Undergraduate Mathematics Conference in Washington

Abstracts of Talks

April 21-22, 2012

Title: A Model for the Population of the Blue Crab in the Chesapeake Bay
Speaker: Timothy Becker, College of William and Mary
Faculty Advisor: Junping Shi

Abstract
We model the population of the Blue Crab in the Chesapeake Bay by using differential equations. Blue crabs are inherently cannibalistic of juveniles, while also in competition with juvenile blue crabs for resources. These differential equations describe the intraguild predation consistent in the blue crab food web, as well as the cannibalistic nature of the blue crab. We introduce an aging and birth rate to alter an intraguild predation model to fit the cannibalistic nature.

Title: Synchronous Oscillatory Solutions in a Two Patch Predator-Prey Model.
Speaker: Matthew Becker, College of William and Mary
Faculty Advisor: Junping Shi

Abstract
A coupled two-patch Rosenzweig-MacArthur system of predator-prey interaction is investigated. Synchronization is the process in which oscillations attain the same frequency based on their coupling. The Moran Effect, the name for the seemingly random changes seen in nature, has been attributed as a cause of synchrony, while scholars have studied other influences to ascertain causal relationships between any of them and a synchronous solution. We use a correlation function similar to Pearson's Correlation, one of the most well known correlation measures in statstical analysis, to measure the synchrony and consider what factors have greater influences upon a synchronous oscillatory solution.

Title: Mathematical Modeling and Analysis of a Non-linear Large Deformation Plate Model with Applications to Micro-Air Vehicles
Speaker: James Cameron, George Mason University
Faculty Advisor: Padmanabhan Seshaiyer

Abstract
In this work we consider the development of a computational methodology to study stability and nonlinear dynamics of large deformation plate models. The main application will be the computational modeling of flexible wing designs for Micro Air Vehicles. Using a geometrically

nonlinear Green strain-displacement formulation, a materially linear constitutive stress-strain formulation, and a Hamiltonian energy approach, we develop the governing differential equations for the axial and transverse displacements of the plate. We also develop an appropriate energy norm for a class of boundary conditions and prove a stability estimate. The model developed will also be numerically validated for benchmark applications.

Title: Effects of Non-Independent Behavior on a Macroeconomic Model
Speaker: Nicholas Chaung, George Mason University
Faculty Advisor: Harbir Lamba

Abstract
The standard macroeconomic models in use by numerous banking institutions today, Dynamic Stochastic General Equilibrium (DSGE) models, attempt to calculate macroeconomic variables based on microeconomic principles. By assumption, the agents in these models behave independently, which causes the calculated values for the macroeconomic variables to have near-normal distributions. However, in real economies, very large fluctuations occur far more frequently than a normal distribution would predict and the frequency distributions of macroeconomic variables show fat tails. A plausible explanation for these fat tails is that agents do not always act independently. We incorporate herding, one such non-independent behavior, and observe its effect upon the simulations.

Title: Global Dynamics of Pulse-Coupled Oscillators
Speaker: Allison Corish, College of William and Mary
Faculty Advisors: Sarah Day, M. Drew LaMar

Abstract
Networks of pulse-coupled oscillators can be used to model systems from firing neurons to blinking fireflies. Many past studies have focused on numerical simulations and locating the synchronous state of such systems. In this project, we construct a Poincare map for a system of three pulse-coupled oscillators and use rigorous computational techniques and topological tools to study asynchronous dynamics. We will present sample results, focusing on periodic behavior in the system.

Title: Stability Analysis of Non-Linear Plate Deformations.
Speaker: Charles Daly, George Mason University
Faculty Advisor: Dr. Padmanabhan Seshaiyer

Abstract
An analysis of the governing equations of motions for a non-linear plate deformation derived from Kirchhoff-Love plate theory. In this analysis we try to guarantee the stability of a non-linear plate deformation under certain conditions.

Title: Mathematical Definition Changing Mathematical Practice: A Case Study of Polyhedra
Speaker: Landon Elkind, The George Washington University
Faculty Advisor: Dr. Lowell Abrams

Abstract
Intuitively, a polyhedron is a geometrical figure in three-dimensional space that is both regular and bounded by line segments, called edges. The Platonic solids are also convex. However, these properties do not seem to be necessary: polyhedra may be non-convex, semi-regular, and even infinitely large. The variety here is surprising. How is it that all such shapes are denoted by the term 'polyhedra'? The answer lies in the definition of a polyhedron. I have been awarded a summer fellowship to study how the definition changed over time, and what exactly was captured by the definition in each of its major formulations. I will show by example the diversity of polyhedra, and then give a brief outline of the history of these mathematical objects.

