Organizational Capital, Technology Adoption and the Productivity Slowdown: Technical Appendix*

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Abstract

This appendix contains proofs and other auxiliary results.

A Learning and Signalling

This section outlines the signalling interpretation of the learning curve, similar to Jovanovic and Nyarko (1996). Each period there is a continuum of managerial inputs ("decisions") $j \in [0,1]$. The productive process is characterized by a value $\overline{\theta}_j \in \Re$ for each decision dimension j, that is drawn from a normal distribution with known mean and standard deviation σ^2. Let $\overline{\theta} : [0,1] \to \Re$ be the collection of $\overline{\theta}_j$.

For each j, for each date a, establishments must make a decision $q_{ja} \in \Re$ that is an input into the production process. Let θ_{ja} represent the optimal choice, which is specific to $j \in [0,1]$ and also to the period a. The exact value of θ_{ja} is, however, unknown until after the decision has been made. Let $\overline{\theta}_j$ be the (unknown) mean value of θ_{ja}. Then,

$$\begin{aligned} \theta_{ja} &= \overline{\theta}_j + u_{ja},\ u_{ja}\tilde{}N(0,\sigma_u) \\ \rho(u_{ja},u_{js}) &= 0, s \neq a,\ \rho(u_{ja},u_{kt}) = 0, k \neq j \end{aligned} \tag{1}$$

There is a payoff function $\pi(q_{ja},\theta_{ja})$ that depends on both the target and the assay:

$$\pi(q_{ja},\theta_{ja}) = \left(1 - (\theta_{ja} - q_{ja})^2\right)$$

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The plant learns from observing the ex-post realizations of θ_{ja} over time. Given the assumption of normal disturbances, at any given date, the information set relevant for dimension j can be represented as the prior variance over $\overline{\theta}_j$, denoted σ^2_{ja}.

The establishment is risk-neutral, so it will always select $q_{ja} = E[\theta_{ja}|\sigma^2_{ja}] = E\left[\overline{\theta}_j|\sigma^2_{ja}\right]$. Then, the realized payoff π^* becomes

$$\pi^*(q_{ja}, \theta_{ja}) = \left(1 - (\theta_{ja} - E\left[\overline{\theta}_j|\sigma^2_{ja}\right] + u_{ja})^2\right)$$

If at time t a plant's prior over $\overline{\theta}_j$ is normal with variance σ^2_{ja}, then after observing θ_{ja} Bayes' rule yields *posterior* variance $\psi(\sigma^2_{ja}) \equiv \frac{\sigma^2_u \sigma^2_{ja}}{(\sigma^2_u + \sigma^2_{ja})}$. If there is no disruption, in that $\overline{\theta}_j$ remains the same, the information set next period $\sigma^2_{j,a+1}$ is then characterized by $\sigma^2_{j,a+1} = \psi(\sigma^2_{ja})$. At the moment that an establishment is born, it is characterized by an information set $\sigma^2_{j0} = \{\sigma^2\}\, \forall j$, where σ^2 is the unconditional variance of $\overline{\theta}_j$.

Let $\Omega(a) \equiv \int_{[0,1]} \pi(q_{ja}, \theta_{ja}) dj$ be the aggregation over decisions. The reduced-form learning function is then the solution to the following difference equation:

$$\begin{aligned} \sigma^2_{j0} &= \sigma^2,\, \sigma^2_{j,a+1} = \psi(\sigma^2_{ja}) \forall a \geq 1 \\ \Omega(a) &= E\left[\pi(q_{ja}, \theta_{ja})|\sigma^2_{ja}\right] = 1 - (\sigma^2_{ja} + \sigma^2_u) \end{aligned}$$

This formulation of OC is open to any learning-based interpretation of the notion of OC, so long as the object of learning is the strategy space per se: the "how" rather than the "what". Consider an assignment problem between capital and workers with idiosyncracies. Productivity then depends on signals that managers receive regarding match quality. Since the establishment manages a continuum of inputs, the space of possible assignment strategies has the same cardinality as the real line, and as the set of functions $\overline{\theta} : [0,1] \rightarrow \Re$. Thus I model the information structure in the manner of equation (1), applying the appropriate homeomorphism and concentrating on the properties of $\overline{\theta}$ and θ_a. I interpret this as a "reduced form" approach. As for the disruptive shock, an incumbent's information set in case of updating returns to $\sigma^2_{j0} \forall j$.

