

# Between Alpha and Beta: Modeling Luck, Talent and Size in the Hedge Fund Industry\*

PRELIMINARY, COMMENTS WELCOME

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## Abstract

Widely emphasized features of the hedge fund industry include the limited supply of manager “talent” and the inability of managers to operate funds of unlimited size. We jointly estimate the impact of size on the returns to hedge funds, and characterize the distribution of hedge fund manager “talent”. We find that a 10 percent increase in hedge fund size leads to a decrease in returns of over 20 basis points. The distribution of manager talent appears not to change noticeably among cohorts – suggesting that it is a scarce resource – and both talent and luck have similar contributions to cross-sectional variation in hedge fund returns.

Given these findings, we develop a theoretical model of the hedge fund industry which corresponds closely to the empirical specification, to study the response of the industry to changes in regulation. We find that an increase in the cost of entry can cause a large reduction in the number of active hedge funds while leaving industry profits and industry assets under management relatively unaffected. By contrast, modest decreases in leverage ratios or increases in the cost of leverage have little influence on the number of funds, but decrease hedge fund assets and profits significantly. The model proposes a resolution of the tension between the widespread belief in the importance of talent in hedge funds versus the difficulty of empirically identifying systematic differences in hedge fund returns.

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*In poker, luck can dominate in the short term, but overall, skill is the dominant factor. Poker is, without question, a game of skill.*

Anthony Curtis, Editor, "The Las Vegas Advisor."  
Quoted in "Poker: luck or skill?" *Los Angeles Times*, July 11, 2007

## I. INTRODUCTION

The hedge fund industry has received a good deal of interest, because it is thought to have characteristics different from those of the rest of the financial sector and because increased regulation of hedge funds has become a prominent topic of current policy discussion. Understanding the likely impact of increased regulation on the hedge fund industry requires, implicitly or explicitly, a theoretical model with particular attention to the special features that might distinguish hedge funds from many other asset classes.

Examples of these features are as follows. First, the literature often emphasizes the importance of "alpha" – fund-specific returns that cannot be accounted for by industry-wide factors, often identified with a combination of manager talent (or the quality of the fund's strategy) and with luck. Second, hedge funds often pursue investment strategies that are based on exploiting arbitrage opportunities, liquidity advantages, and other market imperfections. As a result, their investments are unlikely to be *scalable*: the returns to a particular trading strategy may decline with the size of the investment position, as argued by Goetzmann, Ingersoll and Ross (2003), Getmansky (2004), Ammann and Moerth (2005), Jones (2007) and Lo (2008), among others. Sure enough, hedge funds typically close to new investments after raising a certain amount of funds, a feature that is often interpreted as indicating the existence of an "optimal size" for a given hedge fund – a natural outcome in an environment with decreasing returns to scale.

These two features are closely linked. Scarcity of manager talent is of no consequence in a world of constant returns, since all resources could simply be allocated towards the best manager. Conversely, decreasing returns need not limit returns if the pool of manager talent is unlimited, as returns could be raised by spreading given resources more thinly over a larger number of managers. Thus, any attempt to econometrically or theoretically model one of these features requires modeling the other.

This paper develops an empirical model that decomposes alpha into components due to size, talent, and luck. Using data on hedge fund assets under management (AUM) and returns over the period 1994-2005, we estimate the sensitivity of returns to hedge fund size, and characterize the empirical distribution of hedge fund manager talent. Then, we develop a closely related theoretical model in which hedge fund size is endogenous. The model features a set of potential fund managers, but which of those managers operate hedge funds in equilibrium is endogenous. Finally, we apply the model to assess the sensitivity of the hedge fund industry to certain changes in the regulatory or financial environment: an increase in entry costs, an increase in the cost of leverage, and a decrease in leverage ratios. These are among some of the regulatory changes raised in the ongoing policy debate.

We find robust evidence that, controlling for managerial talent, larger funds tend to underperform smaller funds. This is true when we identify “alpha” using the Fung and Hsieh (2001, 2004) hedge fund return factors, and also when accounting for any other unobserved sources of variation in returns across styles and across time using fixed effects. Pooling all hedge funds in the database, a 10 percent increase in assets under management (AUM) is related to a decrease in returns of over 20 basis points. This decrease varies in magnitude across hedge fund styles, rising to almost 3 times as much for Emerging Markets hedge funds.

