11) and estimate new CDO equity tranche prices. This allows us to examine how an increase in correlation affects CDO equity tranche values even though physical default probabilities remain unchanged. Furthermore, we explore whether the increase in risk neutral default probabilities through a rise in correlation can be detrimental to the CDO equity tranche values. Our valuation technique in this paper employs Monte-Carlo simulations, which is described in the next subsection.

2.2. Methodology

Our reference portfolio consists of 125 names, as in the CDX.NA.IG index. We consider a tranching structure similar to the CDX.NA.IG index such that the first-loss equity tranche is a 0-3 tranche i.e. responsible for covering losses from 0 percent to 3 percent in the reference portfolio. For pricing purposes, we model the equity tranche as earning a running spread of five percent and trading on an upfront fee basis¹² – once again to capture the characteristics of the actual CDX.NA.IG 0-3 tranche that trades on similar terms. Using the model discussed above, we generate default distributions for different levels of correlation and spread. We consider initial correlations from 0 to 50 percent in increments of 5 percent, and initial index spreads of 25, 50, 100, 250 and 500 basis points. We also use actual CDS spreads on September 22, 2005.¹³ We assume a fixed recovery rate on defaulted names of 50 percent and consider the LIBOR term structure as on September 22, 2005 as the risk-free term structure for discounting the cash flows of the tranches. Accordingly, we obtain model upfront prices of the equity tranche that are reported in Table 1.

We next transform risk neutral probabilities of the underlying obligations in the reference portfolio to physical probabilities using equation (10). We consider market price of risk as 20 percent, 40 percent and 60 percent¹⁴. Keeping the physical default probabilities constant, we increase asset correlations by 1 and 5 percentage points. We then calculate the corresponding risk neutral probabilities using equation (11) and use these ‘new’ risk-neutral probabilities for valuing CDO equity tranches. We explore the impact of an increase in correlation across assets on equity tranche values as a function of certain portfolio and market characteristics, i.e. as a function of the initial asset correlations and spreads, as well as the market price of risk. As a result, we examine 198

cases (11 initial correlations x 6 initial spread levels x 3 market price of risk = 198). We run 200,000 simulation trials to value each equity tranche.

3. CDO Equity Tranches: Can They Be Short on Correlation?

We have highlighted earlier the general perception in structured credit markets that CDO equity is always long on correlation. An increase in the correlation across assets is considered beneficial for CDO equity since its losses are capped whereas the upside potential increases when correlation rises. As an example, consider a physical default distribution for a portfolio of 125 credits over a 5 year horizon assuming that annualized physical default probability of each credit is 2 percent. Figure 1 shows this distribution when homogenous pair-wise correlations increase from 0 percent to 30 percent in 10 percent increments. As evident from the figure, when correlations increase, the thickness in the positive and negative tails of the distribution increases. The greater likelihood of zero or very few defaults with higher correlation, holding everything else constant, makes correlation beneficial for the first loss tranche investor i.e. makes CDO equity long on correlation. However, we have made the case that as correlation across assets increases, so does their systematic risk and hence their corresponding risk neutral probability of default, even though physical probability of default may not change. Therefore, it is important to examine how an increase in asset correlations affects CDO equity tranches by also considering the impact of the increased correlations on risk neutral default probabilities.

We first obtain the results of the base model, where the default probabilities are determined using the parameters mentioned in the previous section. The equity tranche

values for the base model are given in Table 1. In Table 1, the 0-3 equity tranche prices are quoted as upfront fee for different levels of correlation and index spreads. The 125 credits comprising the index are considered to be homogenous. They have the same pairwise correlation and CDS spread is equal to the index spread. One can observe that for a given index spread, an increase in correlation across assets is beneficial to the CDO equity tranche investor as the price of the CDO equity tranche declines, reflecting a lower risk or lower loss expectation. However, this interpretation assumes that increasing the correlation across assets has no impact on individual assets' systematic risk or their spread.

