Can CDO Equity Be Short on Correlation?*

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Abstract

By examining the impact of an increase in correlation among underlying assets on the value of a collateralized debt obligation (CDO) equity tranche, we show that, contrary to general perception, CDO equity can be short on correlation. Specifically, when the underlying reference portfolio comprises high quality assets (assets with low probability of default) or diverse assets (assets with low correlations), the upfront price of a CDO equity tranche can increase with correlation. In these instances, as asset correlations rise, the corresponding systematic risk of the underlying reference obligations also increases. As a result, a rise in correlation can lead to cases where a corresponding increase in risk-neutral default probabilities outweighs the beneficial effect of having fewer or no defaults.

JEL Classification: G11, G12, G13

Keywords: Credit risk, Collateralized debt obligations, Monte-Carlo simulation, Gaussian copula model

1. Introduction

CDO trading volume has increased considerably in recent years, especially with the creation of standardized tranched products that refer to credit indices such as the Dow Jones CDX North America Investment Grade and High Yield indices (CDX.NA.IG and CDX.NA.HY respectively) and equivalent iTraxx indices in Europe. CDO equity tranches are particularly popular in the market. Since they are the riskiest first loss tranches, they earn high spreads or upfront payments to compensate for the additional risk. Both for the CDX North America Investment Grade and iTraxx Europe Investment Grade index tranches, the equity tranche is denoted as a 0-3 tranche[1] and is responsible for covering the first three percent of losses in the underlying index that typically comprises an equally weighted portfolio of 125 reference obligations.

Equity tranches are the junior-most tranches in the waterfall structure of a CDO. They are perceived as being long on correlation, i.e. an increase in correlation across underlying credits reduces the risk to the equity tranche investor. For example, Bond Market Association's Synthetic CDO Primer (2005) states that the seller of equity tranche protection wants correlation to increase, and equity tranche spreads compress as correlation rises.[2] The common explanation for this is that any increase in correlation increases the likelihood of extreme events in the underlying portfolio's loss distribution. Since losses on equity tranches are capped at the detachment point of the tranche, an increase in the likelihood of positive events (no or very few defaults) should be beneficial

[1] A CDO tranche is responsible for covering underlying portfolio losses that lie between its attachment and detachment points. The 0-3 tranche has an attachment point of zero percent and detachment point of three percent.

[2] Synthetic CDO Primer (2005), Bond Market Association, page 44.

to an equity tranche i.e. the value of equity tranche increases, while a corresponding increase in the likelihood of negative events (very high number of defaults) should not hurt it (See Hull and White (2004) for a detailed discussion).

In this paper, we show that when correlations across assets increase, a CDO equity tranche does not necessarily become less risky. On the contrary, the value of an equity tranche can decrease (i.e. the fair spread or upfront price can increase) as correlations increase. The reason is that even though an increase in correlations across assets does not increase the physical likelihood of default for the underlying reference obligations, it increases their systematic risk and the risk neutral probability of default. In certain cases, this increase in risk neutral default probabilities can outweigh the beneficial impact of high correlation on the equity tranche, thereby reducing its value.

To examine how CDO equity value is affected by an increase in correlation across assets, we conduct a Monte-Carlo simulation analysis. Our model is the industry standard one-factor Gaussian copula model of CDO pricing introduced first by Li (2000).[3] We consider a reference portfolio of 125 credit default swaps (similar to the CDX.NA.IG index), each with a tenor of 5 years. We explore the pricing of the 0-3 tranche (equity tranche) on this portfolio.[4] Following industry practice, we assume a certain initial identical correlation across all assets and obtain the risk neutral probability of default from the parameters of the model. Then, using the Merton (1974) model, we transform risk neutral default probabilities to physical default probabilities. Keeping the physical default probabilities constant, we increase the correlation across all assets. Since

[3] See Burtschell, Gregory, and Laurent (2005) for a comparison of alternative CDO pricing models and Burtschell. Gregory, and Laurent (2007) for a comparison of stochastic and local correlations.

[4] The standardized tranches on the CDX.NA.IG index have detachment points at 3%, 7%, 10%, 15%, 30%, and 100%.

correlation is directly tied to the systematic risk of an asset in our model, this leads to an increase in the risk neutral probability of default although there is no change in the physical default probability. We then examine the impact of an increase in correlation on the value of standardized CDO equity tranches.

Our results suggest that when correlation across assets increase, the detrimental effect of an increase in risk neutral default probabilities on the value of a CDO equity tranche can very often be substantial. Specifically, when underlying assets are of high quality or when the initial correlation across assets is low, CDO equity tranche value decreases when correlations rise. Thus CDO equity can be *short on correlation*.

Since the general perception in credit markets is that CDO equity is always long on correlation while more senior tranches are not, equity tranches are used not only for investment purposes but also in combination with more senior tranches for hedging against correlation movements. The implication of our findings is that not all equity tranche trades provide effective correlation hedging. In fact, in some cases, the 'hedge' might actually increase the correlation risk.