We estimate manager talent for each hedge fund using fund-level fixed effects. The resulting distribution of hedge fund talent displays fat tails, as does the distribution of the random component of returns (“luck”). These findings are consistent with leptokurtosis in returns, as found by Ackermann et al (1999). We find that luck displays very little persistence. Interestingly, the distribution of entrant talent does not appear to change noticeably over time, suggesting inflows from a stable pool of manager talent.

These features – decreasing returns to size and a stable pool of potential entrants – are the basis of our theoretical model of the hedge fund industry. In the model, entrants differ in terms of their talent (the quality of their manager or their strategy), and after entry receive stochastic shocks to their returns. At any date they may choose to leave the industry, or to raise funds at a cost and invest them in a decreasing-returns investment strategy. The equilibrium *industry* size is determined by the presence of negative externalities across funds. This feature captures the notion that hedge funds may engage in similar strategies and hold similar positions so that increased investments in the industry (or in a given style) may lower the returns of other funds in the industry or style. That there is such “congestion” in the hedge fund industry is widely believed anecdotally and in the trade literature, although the academic literature has not focused on this issue.<sup>1</sup> When the stochastic component of returns is not very persistent, the optimal AUM at any given fund in the model is constant over time, suggesting an explanation for the observation that hedge funds close to new investment after reaching a certain size: the current impact of luck on returns does not indicate that higher returns are any more (or less) likely in the future.

In the model, more talented managers generate higher returns for a given level of AUM. Still, the fact that there are diminishing returns to scale implies that, in equilibrium, there may be no persistent differences in hedge fund returns even when there do exist large and persistent differences in talent. Foster, Stine and Young (2008) and others identify “alpha” with the

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<sup>1</sup> See Getmansky (2004), Khandani and Lo (2007) and Chan et al (2007). A recent example of this phenomenon is the October 28, 2008 spike in the value of Volkswagen shares after numerous hedge funds were forced to unwind large short positions because the value of these positions exceeded the value of available outstanding Volkswagen shares – leading Volkswagen to become (briefly) the largest company in the world by market value.

fund's ability to systematically exceed the market return, but are unable to find robust evidence of alpha thus defined. Our model can account for this negative result: in a world of decreasing returns to investment positions, more talented managers optimally operate larger funds. Thus, the model suggests that our econometric strategy with fund-level fixed effects (that conditions on AUM) may identify aspects of manager talent that are harder to detect when focusing mainly on differences in returns.

Even before the subprime crisis, the regulation of hedge funds was a widely debated topic – due to unease about systemic risk and lack of transparency in the industry. For example, Foster and Young (2008) argue that it is difficult to construct an incentive structure that distinguishes between skilled and unskilled hedge fund managers in the absence of investor knowledge about the strategies being pursued. By contrast, Brown et al (2008a) study the impact on hedge funds and investor behavior of the (subsequently repealed) requirement that all hedge fund managers register as investment advisors with the United States Securities and Exchange Commission. A concern among practitioners is that additional regulation such as reporting requirements would increase entry costs of the industry, significantly reducing returns. We use our model to assess the impact on the industry of an increase in entry costs. The model suggests that a significant increase in entry costs might have a large impact on the *number* of hedge funds, yet have little impact on the assets under management and profits in the industry as a whole. The reason is that the increase in operating costs raises the bar on the profitability required for hedge funds to enter and survive: however, this would affect mainly the funds with the lowest manager talent, which comprise only a small fraction of total industry AUM. We also investigate the long term impact on the industry of a significant rise in the cost of leverage, and allowable leverage ratios. We find that changes in these variables can have a very large impact on industry AUM and on total profits – although they do not affect rates of return, because lower profitability is offset by the fact that hedge funds optimally manage smaller portfolios.

To our knowledge, this is the first paper to characterize the distribution of “manager talent” in the hedge fund industry, and to develop an industry model suitable for industry policy analysis. In complementary work, Zhong (2008) also studies fund size and the distribution of “alphas” – where “alpha” is defined as returns to hedge funds that cannot be explained by industry-wide factors – but does not define manager talent *net* of size and of “luck”, and defines size as a flow (the change in AUM) rather than a stock (the level of AUM).