We therefore explore how an increase in asset correlations affect CDO equity values by also considering the impact of correlations on risk neutral default probabilities. It is also important to note that correlations generally increase when markets decline and are usually associated with an increase in the market price of risk during stressed times, as evidenced in the recent financial crisis.¹⁵ Thus, while we consider three scenarios corresponding to 20, 40 and 60 percent as the market prices of risk in our simulations, the more 'representative' real-world scenario is the one where correlations increase when market price of risk is high i.e. 60 percent.

We re-estimate model prices for the 0-3 equity tranche for a 1 percent and 5 percent increases in correlation and examine the impact of the increase in correlation on the value of CDO equity tranches. The underlying assumption is that the 'real-world' or physical probabilities of default between the case in Table 1 and current cases do not change. Increasing the correlation across assets, however, increases the systematic risk of the underlying assets in our model as in equation (11). This implies that even though

physical default probabilities do not change, risk neutral probabilities increase as long as there exists a certain (greater than zero) market price of risk.

Table 2 Panel A shows the results when the market price of risk is assumed as constant at 40 percent. The table displays equity tranche prices for a given initial index spread and correlation as well as prices when correlation across assets increases by 1 percent and 5 percent. As observed in the table, when the initial index spread is low i.e. the index constituents are high quality names, an increase in correlation always leads to an increase in equity tranche price – highlighted by bold fonts in the table. In these cases, the detrimental effect of an increase in risk neutral default probability due to a rise in systematic risk outweighs the beneficial impact of an increase in the real-world likelihood of having few or no defaults. Thus CDO equity is *short on correlation*. On the other hand, when the initial index spread is high i.e. the index constitutes high yield names, an increase in correlation has a beneficial impact on equity tranche values as long as the initial correlation level is not too low. These cases are consistent with the general perception that CDO equity is long in correlation.

Note that when the initial index spread is low implying that the constituents are high grade names and the physical default probability P_t is small, the term $a\Lambda\sqrt{t}$ in equation (11) gets a higher relative weight. Therefore, the marginal impact of an increase in correlation across assets on the risk neutral default probability is large. In this case, the negative impact of an increase in risk neutral default probability due to an increase in correlation overcomes the positive impact of an increase in the probability of zero or small number of defaults. As a result CDO equity becomes short on correlation. When the initial index spread is high implying physical default probability itself is high, the

term $a\Lambda\sqrt{t}$ loses its relative impact on risk neutral default probabilities in equation (11). In such cases, an increase in correlation across assets is beneficial for CDO equity tranche value making it long in correlation. Also as shown in Figure 2, for a given physical probability of default, increasing correlation leads to an increase in the risk neutral probability of default. One can also observe that for a given level of correlation, the slope of the risk neutral default probability decreases as physical default probability increases. Hence the ratio of the risk neutral to physical default probability is higher for lower values of physical default probability but eventually converges to one as the physical default probability increases to 100 percent.

As the market price of risk decreases, risk-neutral default probabilities and physical default probabilities converge to each other. We therefore estimate CDO equity fair-prices with a reduced market price of risk to explore whether even in the presence of less risk-averse sentiment in the market, CDO equity can still be short on correlation. Table 2 Panel B shows the values of CDO equity tranches when market price of risk is 20 percent and when underlying asset correlations increase by 1 percent and 5 percent. Supporting our previous findings, we observe that when the initial portfolio has low average spread, i.e. high credit quality, an increase in underlying asset correlations has a detrimental effect on the value of an equity tranche when the initial underlying correlations are not very high. In fact, even for poor quality or high spread portfolios, when the initial correlation levels are low i.e. when the initial portfolio is diversified, an increase in correlation makes the equity tranche more risky. Our previous finding that the equity tranche can be short on correlation either when the underlying CDO reference pool is of high quality or when it comprises assets with low correlations appears to hold even

in cases where the market price of risk is less than historically observed levels of 40 to 50 percent.