The rest of the paper is organized as follows: Section 2 explains the underlying model and the simulation methodology. Section 3 presents and discusses the impact of an increase in correlation across assets on CDO equity tranche value . Section 4 concludes.

2. Model and Simulation Methodology

2.1. Model

The industry standard default time model that is used to price CDOs captures default dependency through a one-factor Gaussian copula function. The objective of the standard default time model is to determine the timing of losses on a collateral portfolio by taking into account the default probabilities of underlying assets, their correlations and assumptions about recovery in the instance of default. The one-factor Gaussian copula model introduced by Li (2000) is popular in the industry since it is easy to implement and computationally efficient.

Assume that there are n firms or assets. Let t_i be the default time of i^{th} firm and $P_i(t)$ be the cumulative probability of firm i defaulting before time t, i.e. probability that $t_i < t$.[5] Define X_i , i=1,2..n, as follows:

$$X_i = a_i S + \sqrt{1 - a_i^2} Z_i , \tag{1}$$

Here, S (systematic factor) and Z_i (idiosyncratic factor) are independent and have standard normal distributions. By construction, the correlation between X_i and X_j is $a_i a_j$, and X_i has a standard normal distribution. Using this model, we transform the normal variates X_i to uniform variates U_i such that $U_i = \Phi(X_i)$, where Φ is the standard cumulative normal distribution function. In this model, if U_i is greater than $P_i(t)$, then firm i does not default by time t. If $P_i(t_1) < U_i < P_i(t_2)$, then firm i defaults between time t_1 and t_2.

The simplifying assumptions of the industry standard one-factor Gaussian copula model can be summarized as follows: (i) systematic and idiosyncratic factors have standard normal distributions; (ii) all assets have the same pair-wise correlation, thus $a_i = a_j = a$, for all i, j ($a \geq 0$); (iii) all assets have a fixed recovery rate; (iv) all assets have the same spread, s, where this spread is equal to the average spread on the portfolio (the

[5] Laurent and Gregory (2005) provide a detailed discussion of the model.

index spread); and (v) the default intensity, λ_i, is the spread per unit of *LGD* (loss given default, i.e. (1 - *recovery*)). Accordingly, the default intensity, λ_i, and risk neutral probability of default of firm *i*, $P^Q{}_i$, are as follows:

$$\lambda_i = \lambda = s \,/\, LGD, \qquad \text{for all } i=1,2,..n \tag{2}$$

$$P^Q{}_i = P^Q = 1 - \exp(-\lambda t), \qquad \text{for all } i=1,2,..n \tag{3}$$

Since *s* and *LGD* are assumed to be the same for all assets, default intensity and risk neutral probability of default is identical for all assets in the portfolio.

In this paper, our objective is to examine how an increase in correlation across assets affects the value of CDO equity tranches even when physical default probabilities do not change. Therefore, we first transform risk neutral default probabilities derived from spreads to physical or 'real-world' default probabilities using the results of the Merton (1974) model. In Merton's model, the value of a firm, *V*, follows a geometric Brownian motion with mean μ and volatility σ, and where $W(t)$ is a Weiner process.

$$\mathrm{d}V = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W(t) \tag{4}$$

If we assume that *r* is the risk-free rate, then for a debt with face value of \$1 maturing at time *t*, the physical and risk neutral cumulative probabilities of default between time 0 and *t*, P_t and $P^Q{}_t$, respectively, are:

$$P_t = \Phi\left(\frac{-\ln(V) + (\mu - 0.5\sigma^2)t}{\sigma\sqrt{t}}\right) \tag{5}$$

$$P^Q{}_t = \Phi\left(\frac{-\ln(V) + (r - 0.5\sigma^2)t}{\sigma\sqrt{t}}\right) \tag{6}$$

Using above equations, we can express physical default probability and risk neutral default probability as follows:

$$P_t = \Phi\left(\Phi^{-1}(P_t^Q) - \left(\frac{\mu - r}{\sigma}\right)\sqrt{t}\right) \quad (7)$$

$$P^Q_t = \Phi\left(\Phi^{-1}(P_t) + \left(\frac{\mu - r}{\sigma}\right)\sqrt{t}\right) \quad (8)$$