As mentioned, previous work has uncovered evidence of a negative size-return link. However, these findings could be sensitive to certain econometric concerns, and to the definition of the notion of “size” in each study. For example, Ammann and Moerth (2005) and Jones (2007) sort funds into size bins, without conditioning on style factors. Consequently, their results could potentially be capturing differences in returns across styles with different typical fund sizes rather than a size effect per se. Furthermore, if size and manager talent are correlated, then studies that do not distinguish econometrically between manager effects and size may lack an important set of missing variables. In addition, since size is measured in terms of assets under

management, the appropriate measure of size may vary over time – due to inflation, but also due to other factors, such as the expansion or contraction of available investment opportunities due to changes in the underlying asset markets in which hedge funds invest. An additional concern is raised by Getmansky, Lo and Makarov (2004), who document the existence of serial correlation in asset returns. Serial correlation could lead to inconsistent estimates in the absence of appropriate corrections. Using panel econometric methods, we find evidence of a size effect when accounting for all of these considerations.

Our industry model is adapted from the Hopenhayn (1992) framework of firm entry and exit. This framework is employed by Cooley and Quadrini (2001) to assess the impact of financial frictions on firm dynamics: however, we are not aware of studies that use such models of industry dynamics to analyze the financial sector itself, nor the hedge fund industry in particular. As we show, there is a tight link between our empirical specification and the structure of the model that makes it well suited to the current analysis. The Hopenhayn (1992) framework has been applied to study the effect of various forms of regulation, such as firing costs in Hopenhayn and Rogerson (1993) and Veracierto (2008), or entry costs in Samaniego (2009). However, to our knowledge, it has not been used to study the potential sensitivity of the financial sector to changes in the policy environment.

Section II provides a summary of the related literature. Section III describes the estimation procedure and results. Section IV describes the model, and Section V calibrates the model, using it to assess the impact of the costs of additional regulations upon the hedge fund industry. Section VI concludes with suggestions for future work.

## **II. THEORETICAL AND EMPIRICAL BACKGROUND**

We begin with a brief discussion of some distinguishing features of the hedge fund (HF) industry. These include features of the industry that we wish to model in order to understand its structure and its response to policy, as well as features of the industry that affect the appropriate econometric specification. Related discussions may be found in Getmansky (2004), Malkiel and Saha (2005), and Center for International Securities and Derivatives Markets (2006), among other sources.

### **A. Characteristics of Hedge Fund Returns**

Several features of hedge funds distinguish them from more traditional portfolio managers (such as mutual funds). In particular:

- A. HFs are marketed as following a specific “investment style” or “investment strategy” (see Table 1 for a list), and are additionally seen as deriving a significant part of their return from active portfolio management. See Ackermann et al (1999).
- B. HFs are often thought of as being uncorrelated with traditional asset classes (equities, debt, etc.) See Fung and Hsieh (2001, 2004). They are generally free to take long or short positions, to use derivatives, as well as to invest in relatively illiquid assets (e.g.

- real estate). They also face few investment restrictions, and those they do face are often determined by their own internal risk management guidelines and “best practice” norms. As a result they may be thought of as being essentially unrestricted in their investment strategies, as in the model of Ilyina (2007).
- C. HFs are absolute-return driven, i.e., the performance of HF managers is not measured relative to any benchmark (notwithstanding a proliferation of HF indices and index providers in recent years).
  - D. Mindful of their size and the need to stay “lean and mean”, HFs tend to close down to new investors and/or to return part of their earnings whenever they reach certain size. See Ackermann et al (1999). In addition, Malkiel and Saha (2005) find that under-performers tend to disappear fairly quickly from the list of “investable” HFs.
  - E. HFs do not have to disclose their portfolio holdings to regulators, although they have to provide some information on the risk/return profile of their portfolios to end-investors. See Brown et al (2008a).
  - F. There is evidence that hedge fund strategies can overlap, and that a given hedge fund may negatively influence returns at other hedge funds with similar positions. A well known example is the October 28, 2008 rush for Volkswagen equity by hedge funds who had short-sold its assets. See also Khandani and Lo (2007) on profit spillovers across “quant” hedge funds in 2007. Getmansky (2004) argues that returns are lower in fund styles with more intense competition, and Chan et al (2007) argue that large inflows into the hedge fund industry were one cause of deteriorating returns in 2004.