We next investigate the impact of a change in the market price of risk in the other direction. As market price of risk increases, risk-neutral default probabilities and physical default probabilities diverge even more than in the 40 percent case. One can therefore anticipate more instances where CDO equity is short on correlation. Table 2 Panel C shows the values of CDO equity tranches when the market price of risk is 60 percent and the underlying asset correlations increase by 1 percent and 5 percent. We observe more cases where CDO equity tranche is short on correlation either when the underlying CDO reference pool is of high quality or comprises assets with low initial correlations. Thus when market price of risk is at more elevated levels than observed historically, CDO equity can be short on correlation more often. Note that in our simulations we control for the level of the physical probability of default. In real life, however, increased correlations are typically observed during market declines which also correspond to higher defaults, credit quality deteriorations as evidenced by ratings downgrades and a general increase in default expectations in the market. This observation suggests that the adverse affect of correlation on CDO equity tranches is likely to be more severe than that is observed by simulation.

To provide some empirical support for our findings, we first use actual CDS spreads observed for the 125-name basket of constituents that comprise the CDX.NA.IG.5 index to examine the impact of an increase in asset correlations on the value of the 0-3 equity tranche. Our CDS data are from Markit and are as of the September, 22, 2005. Table 3 shows the 'fair' prices for the 0-3 equity tranche as well as

the prices for this tranche when underlying asset correlations are increased by 1 percent and 5 percent, respectively. Once again we consider the market price of risk as 20 percent, 40 percent, and 60 percent. For the investment grade universe, the results seem unequivocal: CDO equity is short on correlation under historically observed market price of risk levels between 40 percent and 60 percent. Even in more benign conditions – corresponding to a market price of risk of 20 percent in our analysis, CDO equity can become more risky if the underlying reference obligations come from a diverse pool i.e. have low initial correlations. In these instances, any increase in correlation leads to a relatively large change in the systematic default risk of these underlying reference entities. As a result, CDO equity tranches can be *short on correlation*.

We next look at the performance of CDO equity tranches during the recent financial crisis when correlations increased substantially.¹⁶ Specifically, we look at the CDX.NA.IG.8 0-3 tranche for which we have data available¹⁷ from March, 20 2007 (when series 8 came into existence) up to September 30, 2008, a period that includes the ‘peak’ of the crisis corresponding to the bankruptcy filing by Lehman Brothers on Sep 15, 2008. To examine whether we indeed observe CDO equity being short on correlations, we analyze the correlations implied by the 0-3 equity tranche prices vis-à-vis the 0-3 tranche price itself.

While we acknowledge that the implied correlation for an equity tranche as measured from its spread or price may not be a very good measure of the ‘true’ underlying asset return correlations, it is nonetheless a common measure of correlation used in the marketplace. Moreover, implied correlations are highly correlated with ‘true’ correlations in the sense that an increase (decrease) in ‘true’ correlations results in a

higher (lower) model implied correlation¹⁸. In this respect, we examine the behaviour of tranche correlations and the corresponding CDO equity prices during the recent market downturn where both systematic risk and market price of risk rose to high levels.

Figure 3 shows the tranche correlation versus the upfront price for the 0-3 equity tranche for our sample period. As expected, tranche correlations increase as the financial crisis unfolds and the level of systematic risk and risk aversion increases in the market. However, contrary to the expectation of CDO equity becoming less risky as correlations increase, the upfront price of the 0-3 tranche moves in tandem with the correlation measure. In other words, higher correlations demand higher premiums from the sellers of protection on the 0-3 equity tranche suggesting that CDO equity was *short on correlation* during the financial crisis.

It is important to note that since historically observed increases in correlation occurred mainly under stressed market conditions, rising correlations almost always lead to higher instances of joint defaults but rarely translate to higher probability of upside events. This is because as markets start moving up, correlations decline as idiosyncratic risk becomes the predominant form of credit risk in the market. Thus the higher possibility of few or no defaults in a credit portfolio model due to increasing correlations, which is the basis of the perception of CDO being long on correlation, is likely not to be realized in practice. In other words, correlation is endogenously related to market movements and to increase correlations assuming *ceteris paribus* conditions leads to the misperception that the upside potential benefiting CDO equity increases making the tranche long on correlation.²⁰

4. Conclusion

The recent financial crisis has brought to the fore the importance of understanding the risks behind structured finance products and especially CDOs. There is a general perception among participants in structured credit markets that correlation is beneficial for CDO equity tranche investors – a notion which is referred to as CDO equity being long on correlation. This view is based on the argument that an increase in correlation among the underlying reference assets of a CDO leads to a higher incidence of zero or few defaults in the collateral pool that increases the value of an equity tranche in a CDO structure. In this paper, we show that, contrary to the general market perception, CDO equity can in fact be *short on correlation*.