We consider a CAPM[6] type model where the excess return on asset *i* is a linear function of the systematic factor *S*. Since the correlation of asset *i* with *S* is *a* in the above model, we have:

$$\frac{\mu - r}{\sigma} = a\Lambda \quad (9)$$

Here, Λ is the market price of risk. Using equations (7), (8) and (9), we transform risk neutral and physical default probabilities to each other as follows:

$$P_t = \Phi\left(\Phi^{-1}(P_t^Q) - a\Lambda\sqrt{t}\right) \quad (10)$$

$$P^Q_t = \Phi\left(\Phi^{-1}(P_t) + a\Lambda\sqrt{t}\right) \quad (11)$$

After determining the risk neutral default probability, P^Q_t from a given (market observed) spread as in equation (3), we convert this probability to a physical default probability P_t using equation (10). Then, keeping this physical probability constant, we increase correlations by increasing *a* i.e. the loading on the systematic factor. Accordingly, we find 'new' corresponding risk neutral probabilities from equation (11) and estimate new CDO equity tranche prices. This allows us to examine how an increase in correlation affects CDO equity tranche values even though physical default probabilities remain unchanged. Furthermore, we explore whether the increase in risk neutral default probabilities through a rise in correlation can be detrimental to the CDO

[6] See Sharpe(1964), Lintner (1965).

equity tranche values. Our valuation technique in this paper employs Monte-Carlo simulations, which is described in the next subsection.

2.2. Methodology

Our reference portfolio consists of 125 names, as in the CDX.NA.IG index. We consider a tranching structure similar to the CDX.NA.IG index such that the first-loss equity tranche is a 0-3 tranche i.e. responsible for covering losses from 0 percent to 3 percent in the reference portfolio. For pricing purposes, we model the equity tranche as earning a running spread of five percent and trading on an upfront fee basis[7] – once again to capture the characteristics of the actual CDX.NA.IG 0-3 tranche that trades on similar terms. Using the model discussed above, we generate default distributions for different levels of correlation and spread. We consider initial correlations from 0 to 50 percent in increments of 5 percent, and initial index spread of 25, 50, 100, 250 and 500 basis points. We also use actual CDS spreads on September 22, 2005.[8] We assume a fixed recovery rate on defaulted names of 50 percent and consider the LIBOR term structure as on September 22, 2005 as the risk-free term structure for discounting the cash flows of the tranches. Accordingly, we obtain model upfront prices of the equity tranche that are reported in Table 1.

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[7] Market convention is to decompose the fair spread on the CDX.NA.IG equity tranche into two components – a fixed annual spread of five percent that is earned on the notional balance of the equity tranche, called the *running spread*, and the remainder that is paid in advance as *upfront fee*. One can therefore view the *upfront fee* as the present value of the fair spread minus five percent running spread cash flow stream.

[8] The CDX indices get rebalanced every six months, usually the 20th of March and September, with each rebalancing producing a new series denoted by a numeral at the end of the index name. September 22, 2005 represents the first Wednesday (i.e. mid-week) after series 5 i.e. CDX.NA.IG.5 came into existence.

We next transform risk neutral probabilities of the underlying obligations in the reference portfolio to physical probabilities using equation (10). We consider market price of risk as 20 percent, 40 percent and 60 percent[9]. Keeping the physical default probabilities constant, we increase asset correlations by 1 and 5 percentage points. We then calculate the corresponding risk neutral probabilities using equation (11) and use these 'new' risk-neutral probabilities for valuing CDO equity tranches. We explore the impact of an increase in correlation across assets on equity tranche values as a function of certain portfolio and market characteristics, i.e. as a function of the initial asset correlations and spreads, as well as the market price of risk. As a result, we examine 198 cases (11 initial correlations x 6 initial spread levels x 3 market price of risk = 198). We run 200,000 simulation trials to value each equity tranche.

3. CDO Equity Tranches: Can They Be Short on Correlation?

Structured credit markets perceive CDO equity tranche to be long on correlations. An increase in the correlation across assets is considered to be beneficial for the CDO equity tranche value, since the losses are capped whereas the upside potential increases when correlation rises. As an example, consider the physical default distribution for a portfolio of 125 credits over a 5 year horizon assuming that annualized physical default probability of each credit is 2 percent. Figure 1 shows the physical default distribution when homogenous pair-wise correlations increase from 0 percent to 30 percent with 10 percent increments. As shown in Figure 1, when correlations increase, the thickness in the tails of the distribution increases. The greater likelihood of zero or very few defaults

[9] Moody's KMV estimates a market price of risk on a daily basis from bond and CDS prices. The Moody's KMV estimate has historically varied in the range 0.4 to 0.6.

with higher correlation, *holding everything else constant*, makes correlation beneficial for the first loss tranche investor i.e. makes CDO equity long on correlation. However, as correlation across assets increases, so does their systematic risk and hence their corresponding risk neutral probability of default, even though physical probability of default does not change. Therefore, it is important to examine how an increase in asset correlations affects CDO equity tranches by considering the impact of the increased correlations on risk neutral default probabilities.