Title: Modeling the Impact of Flooding on Cholera in Haiti, 2010-present
Speaker: Callie Freitag, The George Washington University
Faculty Advisors: Svetlana Roudenko and Stephen Tennenbaum

Abstract
We propose a model with two infective classes and a bacterial reservoir in order to model the cholera epidemic in Haiti. Our model builds on previous research but also incorporates current data and in order to provide updated predictions regarding endemic cholera in Haiti.
Although we suspect flooding exacerbates the spread of waterborne cholera, little research has been done on the topic. We seek to model the impact of flood events on the epidemic by introducing oscillating terms for the contact rate (α) and contamination rate (ξ). To determine the timing and magnitude of the effect of flooding, we analyze weekly cholera hospitalizations and cholera-related deaths versus weekly rainfall, by region. Once we have determined the parameters of the effect, we will insert the oscillating terms into our mathematical model and simulate the course of the epidemic through Haiti's two rainy seasons, one beginning in May and the second in late October. We will additionally simulate the effect of potential intervention strategies, including vaccination, on the magnitude of the epidemic.

Title: A Differential Equations Analysis of Pandemic Disease Spread in an Apocalyptic State
Speaker: Robert Grell, American University
Faculty Advisor: Dr. Stephen D. Casey

Abstract
The biological threat of a pandemic disease eruption has become all too real in the current state of world affairs. Although adorned by Hollywood tycoons, the threatening danger of biochemical disease warfare is a reality. If a weaponized virulent warhead containing a highly infectious contagion were to detonate on American soil, what are the chances of controlling the disease? Depending on the infectious range used of diseased individuals, as well as the radius and population density of the initial disease outbreak, we can model different scenarios that adjust these density parameters using differential equations. This will enable us to more accurately create a plan in which survival and suppression is most effective. In population-dense areas such as the District of Columbia, can a highly transmissive virus even be contained? If the contagion is designed to create a highly aggressive host, survival may not be possible. At this point, if the contagion has no known suppressor or cure, in order to save the most lives, the only possibility may be to isolate and eliminate everything within the infected radius.

Title: A Population Activity Model of Cortico-Striatal Circuitry Underlying Behavioral Inhibition in Rats.

Speaker: Esther Jackson, George Mason University
Faculty Advisor: Dr. Timothy Sauer

Abstract

Impulsivity, often characterized as a failure of inhibition, critically contributes to the symptoms of several psychological disorders, including attention deficit hyperactivity disorder and drug use. Failure of inhibition can be assessed with tasks that require the ability to stop an initiated response, such as the stop signal reaction time task. Numerous studies have implicated the basal ganglia in action selection and motor control. We constructed a neural network modeling framework that can assess the specific neurobiological mechanisms subserving impulsivity, consistent with known anatomy and physiology of the basal ganglia. A minimal formulation includes six groups of nuclei: the cortex, striatal D1 receptor expressing neurons, striatal D2 receptor expressing neurons, the globus pallidus, the subthalamic nucleus (STN), and the substantia nigra reticulum (SNr). A stochastic Wilson-Cowan-type system of nonlinear differential equations depicts the spiking and synaptic activity of these six neural populations. In our additive network, we considered a cortical drive that contains Gaussian white noise terms, which functions as the source of our stochasticity. Three different channels represent subpopulations of neurons corresponding to actions that may be selected, as determined by suppression of SNr activity. Parameter studies are performed by analyzing model predictions of action selection probability and latency of response, and correlated with performance in measures of behavioral inhibition, such as the stop signal reaction time task.

Title: A Population Activity Model of Cortico-Striatal Circuitry Underlying Behavioral Inhibition in Rats.