Changes in correlation not only affect the physical default distribution of a CDO pool of assets but also the risk neutral default probabilities of the underlying assets since correlation is tied to the systematic risk of a credit obligation. For a CDO equity tranche, the impact of these two factors on the tranche value is in opposite directions. While higher likelihood of zero or no physical defaults benefits it, higher risk neutral default probabilities due to an increase in systematic risk increases its risk. Using a simulation study with standard one-factor Gaussian copula model and a structure similar to the CDX.NA.IG index tranches, we show that CDO equity price can actually increase with correlation i.e. CDO equity can be short on correlation. Furthermore, we support this finding with empirical evidence from the recent financial crisis.

We find that CDOs that hold high quality investment grade names in their collateral portfolio are more likely to have equity tranches that are short on correlation. For poorer quality collateral pools comprising high yield or sub-investment grade assets,

CDO equity tranches could benefit from an increase in correlations among the underlying assets, unless the initial correlation across assets are very low. The notion of equity tranche being long on correlation could thus be valid for such types of CDOs.

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Table 1
CDO Equity Prices for the Base Model

This table reports CDO equity tranche (0-3 tranche) upfront prices obtained from a one-factor Gaussian copula model for different initial correlations and index spreads. Running spread is assumed as 5 percent. The reference portfolio consists of 125 credits. Recovery rate is assumed as 50 percent. The index spreads are given in the first column and reported in basis points. The initial correlations vary between 0 to 50 percent with 5 percent increments. The CDO equity upfront prices are given in points.

Spread (bps)	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
25	19.05	18.98	18.93	18.66	18.36	17.45	16.55	15.33	13.77	12.01	10.07
50	54.53	54.09	53.18	51.49	49.43	46.77	44.00	41.01	37.75	34.31	30.67
100	83.86	83.48	82.54	80.79	78.53	75.49	72.19	68.43	64.42	59.92	55.27
250	93.93	93.86	93.7	93.38	92.94	92.23	91.25	89.84	87.9	85.41	82.36
500	96.99	96.95	96.87	96.71	96.49	96.18	95.78	95.27	94.58	93.67	92.42

Table 2
CDO Equity Prices When Correlations Across Assets Increase

This table reports CDO equity upfront prices obtained when correlations increase by 1 and 5 percent without any change in the physical probability of default. Running spread is assumed as 5 percent. Base model prices for comparison are given in Table 1. Cases where equity becomes more risky when correlation increases are in bold. The reference portfolio consists of 125 credits. Recovery rate is assumed as 50 percent. The index spreads are given in the first column and reported in basis points. The initial correlations vary between 0 to 50 percent with 5 percent increments. The CDO equity upfront prices are given in points. Panel A, B and C reports the upfront prices of CDO equity values when market price of risk is considered as 40%, 20%, and 60%, respectively.

Panel A: Market Price of Risk = 40%											
	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Spread = 25 bps											
+1% Correlation	19.91	19.82	19.74	19.42	19.03	18.02	17.02	15.71	14.05	12.21	10.2
+5% Correlation	23.49	23.23	22.95	22.28	21.52	20.1	18.72	17	14.99	12.82	10.49
Spread = 50 bps											
+1% Correlation	55.61	54.99	53.92	52.07	49.87	47.09	44.23	41.15	37.82	34.29	30.59
+5% Correlation	59.49	58.1	56.4	53.95	51.27	48.07	44.86	41.44	37.81	33.99	30.03
Spread = 100 bps											
+1% Correlation	84.15	83.77	82.63	80.9	78.39	75.42	72.08	68.22	64.08	59.62	54.98
+5% Correlation	85.26	84.36	82.98	80.79	78.16	74.83	71.27	67.27	63.02	58.25	53.35
Spread = 250 bps											
+1% Correlation	94.01	93.92	93.73	93.4	92.92	92.18	91.14	89.68	87.66	85.1	81.97
+5% Correlation	94.28	94.1	93.83	93.4	92.8	91.88	90.64	88.91	86.6	83.73	80.28
Spread = 500 bps											
+1% Correlation	97.02	96.97	96.88	96.71	96.48	96.15	95.74	95.2	94.49	93.54	92.23
+5% Correlation	97.12	97.03	96.89	96.68	96.39	96.01	95.53	94.9	94.04	92.91	91.36