These observations suggest the following “stylized facts” that are important for modeling HF industry dynamics, both econometrically and theoretically:

1. **An important determinant of HF returns could be the fund manager’s “talent”** (based on Observations A and B). Getmansky, Lo and Makarov (2004) document persistence in returns and interpret it as liquidity, but to some extent persistence in returns could also reflect fund-level fixed effects related to “manager talent” or the quality of the fund’s trading strategy.
2. **In contrast with traditional asset management, HF strategies may not be “scalable”** (Observation B). In particular, many HF investment styles focus on exploiting specific arbitrage opportunities or profitable opportunities in less liquid markets (low liquidity can be viewed as a particular source of non-scalability.) Thus, the size of an individual HF (as well as the aggregate capital base of all HFs following the same strategy) is likely to have an impact on HF returns. See for example Ammann and Moerth (2005) and Lo (2008).
3. **A given hedge fund can be thought of as being characterized by an optimal size** (Observation D).
4. **Entry and exit are common, so that survival bias and reporting bias are potential concerns** (Observations D and E).
5. **There are negative externalities across hedge funds** as funds with similar styles may erode each others’ returns (Observation F).

## B. Characteristics of Hedge Fund Industry Dynamics

The above stylized facts relate to the determinants of hedge fund returns. There are a few more facts – regarding industry dynamics – that are also important for any effort to model the hedge fund industry. These concern the factors that lead to the entry and exit of hedge funds.

Regarding exit, there is a broad literature that examines the causes of liquidation in hedge funds, including Getmansky et al (2004) and Liang and Park (2009). The latter paper finds that about a third of exits can be attributed to low fund profitability, whereas two thirds of exits appear to occur for other reasons, including changes in the career concerns of the fund manager, operational risks (including fraud), etc. Thus, at least a portion of exits are due to factors unrelated to profitability. As for the remainder, Getmansky et al (2004) find that profits at hedge funds tend to deteriorate within 12 months before exit, suggesting that low profitability is often linked to a change in the nature of the fund’s strategy that could not likely have been predicted more than a year in advance and that leads to exit.

Perhaps surprisingly, the *entry* of hedge funds is a topic that has received considerably less attention. One fact that stands out in the literature is the fact that the costs to the investor of contracting a hedge fund appear to be very low relative to the profits of a typical hedge fund. For example, Brown et al (2008b) estimate the typical due diligence cost to the investor of contracting a hedge fund to be in the range \$50,000-100,000. Strachman (2007) also reports the cost of starting a hedge fund to be about \$50,000. From the perspective of a fund of funds, Brown et al (2008b) interpret the costs of due diligence as being potentially substantial as funds of funds typically review several hedge funds before selecting one, and they propose a cost ten times as large on the assumption that a fund of funds contracts one in ten reviewed funds. Even so, these costs are far lower than the profits at the average hedge fund. As a result, a typical free entry condition used in industrial organization models of the kind whereby the cost of setting up a hedge fund equals its expected return is unlikely to be appropriate.

Rather, the size of the hedge fund industry appears limited by the availability of *sufficiently skilled fund managers*. Thus, the supply of potential fund managers can be viewed as being largely exogenous, so that the distribution of active fund managers equals the distribution of potential managers *censored* by the cost of entry. Several other observations support this view (and we provide further evidence later). First, the centrality of “alpha” (interpreted as manager talent or as a strategy’s quality) to discussions of hedge fund performance, and the existence of heterogeneity in “alpha”, are itself evidence that manager talent (or strategy quality) is in limited supply. Second, the press, the hedge fund trade literature and hedge fund blogs can be seen to often note that large increases in the creation of hedge funds appear related to the entry of (on average) less talented fund managers.<sup>2</sup>

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<sup>2</sup> See for example “Gunslingers No More: The Cautious Cash In”, New York Times, May 22, 2005, by Nina Munk, and “Are There Too Many Hedge Funds?” Safehaven.com, by John Maudlin, July 09, 2005. See also discussions in Chan et al (2007), Zhong (2008) and Lo (2009).