Speaker: Kiah Hardcastle, College of William and Mary
Faculty Advisor: Gregory D. Smith, Joshua A. Burk

Abstract

Impulsivity, often characterized as a failure of inhibition, critically contributes to the symptoms of several psychological disorders, including attention deficit hyperactivity disorder and drug use. Failure of inhibition can be assessed with tasks that require the ability to stop an initiated response, such as the stop signal reaction time task. Numerous studies have implicated the basal ganglia in action selection and motor control. We constructed a neural network modeling framework that can assess the specific neurobiological mechanisms subserving impulsivity, consistent with known anatomy and physiology of the basal ganglia. A minimal formulation includes six groups of nuclei: the cortex, striatal D1 receptor expressing neurons, striatal D2 receptor expressing neurons, the globus pallidus, the subthalamic nucleus (STN), and the substantia nigra reticulum (SNr). A stochastic Wilson-Cowan-type system of nonlinear differential equations depicts the spiking and synaptic activity of these six neural populations. In our additive network, we considered a cortical drive that contains Gaussian white noise terms, which functions as the source of our stochasticity. Three different channels represent subpopulations of neurons corresponding to actions that may be selected, as determined by suppression of SNr activity. Parameter studies are performed by analyzing model predictions of

action selection probability and latency of response, and correlated with performance in measures of behavioral inhibition, such as the stop signal reaction time task.

Title: The Laplacian on Isotropic Graphs: Solutions and Applications
Speaker: Patrick King, College of William and Mary
Faculty Advisor: Junping Shi

Abstract
The quantum graph has been demonstrated to be widely applicable in the field of mathematical physics. The Laplacian in particular has use everywhere from quantum mechanics to wave phenomena. Restricting our study to several classes of isotropic quantum graphs, where every edge is the same length, we present complete solutions of several of these classes of graphs.

Title: Indications: A Matching-Like Structure on Directed Graphs
Speaker: Allison Oldham, College of William and Mary
Faculty Advisor: Professor Rex Kincaid

Abstract
A matching, as dened on an undirected graph, is a collection of non-adjacent edges. Adjacency and, as a result matching, does not easily extend to directed graphs. However, following Liu et al. (2011)we dene an indication, a matching-like collection of directed edges. Further, we outline a method for nding a maximum cardinality indication in a directed graph. We then explore several properties of indications as well as some of the interactions between these properties. Finally, we discuss an analogue of the Marriage Problem on a directed graph whose solution is a complete indication. Algorithms and proofs for the results rely heavily on converting a directed graph into an undirected bipartite graph.

Title: Searching for the Implied Market Utility Function
Speaker: Aniket Panjwani, George Mason University
Faculty Advisor: Tim Sauer

Abstract
Economists model peoples' attitudes towards financial assets using utility functions; an asset that gives higher `expected utility' is more desirable. In this research, we modify the parametric time series estimation procedure in Detlefsen (2007) to acquire estimates of the market's utility function. We then implement our estimation strategy with various time periods and data sets. The estimates give us a better understanding of the market's attitudes towards risk and investors' actions in financial markets.
Title: Optimizing Plasmonic Effects for a More Efficient Nanoscale Biophotovoltaic Device
Speaker: Jason Pina, The George Washington University
Faculty Advisor: Igor Griva

Abstract
Biophotovoltaics utilize biological components from photosynthetic systems for the normal charge separation process in a photovoltaic device. Such devices hold great promise as they are highly efficient in their biological environment at converting photons into free charge. However

when utilized in artificial photovoltaic devices, their efficiency is quite low, even when compared with conventional solar cells. One way to increase their efficiency is to use plasmonic effects to augment the intensity of incident light in the charge separation area. Plasmonic effects occur when electromagnetic waves at a conductor-dielectric interface causes the oscillation of free elections relative to the atomic lattice, allowing the electrons to act as a plasma. In our current computational study, we are investigating the use of plasmonic effects with silver and gold to optimize the delivery of light to a film of bacterial photosynthetic reaction centers in an attempt to increase the efficiency of a biophotovoltaic nanodevice.

Title: Minimizing Surface Area of Bubbles and Bubble Films
Speaker: Charles Rong, Winston Churchill High School
Faculty Advisor: Simon Cox

Abstract
Bubbles, besides being fun to play with, have many interesting physical and mathematical properties. They will adopt the shape of minimal energy, and therefore minimal surface area for a given volume. A well known open problem in mathematics is to find the most effective shapes given n regions to enclose.

In this talk, we look at 2-dimensional analog of the above problem. In particular, we study efficient ways to divide a rectangle into three equal areas. We provide an estimation for rectangles of various shapes. This project was motivated by a question from Dr. Simon Cox of Aberystwyth University.