Panel B: Market Price of Risk = 20%

	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Spread = 25 bps											
+1% Correlation	19.48	19.39	19.32	19	18.62	17.64	16.67	15.38	13.75	11.93	9.93
+5% Correlation	21.24	21.01	20.80	20.22	19.55	18.25	16.99	15.4	13.50	11.44	9.21
Spread = 50 bps											
+1% Correlation	55.07	54.46	53.41	51.58	49.41	46.66	43.82	40.76	37.45	33.95	30.27
+5% Correlation	56.88	55.59	53.98	51.65	49.10	46.02	42.92	39.61	36.08	32.38	28.53
Spread = 100 bps											
+1% Correlation	84.03	83.52	82.47	80.60	78.23	75.12	71.76	67.93	63.87	59.32	54.62
+5% Correlation	84.44	83.46	81.96	79.65	76.91	73.51	69.87	65.85	61.56	56.80	51.92
Spread = 250 bps											
+1% Correlation	93.97	93.87	93.69	93.35	92.87	92.12	91.07	89.59	87.55	84.97	81.83
+5% Correlation	94.08	93.89	93.62	93.17	92.54	91.57	90.25	88.44	86.03	83.05	79.50
Spread = 500 bps											
+1% Correlation	97.00	96.95	96.86	96.69	96.46	96.13	95.72	95.18	94.45	93.49	92.17
+5% Correlation	97.04	96.95	96.81	96.58	96.29	95.89	95.40	94.75	93.86	92.67	91.06

Panel C: Market Price of Risk = 60%

	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Spread = 25 bps											
+1% Correlation	20.35	20.24	20.17	19.84	19.42	18.39	17.38	16.04	14.36	12.5	10.47
+5% Correlation	25.81	25.52	25.17	24.41	23.54	22.00	20.49	18.63	16.51	14.24	11.79
Spread = 50 bps											
+1% Correlation	56.15	55.52	54.42	52.56	50.32	47.52	44.64	41.53	38.18	34.63	30.9
+5% Correlation	62.02	60.55	58.75	56.19	53.39	50.08	46.78	43.26	39.55	35.6	31.55
Spread = 100 bps											
+1% Correlation	84.38	83.90	82.87	81.05	78.73	75.66	72.32	68.51	64.46	59.91	55.21
+5% Correlation	85.99	85.19	83.91	81.85	79.33	76.10	72.60	68.63	64.42	59.67	54.75
Spread = 250 bps											
+1% Correlation	94.05	93.96	93.78	93.44	92.97	92.24	91.21	89.77	87.77	85.22	82.12
+5% Correlation	94.47	94.29	94.04	93.62	93.05	92.18	91.00	89.36	87.15	84.38	81.03
Spread = 500 bps											
+1% Correlation	97.04	96.99	96.9	96.73	96.5	96.17	95.76	95.23	94.52	93.58	92.28
+5% Correlation	97.20	97.11	96.98	96.77	96.49	96.12	95.66	95.05	94.22	93.13	91.65

Table 3
CDO Equity Prices Using Actual Credit Default Swap Spreads

This table reports the CDO equity tranche (0-3 tranche) upfront prices when correlation is increased by 1 percent and 5 percent and actual credit default swap (CDS) spreads are used. Running spread is assumed as 5 percent. The first row gives the base model prices of CDO equity tranches. The CDS data come from Mark-It and are the closing prices for the CDX.NA.IG Series 5 constituents on September 22, 2005. Cases where equity becomes more risky when correlation increases are in bold. The reference portfolio consists of 125 credits. Recovery rate is assumed as 50 percent. The initial correlations vary between 0 to 50 percent with 5 percent increments. The CDO equity upfront prices are in points.