We first obtain the results of the base model, where the default probabilities are determined using the parameters mentioned in the previous section. The equity tranche values for the base model are given in Table 1. In Table 1, the 0-3 equity tranche prices are quoted as upfront fee for different levels of correlation and index spreads. The 125 credits comprising the index are considered to be homogenous. They have the same pair-wise correlation and CDS spread is equal to the index spread. One can observe that for a given index spread, an increase in correlation across assets is beneficial to the CDO equity tranche investor, since the price of the CDO equity tranche declines, reflecting a lower risk or lower expected loss. However, this interpretation assumes that increasing the correlation across assets has no impact on the systematic risk or the index spread.

We explore how an increase in asset correlations affect CDO equity values by considering the impact of correlations on risk neutral default probabilities. We consider 20, 40 and 60 percent as market prices of risk. We estimate model prices for the 0-3 equity tranche for a 1 percent and 5 percent increases in correlation and examine the impact of the increase in correlation on the value of CDO equity tranches. The underlying assumption is that the ‘real-world’ or physical probabilities of default between the case in

Table 1 and current cases do not change. Increasing the correlation across assets, however, increases the systematic risk of the underlying assets in our model as in equation (1). This implies that even though physical default probabilities do not change, risk neutral probabilities increase as long as there exists a certain (greater than zero) market price of risk (as can be seen in equation (11)). Thus, keeping recovery rates and the market price of risk constant, a rise in correlation leads to a higher risk-neutral probability of default.

Table 2 Panel A shows the results when the market price of risk is assumed as constant at 40 percent. The table displays equity tranche prices for a given initial index spread and correlation as well as prices when correlation across assets increases by 1 percent and 5 percent. As observed in the table, when the initial index spread is low i.e. the index constituents are high quality names, an increase in correlation always leads to an increase in equity tranche price – highlighted by bold fonts in the table. In these cases, the detrimental effect of an increase in risk neutral default probability due to a rise in systematic risk outweighs the beneficial impact of an increase in the real-world likelihood of having few or no correlated defaults. Thus CDO equity is short on correlation. On the other hand, when the initial index spread is high, an increase in correlation has a beneficial impact on equity tranche values when the initial correlation level is not low. These cases are consistent with the general perception that CDO equity is long in correlation.

When the initial index spread is low i.e. the physical default probability P_t is small, the term $a\Lambda\sqrt{t}$ in equation (11) gets a higher relative weight. Therefore, the marginal impact of an increase in correlation across assets on the risk neutral default probability is large. Thus, when the reference portfolio consists of high quality names, an increase in correlation across assets can be detrimental to the value of equity tranche. In these cases, the negative impact of an increase in risk neutral default probability due to an increase in correlation overcomes the positive impact of an increase in the probability of no or small number of defaults. As a result CDO equity becomes short on correlation. When the initial index spread is high i.e. physical default probability is high, the term $a\Lambda\sqrt{t}$ loses its relative impact on risk neutral default probabilities in equation (11). In these states, an increase in correlation across assets is beneficial for CDO equity tranche value, and thus CDO equity is long in correlation. As shown in Figure 2, for a given physical probability of default, increasing correlation leads to an increase in the risk neutral probability of default. One can also observe that for a given level of correlation, the slope of the risk neutral default probability decreases as physical default probability increases. Hence the ratio of the risk neutral to physical default probability is higher for lower values of physical default probability but eventually converges to one as the physical default probability increases to 100 percent.

As market price of risk decreases, risk-neutral default probabilities and physical default probabilities converge to each other. We therefore estimate CDO equity fair-prices with a reduced market price of risk to explore whether, in the presence of less risk-averse sentiment in the market, CDO equity can still be short on correlation. Table 2 Panel B shows the values of CDO equity tranches when market price of risk is 20 percent and when underlying asset correlations increase by 1 percent and 5 percent. Supporting our previous findings, we observe that when the initial portfolio has low average spread, i.e. high credit quality, an increase in underlying asset correlations has a detrimental effect on the value of an equity tranche when the initial underlying correlations are not very high. In fact, even for poor quality or high spread portfolios, when the initial correlation levels are low, an increase in correlation makes the equity tranche more risky. Our previous finding that the equity tranche can be short on correlation either when the underlying CDO reference pool is of high quality or when it comprises assets with low correlations appears to hold even in cases where the market price of risk is less than historically observed levels of 40 to 60 percent.

We next investigate the impact of a change in the market price of risk in the other direction. As market price of risk increases, risk-neutral default probabilities and physical default probabilities diverge even more than in the 40 percent case. One can therefore anticipate more instances where CDO equity is short on correlation. Table 2 Panel C shows the values of CDO equity tranches when the market price of risk is 60 percent and the underlying asset correlations increase by 1 percent and 5 percent. We observe more cases where CDO equity tranche is short on correlation either when the underlying CDO reference pool is of high quality or comprises assets with low correlations. Thus when market price of risk is at the high end of historically observed levels, CDO equity is short on correlation more often.