Title: Dehydration of a soft contact lens
Speaker: Mihail Sharov, George Mason University
Faculty Advisor: Daniel Anderson

Abstract
Soft contact lenses are made of water-permeable polymer and have an initial water content. Upon insertion on the eye, the lens is positioned between two tear-film layers. The pre-lens tear film (PrLTF) covers the lens, and the post-lens tear film (PoLTF) is between the eye and lens. The rate of evaporation of the PrLTF and the rate of dehydration of the lens depend significantly on the wind speed and the relative humidity of the environment. The rate of evaporation/dehydration is proportional to the wind speed and inversely proportional to the relative humidity. I will adopt a mathematical model created by Francesco Fornasiero and make some modifications in order to mimic more realistic blinking conditions.

Title: Mean-reversion pricing model with hysteresis and positive feed-back
Speaker: Jody T Shipp, George Mason University
Faculty Advisor: Dr. Harbir Lamba

Abstract
We examine changes to a standard pricing model used in finance and economics. This model is a mean-reverting (Ornstein-Uhlenbeck) stochastic process where fluctuations in supply and

demand occur but a drift pushes the price back towards a mean value, giving rise to a Gaussian price distribution.

Hysteresis refers to memory-dependent or non-reversible effects. For example, an agent may switch their investment position due to a price change but, in the presence of non-zero transaction costs, they will not immediately switch back if the price change reverses. We add hysteretic economic agents to an Ornstein-Uhlenbeck process and numerically simulate the system using the Euler-Maruyama method. We then compare the statistics of Ornstein-Uhlenbeck processes with and without hysteresis-type effects.

Title: Partitioning Sets of Permutations
Speaker: Jeff Soosiah, College of William and Mary
Faculty Advisor: Gexin Yu

Abstract
Given a set of permutation matrices, is it possible to partition the set into equal-size subsets so that each have equal matrix sum? If so, we say the set admits a perfect partition. We examine various types of these sets, and in particular, show that there does exist a perfect partition for the set of 6 x 6 permutation matrices which forbid 1s on the main diagonal.

Title: An Analytical Approach to Green Oxidation
Speaker: Diego Torrejon, George Mason University
Faculty Advisor: Dr. Maria Emelianenko

Abstract
Oxidation, a process in which oxygen is added to break pollutants or organic wastes, is important in many industries. However, this process often uses chemicals that can result in the production of hazardous substances, so it is imperative to be able to control the process to make it environmentally safe.

In this talk, we study the problem of suicidal inactivation of enzymes and man-made oxidation catalysts. Based on experimental data obtained from our colleagues at Carnegie Mellon University, we formulate a system of differential equations that models chemical reactions and analyze its numerical and analytical properties. The main goal is to be able to estimate the rates of the reactions based on limited experimental observations. The nonlinear 3-dimensional ODE system under investigation does not allow for an exact solution. However, noticing its similarity with Michaelis-Menten system, we have been able to develop quasi-state approximation of the model that together with perturbation techniques has allowed us to derive approximate solution matching experimental observations to a high degree of accuracy. Analytical results developed using this approach, generalized upon previously known relations between the rate constants, allow for a much deeper understanding and control of the oxidation processes. It is expected that these novel techniques will yield even more far-reaching results when applied to more realistic systems taking into account complex interactions between the catalysts.

Title: Permutations with Extremal Routings on Cycles
Speaker: Alex Valentin, College of William and Mary

Faculty Advisor: Gexin Yu

Abstract
Let G be a graph on n vertices, labeled $v_1,\ldots,v_n$. Suppose that on each vertex there is a pebble, p_j which has a destination of v_j. During each step, a disjoint set of edges is selected and the pebbles on an edge are swapped. The routing problem, $rt(G, \pi)$, asks what the minimum number of steps necessary for any permutation of the pebbles to be routed so that for each pebble, p_i is on v_i.

Li, Lu, and Yang prove that the routing number of a cycle of n vertices is equal to $n-1$. They conjecture that for $n \geq 5$, if $rt(C_n, \pi) = n-1$, then $\pi = (123\cdots n)$ or its inverse. They show that the conjecture holds true for values of n less than 8. We prove here that the conjecture holds for all even n.

Title: Symbolic ARMA Model Analysis
Speaker: Keith Webb, College of William and Mary
Faculty Advisor: Larry Leemis

Abstract
ARMA models provide a parsimonious and flexible mechanism for modeling the evolution of a time series. Some useful measures of these models (e.g., the autocorrelation function or the spectral density function) are tedious to compute by hand. We use a computer algebra system, not simulation, to calculate measures of interest associated with ARMA models.