	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Base	53.02	52.68	51.87	50.32	48.63	46.19	43.72	40.99	38.08	34.89	31.56
Market price of risk = 20%											
+1% Correlation	53.53	53.05	52.13	50.45	48.66	46.13	43.59	40.78	37.83	34.58	31.21
+5% Correlation	55.31	54.28	52.82	50.68	48.51	45.66	42.88	39.82	36.64	33.23	29.68
Market price of risk = 40 %											
+1% Correlation	54.06	53.56	52.62	50.92	49.1	46.55	43.98	41.16	38.18	34.91	31.52
+5% Correlation	57.86	56.72	55.16	52.91	50.6	47.66	44.75	41.6	38.34	34.8	31.17
Market price of risk = 60%											
+1% Correlation	54.89	54.07	53.11	51.39	49.54	46.97	44.39	41.54	38.53	35.23	31.83
+5% Correlation	60.32	59.09	57.45	55.09	52.67	49.63	46.62	43.38	40.04	36.38	32.66

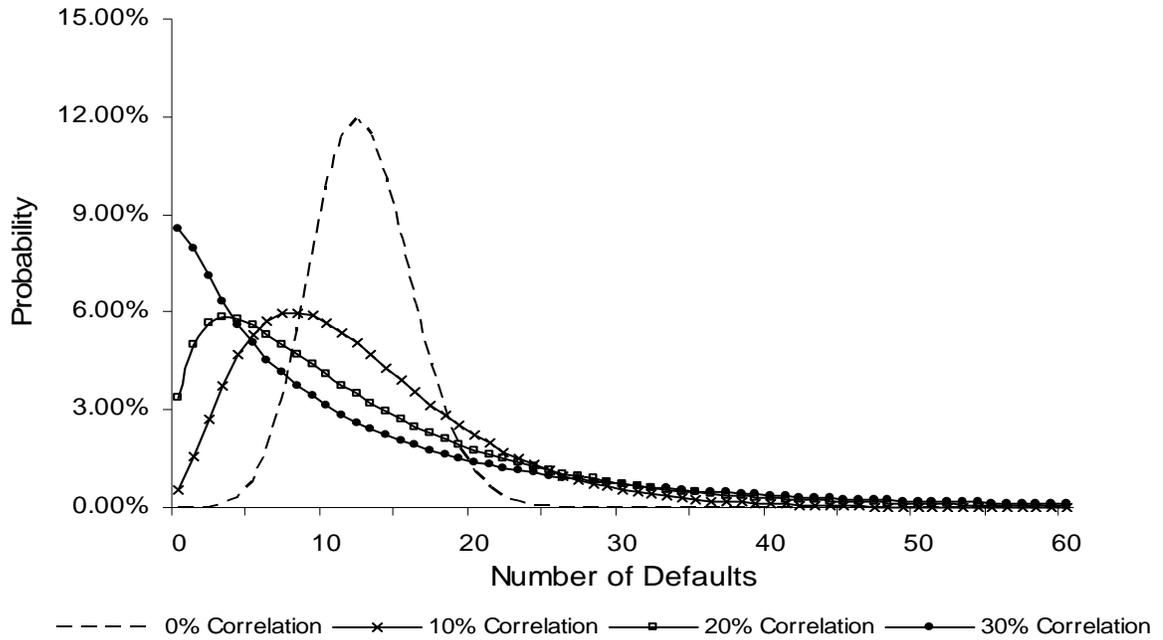


Figure 1: The Impact of Correlation on the Physical Default Distribution

This figure shows the physical default distribution for a portfolio of 125 credits as correlation across credits increases from 0 percent to 30 percent with 10 percent increments. All credits have a constant annualized physical default probability of two percent and the default horizon is five years.