As a final test, we use actual CDS spreads observed for the 125-name basket of constituents that comprise the CDX.NA.IG.5 index to examine the impact of an increase in asset correlations on the value of the 0-3 equity tranche. Our CDS data are from Mark-It and are as-of the September, 22, 2005. Table 3 shows the 'fair' prices for the 0-3 equity tranche as well as the prices for this tranche when underlying asset correlations are increased by 1 percent and 5 percent, respectively. Once again we consider the market price of risk as 20 percent, 40 percent, and 60 percent. For the investment grade universe, the results seem unequivocal: CDO equity is short on correlation under historically observed market price of risk levels between 40 percent and 60 percent. Even in more benign conditions – corresponding to a market price of risk of 20 percent in our analysis, CDO equity can become more risky if the underlying reference obligations come from a diverse pool i.e. have low initial correlations. In these instances, any increase in correlation leads to a relatively large change in the systematic default risk of these underlying reference entities. As a result, CDO equity tranches can be short on correlation.

4. Conclusion

There is a general perception among participants in structured credit markets that correlation is beneficial for CDO equity tranche investors – a notion which is referred to as CDO equity being long on correlation. This view is based on the argument that an increase in correlation among the underlying reference assets of a CDO leads to a higher incidence of zero or few defaults in the collateral pool that increases the value of an equity tranche in a CDO structure. Since equity tranche losses are capped at the detachment point of the tranche, a corresponding increase in instances of very high (correlated) defaults at the other end of the default distribution have no impact on the junior-most equity tranche.

However, changes in correlation not only affect the physical default distribution of a CDO pool of assets but also the risk neutral default probabilities of the underlying assets since correlation is tied to the systematic risk of a credit obligation. For a CDO equity tranche, the impact of these two factors on the tranche value is in opposite directions. While higher likelihood of zero or no defaults benefits it, higher risk neutral default probabilities due to an increase in systematic risk increases its risk. Using a standard one-factor Gaussian copula model and a structure similar to the CDX.NA.IG index tranches, we show that for certain types of CDOs and a range of market prices of risk, CDO equity price can actually increase with correlation i.e. CDO equity can be short on correlation.

We find that CDOs that hold high quality investment grade names in their collateral portfolio are more likely to have equity tranches that are short on correlation. For poorer quality collateral pools comprising high yield or sub-investment grade assets, CDO equity tranches do benefit from an increase in correlations among the underlying assets, unless the initial correlation across assets are very low. The notion of equity tranche being long on correlation appears valid for such types of CDOs.

Given the increasing popularity of correlation trading in today's credit markets, our results have implications for traders who take long or short positions in CDO equity tranches, especially for those investors who focus on the high grade CDO universe. Investors and speculative participants increasingly express views either about correlations moving in certain directions or about realized correlations being different from those 'embedded' in CDO tranche prices over a holding period. As the structured credit market continues to grow and the sophistication of associated instruments increases, it becomes more important for market participants to understand the complex relationships between correlations and their impact on the riskiness of different tranches across a CDO capital structure.

References:

Bond Market Association, *Synthetic CDO Primer*, 2005.

Burtschell, X., J. Gregory, and J.-P. Laurent, 2005, A Comparative Analysis of CDO Pricing Models, Working paper, BNP-Paribas.

Burtschell, X., J. Gregory, and J.-P. Laurent, 2007, Beyond the Gaussian copula: stochastic and local correlation, *Journal of Credit Risk* ,3.

Duffie, D, and N. Gârleanu, 2001, Risk and the Valuation of Collateralized Debt Obligations, *Financial Analysts Journal*, 57, 41-59.

Gregory, J., and J.-P. Laurent, 2003, I Will Survive, *RISK*, June, 103-107.

Gregory, J., and J.-P. Laurent, 2004, In the Core of Correlation, *RISK*, October, 87-91.

Hull, J., and A. White, 2004, Valuation of a CDO and an nth to Default CDS without Monte Carlo Simulation, *Journal of Derivatives*, 2, 8-23.

Laurent, J.-P., and J. Gregory, Basket Default Swaps, CDOs and Factor Copulas, 2005, *Journal of Risk*, 7, 103-122.

Li, D., 2000, On Default Correlation: A Copula Function Approach, *Journal of Fixed Income*, 9, 43-54.

Lintner, J., 1965, The Valuation of Risk assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets, *Review of Economics and Statistics*, 47 , 13-37.

Merton, R. C., 1974, On the Pricing of Corporate debt: The Risk Structure of Interest Rates, *Journal of Finance* 2, 449–470.

Sharpe, W. F., 1964, Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk, *Journal of Finance*, 19 (3), 425-442.

Table 1
CDO Equity Prices for the Base Model

This table reports CDO equity tranche (0-3 tranche) upfront prices obtained from a one-factor Gaussian copula model for different initial correlations and index spreads. Running spread is assumed as 5 percent. The reference portfolio consists of 125 credits. Recovery rate is assumed as 50 percent. The index spreads are given in the first column and reported in basis points. The initial correlations vary between 0 to 50 percent with 5 percent increments. The CDO equity upfront prices are given in points.