B Proofs

Proof of Proposition 1. That W exists, is decreasing and convex in τ and increasing and concave in a is immediate from standard recursive methods and the properties of P. That W is decreasing in τ and that U does not depend on τ implies that Υ is increasing in τ. Finally, increasing a increases $W(v, a, X_t)$ more than it does $W(\tau, a, X_t)$ for any $\tau > v > 0$, so that $W(\tau + 1, a, X_t) - U(a, X_t)$ is decreasing in a. Hence, Υ is increasing in a. These two results prove that the policy is (S, s). That all updating is to the frontier stems from the fact that updating costs are linear in productivity: see Samaniego (2002). ■

Proof of Proposition 2. Given τ, Υ is increasing in a. Given a, Υ is increasing in τ. Hence, the updating frontier must be decreasing and convex in (τ, a) −space – in other words, updating lags are shorter for older firms. ■

Proof of Proposition 3**.** Let a_{t^*} be the age of a plant at the time of the shock, and s the years since t^*. If it has yet to update, a plant's deflated payoff $\widetilde{W}$ is

$$\begin{aligned} \widetilde{W}(T, X_{t^*+s}; a_{t^*}) &= \max_{k_t,n_t} \left\{\gamma^{-s}\Omega(a_{t^*}+s) f\left(k_t, n_t\right) - k_t r(X_{t^*+s}) - n_t w(X_{t^*+s})\right\} \\ &\quad +\eta \max\left\{\widetilde{W}\left(s+1, X_{t^*+s+1}; a_{t^*}\right), W\left(0, 0, X_{t^*+s+1}\right) - \kappa p\left(X_{t^*+s+1}\right)\right\} \\ X_{t^*+s+1} &= \widetilde{\Gamma}(X_{t^*+s}) \end{aligned}$$

which is increasing in a. If establishments update, they must go back to $a = 0$. However, prices are fixed so their continuation value has not changed. ■

Proof of Proposition 4**.** If prices are fixed, then the value of an incumbent does not change. On the other hand, the value of all other types is reduced because the continuation value imposes reversion to $W\left(0, 0, X_{t^*+s+1}\right)$ in case of updating. ■

Proof of Proposition 5**.** Note that $\widetilde{W}$ is the fixed point $f = Bf$ of the functional

$$\begin{aligned} Bf(s, X_{t^*+s}; a_{t^*}) &= \max_{k_t,n_t} \left\{\gamma^{-s}\Omega(a_{t^*}+s) f\left(k_t, n_t\right) - k_t r(X_{t^*+s}) - n_t w(X_{t^*+s})\right\} \\ &\quad +\eta \max\left\{f\left(s+1, X_{t^*+s+1}; a_{t^*}\right), W\left(0, 0, X_{t^*+s+1}\right) - \kappa p\left(X_{t^*+s+1}\right)\right\} \end{aligned}$$

where B is the Bellman operator, and which can be shown to exist via Blackwell's theorem. Given s and t, if f is increasing in a_{t^*} then so is Bf, showing that $\widetilde{W}$ must be: the opportunity cost of updating is increasing with age – whereas $W\left(0, 0, X_{t^*+s+1}\right) - \kappa p\left(X_{t^*+s+1}\right)$ does not. Thus if plants of age a update in a given period, then younger ones would also. ■

C Equilibrium and Simulation

Some care is required regarding the definition of equilibrium. Let μ_t be the measure of plant types, and let Γ be its law of motion. Γ satisfies

$$
\begin{aligned}
\mu_{t+1}\left(T\times A\right) &= \int_{a+1\in A}\int_{\tau+1\in T}\left(1-\Upsilon\left(\tau,a,X_t\right)\right)\left(1-\lambda\left(a\right)\right)d\mu_t\left(\tau,a\right)\\
&\quad+\int_{0\in A}\int_{\tau+1\in T}\Upsilon\left(\tau,a,X_t\right)\left(1-\lambda\left(a\right)\right)d\mu_t\left(\tau,a\right)\\
&\quad+I\left(0\in A\wedge 0\in T\right)\phi\left(e_t\right)
\end{aligned}
$$

Due to the fact that the type space is discrete, and that the updating rule Υ is not in general continuous, labor demand will not be continuous either. Hence, for the definition of equilibrium, I extend the model in a way that convexifies the function space in which the model resides, and regard the model as an approximation to this extension.

Suppose that, in addition to a and τ, plants also vary in terms of idiosyncratic productivity $z\in\Re^{+}$. Thus, a plant's output is

$$
y_t=\gamma_n^t\gamma_s^{t-\tau}z_t\Omega\left(a\right)k_t^{\alpha_k}n_t^{\alpha_n}
$$

z is exogenous, and when plants are born their value of z is drawn from a distribution $\psi\left(z\right)$.