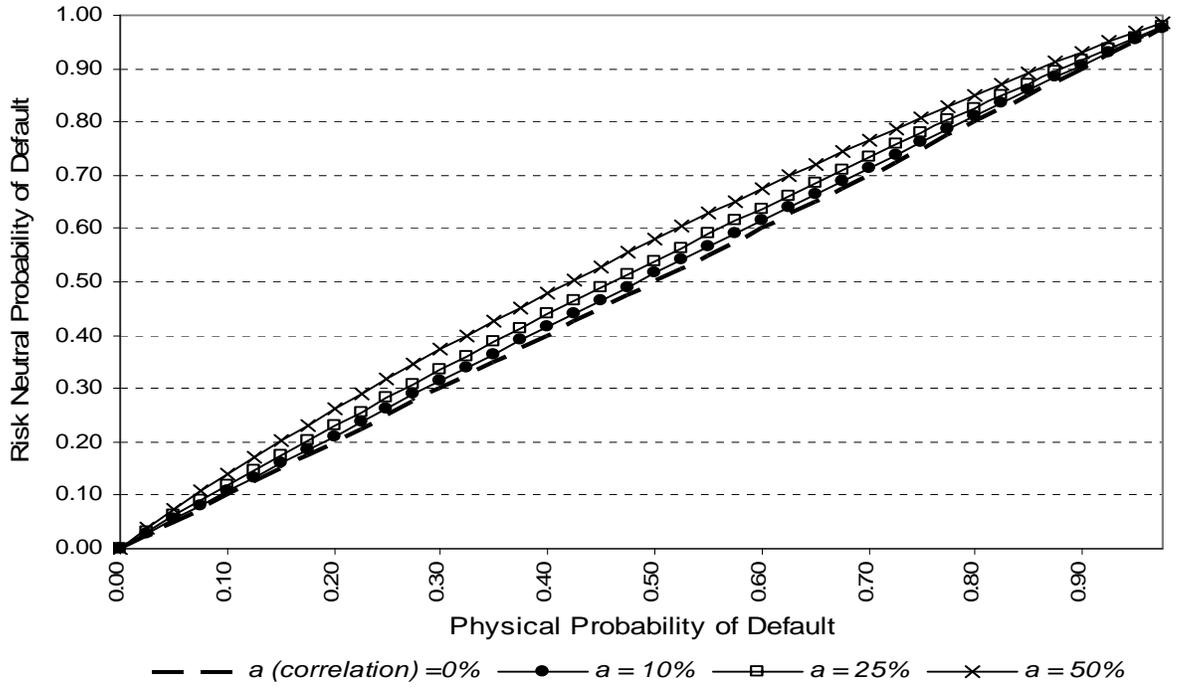


Figure 2: The Impact of an Increase in Correlation on Risk Neutral Default Probabilities

This figure shows the risk neutral probabilities of default corresponding to the physical probabilities of default for different correlation levels (a). Market price of risk is assumed to be 40 percent. When correlation is 0 percent (i.e. $a = 0\%$), risk neutral probability of default is equal to the physical probability of default.

0-3 Equity Tranche Prices and Implied Correlations for CDX.NA.IG.8

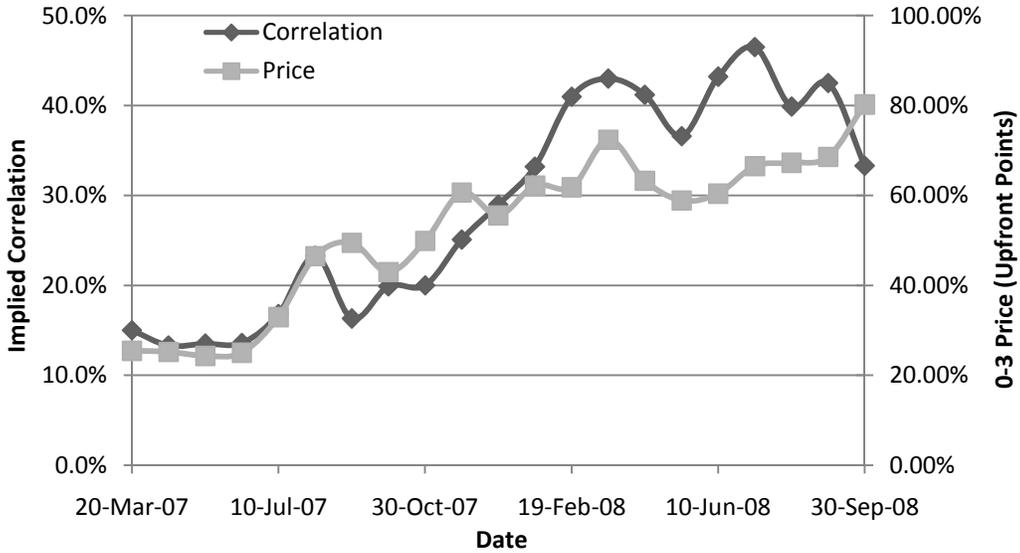


Figure 3: Equity Tranche Correlations versus Equity Tranche Price for CDX.NA.IG.8