	Initial Correlation										
Spread (bps)	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
25	19.05	18.98	18.93	18.66	18.36	17.45	16.55	15.33	13.77	12.01	10.07
50	54.53	54.09	53.18	51.49	49.43	46.77	44.00	41.01	37.75	34.31	30.67
100	83.86	83.48	82.54	80.79	78.53	75.49	72.19	68.43	64.42	59.92	55.27
250	93.93	93.86	93.7	93.38	92.94	92.23	91.25	89.84	87.9	85.41	82.36
500	96.99	96.95	96.87	96.71	96.49	96.18	95.78	95.27	94.58	93.67	92.42

Table 2
CDO Equity Prices When Correlations Across Assets Increase

This table reports CDO equity upfront prices obtained when correlations increase by 1 and 5 percent without any change in the physical probability of default. Running spread is assumed as 5 percent. Base model prices for comparison are given in Table 1. Cases where equity becomes more risky when correlation increases are in bold. The reference portfolio consists of 125 credits. Recovery rate is assumed as 50 percent. The index spreads are given in the first column and reported in basis points. The initial correlations vary between 0 to 50 percent with 5 percent increments. The CDO equity upfront prices are given in points. Panel A, B and C reports the upfront prices of CDO equity values when market price of risk is considered as 40%, 20%, and 60%, respectively.

Panel A: Market Price of Risk = 40%

	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Spread = 25 bps											
+1% Correlation	**19.91**	**19.82**	**19.74**	**19.42**	**19.03**	**18.02**	**17.02**	**15.71**	**14.05**	**12.21**	**10.2**
+5% Correlation	**23.49**	**23.23**	**22.95**	**22.28**	**21.52**	**20.1**	**18.72**	**17**	**14.99**	**12.82**	**10.49**
Spread = 50 bps											
+1% Correlation	**55.61**	**54.99**	**53.92**	**52.07**	**49.87**	**47.09**	**44.23**	**41.15**	**37.82**	34.29	30.59
+5% Correlation	**59.49**	**58.1**	**56.4**	**53.95**	**51.27**	**48.07**	**44.86**	**41.44**	**37.81**	33.99	30.03
Spread = 100 bps											
+1% Correlation	**84.15**	**83.77**	**82.63**	**80.9**	78.39	75.42	72.08	68.22	64.08	59.62	54.98
+5% Correlation	**85.26**	**84.36**	**82.98**	80.79	78.16	74.83	71.27	67.27	63.02	58.25	53.35
Spread = 250 bps											
+1% Correlation	**94.01**	**93.92**	**93.73**	**93.4**	92.92	92.18	91.14	89.68	87.66	85.1	81.97
+5% Correlation	**94.28**	**94.1**	**93.83**	**93.4**	92.8	91.88	90.64	88.91	86.6	83.73	80.28
Spread = 500 bps											
+1% Correlation	**97.02**	**96.97**	**96.88**	96.71	96.48	96.15	95.74	95.2	94.49	93.54	92.23
+5% Correlation	**97.12**	**97.03**	**96.89**	96.68	96.39	96.01	95.53	94.9	94.04	92.91	91.36

Panel B: Market Price of Risk = 20%

	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Spread = 25 bps											
+1% Correlation	**19.48**	**19.39**	**19.32**	**19**	**18.62**	**17.64**	**16.67**	**15.38**	13.75	11.93	9.93
+5% Correlation	**21.24**	**21.01**	**20.80**	**20.22**	**19.55**	**18.25**	**16.99**	**15.4**	13.50	11.44	9.21
Spread = 50 bps											
+1% Correlation	**55.07**	**54.46**	**53.41**	**51.58**	49.41	46.66	43.82	40.76	37.45	33.95	30.27
+5% Correlation	**56.88**	**55.59**	**53.98**	**51.65**	49.10	46.02	42.92	39.61	36.08	32.38	28.53
Spread = 100 bps											
+1% Correlation	**84.03**	**83.52**	82.47	80.60	78.23	75.12	71.76	67.93	63.87	59.32	54.62
+5% Correlation	**84.44**	83.46	81.96	79.65	76.91	73.51	69.87	65.85	61.56	56.80	51.92
Spread = 250 bps											
+1% Correlation	**93.97**	**93.87**	93.69	93.35	92.87	92.12	91.07	89.59	87.55	84.97	81.83
+5% Correlation	**94.08**	**93.89**	93.62	93.17	92.54	91.57	90.25	88.44	86.03	83.05	79.50
Spread = 500 bps											
+1% Correlation	**97.00**	96.95	96.86	96.69	96.46	96.13	95.72	95.18	94.45	93.49	92.17
+5% Correlation	**97.04**	96.95	96.81	96.58	96.29	95.89	95.40	94.75	93.86	92.67	91.06