In this case, Γ must satisfy

$$
\begin{aligned}
\mu_{t+1}\left(A\times T\times Z\right) &= \int_{a+1\in A}\int_{\tau+1\in T}\int_{Z}\left(1-\Upsilon\left(\tau,a,z,X_t\right)\right)\left(1-\lambda\left(a\right)\right)d\mu_t\left(\tau,a,z\right)\\
&\quad+\int_{0\in A}\int_{\tau+1\in T}\int_{Z}\Upsilon\left(\tau,a,z,X_t\right)\left(1-\lambda\left(a\right)\right)d\mu_t\left(\tau,a,z\right)\\
&\quad+I\left(0\in A\wedge 0\in T\right)\phi\left(e_t\right)\psi\left(Z\right)
\end{aligned}
$$

I interpret the behavior of the model economy as corresponding to that of the extended model for the case in which ψ is highly concentrated around $z=1$.

Let time $t=1973$ be the period of shock impact: agents wake up in 1973 and find out that their non-tradeable OC will become obsolete if they update. μ_{1973}, E_{1973} and k_{1973} are already given. If the economy eventually returns to its original state, there should be a date $\widehat{T}$ such that $\mu_{\widehat{T}}$, $E_{\widehat{T}}$ and $K_{\widehat{T}}$ deviate negligibly from their steady state values in their respective spaces. $W(.,.,X_{\widehat{T}})$ will also deviate negligibly from its steady state value, as will decision rules and consequently p,r,w and Γ. I set $\widehat{T}=2073$.

Given a guess for the wage stream for $t\in\left\{1973,...,\widehat{T}\right\}$, the rental rate stream[1], value functions for all periods and decision rules can be computed via backward recursion parting from $W(.,.,X_{\widehat{T}})$ and using the first order conditions. These decision rules in turn allow the measure to be computed recursively *forwards* from μ_0. Imposing the law of motion for aggregate capital that is implied by the plant decision rules, household consumption and

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[1] The value of r_0 is set to equate demand and supply of capital in the period after the shock.

labor supply rules are derived. A new candidate wage stream is generated until the labor market clears in all periods.

D Robustness: Delayed Announcement

I extend the model so that there is a delay between shock *announcement* and the shock itself. The dynamics of this model are similar, except that the collapse in investment is much stronger. The resulting change in the average age of capital can account for the data entirely – Figure (1) displays model dynamics for an 8-year pre-announcement but the same is true for other delays. Startup values and entry increase even $before$ the shock, however, because once the shock occurs young pre-shock incumbents are better off than their older counterparts since they have less to lose – as per Proposition 3.

The main difference is that, after the announcement, updating is synchronized across plants, as it is most profitable to update just before the shock (and hence 4 or 5 years before that date also). Investment drops to 40% below trend just before the shock, but the capital age dynamics are mainly due to earlier changes. This deep a trough would be absent if the date of the shock were unknown but likely in any particular period with a certain probability. I do not compute such a model because it would involve implicitly solving for paths at all possible dates of shock impact simultaneously.

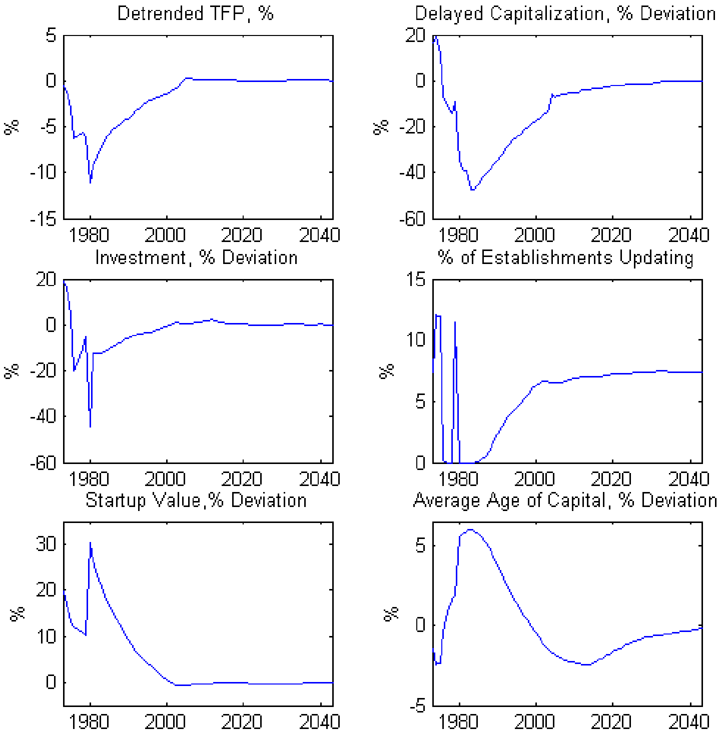

Figure 1: Dynamics with Delayed Announcement