The figure shows the positive relationship between tranche correlation and the upfront price for the 0-3 equity tranche on the CDX.NA.IG.8 for the period March 2007 to September 2008 implying that the equity tranche got riskier when correlations increased and vice versa.

Endnotes:

¹ A CDO tranche is responsible for covering underlying portfolio losses that lie between its attachment and detachment points. The 0-3 tranche has an attachment point of zero percent and detachment point of three percent.

² Synthetic CDO Primer [2005], Bond Market Association, page 44.

³ We thank the referee for this comment.

⁴ Market data available in the form of spreads or tranche prices are an aggregation of future correlation, default probability, recovery, risk aversion and liquidity expectations and thus pose a substantial challenge in disintegrating these observed spreads/prices into various components.

⁵ See Burtschell, Gregory, and Laurent [2005] for a comparison of alternative CDO pricing models and Burtschell, Gregory, and Laurent [2007] for a comparison of stochastic and local correlations.

⁶ The standardized tranches on the CDX.NA.IG index have detachment points at 3%, 7%, 10%, 15%, 30%, and 100%.

⁷ Laurent and Gregory [2005] provide a detailed discussion of the model.

⁸ In this framework, CDS spreads are assumed to price only default risk and other risk factors such as liquidity, demand-supply conditions, etc. are assumed to be negligible.

⁹ Suppose that for a time period from 0 to T , default occurs between t and dt . The present value of expected spread earnings for a constant default intensity λ and discount rate r is $\int_0^T s e^{-(\lambda+r)t} dt$. The present value of

expected loss is $\int_0^T LGD * \lambda * e^{-(\lambda+r)t} dt$. Equating present value of expected loss with the present value of expected spread earnings results in $\lambda = s / LGD$.

¹⁰ We consider Merton's model with constant correlation. Another possibility is to use an asset pricing model with stochastic correlations. However, our conjecture is that since correlations exist due to joint sensitivities of asset returns to common underlying risk factors in the economy, any increase in correlation should increase systematic risk and thus the (risk-neutral) default risk in an alternative asset pricing model as well implying that the evidence would be qualitatively similar to the one presented in this paper.

¹¹ In other words, we assume a relationship similar to the CAPM between the expected asset return, μ , and

the systematic (or market) latent factor S :
$$\mu - r = \frac{Cov(\mu, S)}{Var(S)} (E(S) - r)$$

¹² Market convention is to decompose the fair spread on the CDX.NA.IG equity tranche into two components – a fixed annual spread of five percent that is earned on the notional balance of the equity tranche, called the *running spread*, and the remainder that is paid in advance as *upfront fee*. One can therefore view the *upfront fee* as the present value of the risky fair spread minus five percent running spread cash flow stream.

¹³ The CDX indices get rebalanced every six months, usually the 20th of March and September, with each rebalancing producing a new series denoted by a numeral at the end of the index name. September 22, 2005 represents the first Wednesday (i.e. mid-week) after series 5 i.e. CDX.NA.IG.5 came into existence.

¹⁴ KMV has estimated the market price of risk using corporate bond data and has found that the value “generally falls in a range of .40 to .50 and tends to vary in a well-behaved manner through time”: Page 13 of KMV document titled “Introduction to Creditmark” available at KMV's website http://www.moodyskmv.com/research/portfolioValuation_wp.html

¹⁵ We thank the referee for this comment.

¹⁶ We thank the editor and the referee for this suggestion.

¹⁷ Data is from Markit.

¹⁸ See Ağca, Agrawal and Islam (2008) for a more detailed discussion on the differences between implied correlations and actual correlations as well evidence pointing to the high degree of correlation between implied and actual correlations.

²⁰ We thank the referee for the comments in this section