Panel C: Market Price of Risk = 60%

	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Spread = 25 bps											
+1% Correlation	**20.35**	**20.24**	**20.17**	**19.84**	**19.42**	**18.39**	**17.38**	**16.04**	**14.36**	**12.5**	**10.47**
+5% Correlation	**25.81**	**25.52**	**25.17**	**24.41**	**23.54**	**22.00**	**20.49**	**18.63**	**16.51**	**14.24**	**11.79**
Spread = 50 bps											
+1% Correlation	**56.15**	**55.52**	**54.42**	**52.56**	**50.32**	**47.52**	**44.64**	**41.53**	**38.18**	**34.63**	**30.9**
+5% Correlation	**62.02**	**60.55**	**58.75**	**56.19**	**53.39**	**50.08**	**46.78**	**43.26**	**39.55**	**35.6**	**31.55**
Spread = 100 bps											
+1% Correlation	**84.38**	**83.90**	**82.87**	**81.05**	**78.73**	**75.66**	**72.32**	**68.51**	**64.46**	59.91	55.21
+5% Correlation	**85.99**	**85.19**	**83.91**	**81.85**	**79.33**	**76.10**	**72.60**	**68.63**	64.42	59.67	54.75
Spread = 250 bps											
+1% Correlation	**94.05**	**93.96**	**93.78**	**93.44**	**92.97**	**92.24**	91.21	89.77	87.77	85.22	82.12
+5% Correlation	**94.47**	**94.29**	**94.04**	**93.62**	**93.05**	92.18	91.00	89.36	87.15	84.38	81.03
Spread = 500 bps											
+1% Correlation	**97.04**	**96.99**	**96.9**	**96.73**	**96.5**	96.17	95.76	95.23	94.52	93.58	92.28
+5% Correlation	**97.20**	**97.11**	**96.98**	**96.77**	96.49	96.12	95.66	95.05	94.22	93.13	91.65

Table 3
CDO Equity Prices Using Actual Credit Default Swap Spreads

This table reports the CDO equity tranche (0-3 tranche) upfront prices when correlation is increased by 1 percent and 5 percent and actual credit default swap (CDS) spreads are used. Running spread is assumed as 5 percent. The first row gives the base model prices of CDO equity tranches. The CDS data come from Mark-It and are the closing prices for the CDX.NA.IG Series 5 constituents on September 22, 2005. Cases where equity becomes more risky when correlation increases are in bold. The reference portfolio consists of 125 credits. Recovery rate is assumed as 50 percent. The initial correlations vary between 0 to 50 percent with 5 percent increments. The CDO equity upfront prices are in points.

	Initial Correlation										
	0%	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
Base	53.02	52.68	51.87	50.32	48.63	46.19	43.72	40.99	38.08	34.89	31.56
Market price of risk = 20%											
+1% Correlation	**53.53**	**53.05**	**52.13**	**50.45**	**48.66**	46.13	43.59	40.78	37.83	34.58	31.21
+5% Correlation	**55.31**	**54.28**	**52.82**	**50.68**	48.51	45.66	42.88	39.82	36.64	33.23	29.68
Market price of risk = 40 %											
+1% Correlation	**54.06**	**53.56**	**52.62**	**50.92**	**49.1**	**46.55**	**43.98**	**41.16**	**38.18**	**34.91**	31.52
+5% Correlation	**57.86**	**56.72**	**55.16**	**52.91**	**50.6**	**47.66**	**44.75**	**41.6**	**38.34**	34.8	31.17
Market price of risk = 60%											
+1% Correlation	**54.89**	**54.07**	**53.11**	**51.39**	**49.54**	**46.97**	**44.39**	**41.54**	**38.53**	**35.23**	**31.83**
+5% Correlation	**60.32**	**59.09**	**57.45**	**55.09**	**52.67**	**49.63**	**46.62**	**43.38**	**40.04**	**36.38**	**32.66**

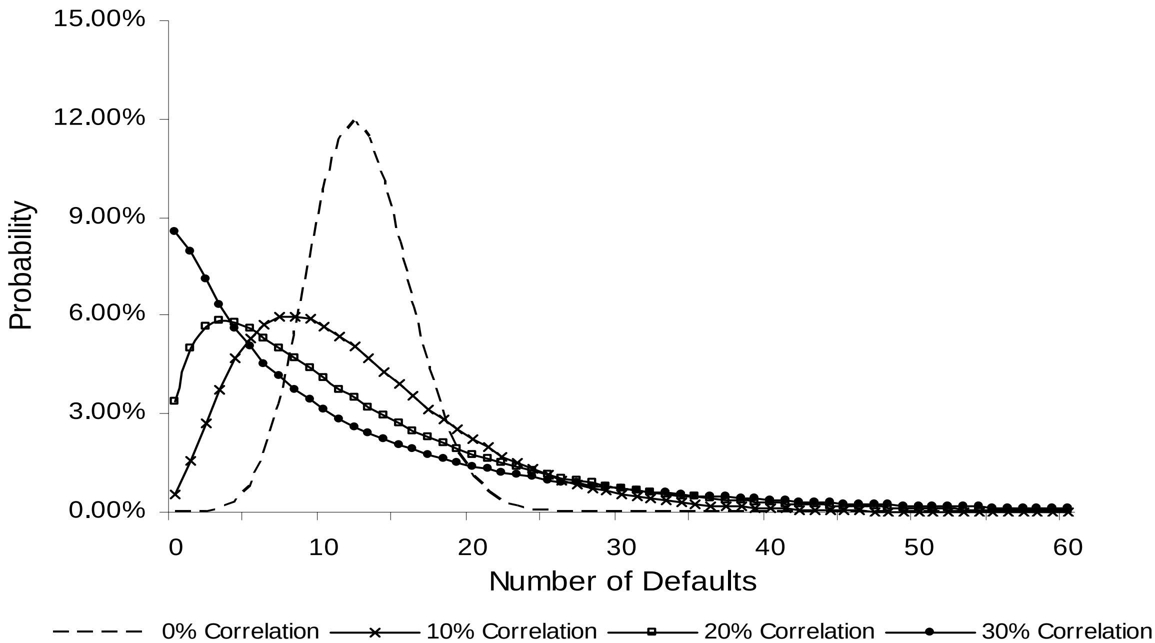

Figure 1: The Impact of Correlation on the Physical Default Distribution
This figure shows the physical default distribution for a portfolio of 125 credits as correlation across credits increases from 0 percent to 30 percent with 10 percent increments. All credits have a constant annualized physical default probability of two percent and the default horizon is five years.

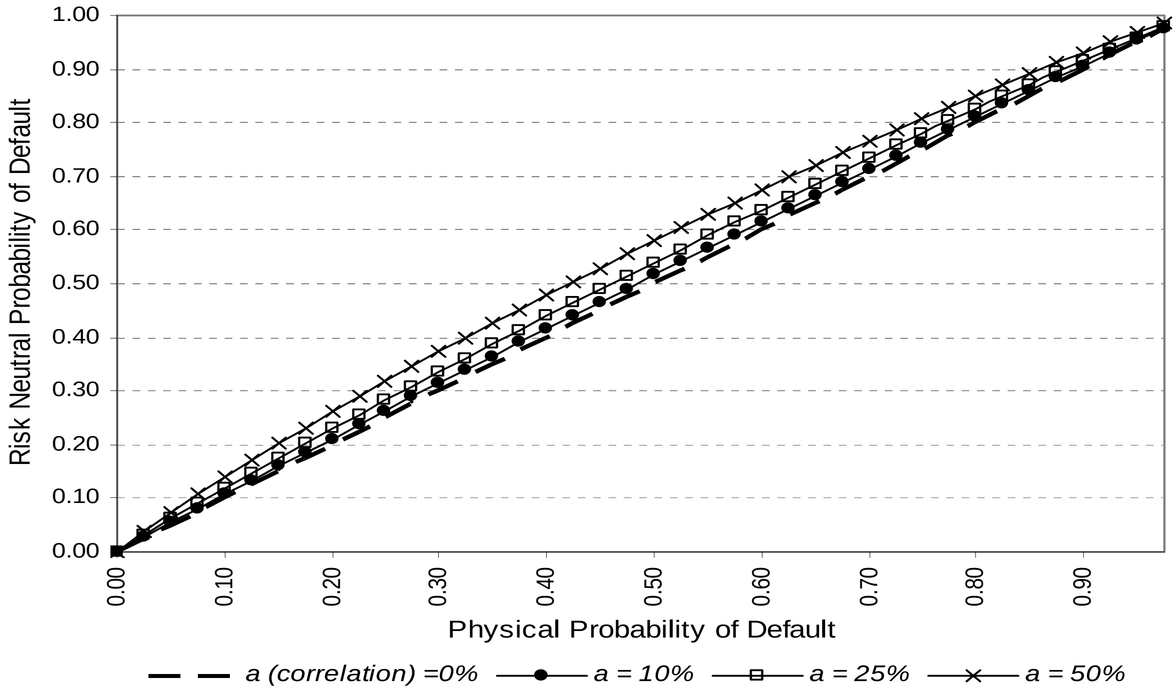

Figure 2: The Impact of an Increase in Correlation on Risk Neutral Default Probabilities

This figure shows the risk neutral probabilities of default corresponding to the physical probabilities of default for different correlation levels (*a*). Market price of risk is assumed to be 40 percent. When correlation is 0 percent (i.e. *a* = 0%), risk neutral probability of default is equal to the physical probability of default.