With these stylized facts in mind, we construct an econometric model of the HF industry. We use panel regression analysis to determine the sensitivity of HF returns to various factors that the literature has identified as being important for style- or for industry-wide returns. Unexplained residuals are defined as the fund's "alpha." In turn, "alpha" is decomposed into effects due to fund size, fund-level fixed effects, and transitory (although possibly serially-correlated) returns. Based on the results of the empirical analysis, we develop a theoretical model of the HF industry. We use the econometric estimates to calibrate the theoretical model, and apply the model to quantify the long-run impact upon the HF industry of exogenous changes in the regulatory environment.

### C. Changes in the Environment of the Hedge Fund Industry

Following the demise of several large macro hedge funds during the EM crises of 1997-98 and the collapse of the global IT bubble in 2000, hedge fund industry growth picked up again, with the size of global hedge fund industry at \$2,300 billion in 2007, compared to around \$400 billion in 1998 – see Maslakovic (2008). This led policy makers to consider new forms of regulation of hedge funds, including limitations on leverage and tighter accounting requirements: see for example European Central Bank (2006). Policy concerns became exacerbated given that it was no longer simply wealthy individuals who were investing in hedge funds, but also institutional investors (pension funds, for example). The bust in the hedge fund industry during the subprime crisis led to further calls for regulation of the industry. Policy suggestions have included increasing transparency requirements, increasing the requirements for starting a hedge fund, and regulating leverage ratios. A countervailing concern is that increased regulation might depress profits in the hedge fund industry, leading many hedge funds to become unviable and hence to exit.

The significant growth of the hedge fund industry over the past decade until about 2007 was to a large extent supported by **increased institutional investor allocations towards 'alternative investments'**, including hedge funds' share in the portfolios of the mature-market institutional investors (such as pension funds, endowments, foundations) at the expense of domestic stocks. This was largely due to three factors: (1) a general dissatisfaction with traditional 'benchmark-based' portfolio management amid poor performance of global markets following the collapse of the IT bubble; (2) the growing asset-liability mismatches of many institutional accounts in mature market countries and hence, the need to pick up yield, (3) aggressive marketing efforts of investment consultants, and (4) low interest rates. Several of these are developments that are reversible. In particular, since mid 2008, there has been a collapse in the willingness to invest in hedge funds and, while it is not clear what part of this phenomenon is permanent and what part is transitory, it is certainly reasonable to suggest that interest rates have been historically low for several years, and that the appetite for risky investments such as hedge funds might abate, as they did with the dramatic rise in LIBOR in late 2008. In addition, some observers have assigned hedge funds an important role in the unfolding of the subprime crisis, leading to further calls for regulation, including increased transparency and limits on leverage.

These facts motivate the policy experiments in which we are interested. First, the possibility of increases in the costs of compliance might increase the “fixed” costs of running a hedge fund. Second, a regime with higher holding costs – due to increased costs of raising leverage as a result of policy or a decreased risk appetite on the part of investors, or due to compliance costs that are connected to the volume of AUM – are also interesting to study. We will also examine leverage restrictions.

### III. EMPIRICAL MODEL

#### A. Basic Structure

In this section, we define an empirical strategy that allows us to identify talent, luck and size in the hedge fund industry.

We begin by defining “alpha” as a hedge fund’s performance *over and above what can be explained by the fund’s “style”*.<sup>3</sup> This follows the definition of Jensen (1968), applied to HFs as in Malkiel and Saha (2005). Then, we decompose alpha into three components: a size effect, a fund-specific fixed effect, and a transitory (though possibly persistent) effect. We refer to the fixed effect as “talent”, and to transitory factors as “luck.” “Size” is defined as assets under management (AUM).

We use the data to explore whether there is a negative relationship between a hedge fund’s return and the size of its AUM (“decreasing returns to scale”.) The idea is that large funds are marginally closer to exhausting available profit opportunities than smaller funds, conditional on other factors. Then, we characterize the distribution of fund-level fixed effects (“talent”), as well as short-lived returns (“luck”). In a later section, we use these results to develop a theoretical model of the hedge fund industry.

#### Estimation of “alpha”

For a given style, the individual fund’s “alpha”,  $\alpha_{it}$ , can be defined as hedge fund returns that cannot be accounted for by style factors. Consider the following return equation:

$$R_{it} = \sum_{k=1}^K \beta_k x_{kt} + \alpha_{it} \quad (1)$$

The index  $i$  represents a hedge fund,  $t$  is the date, and  $k$  is one of  $K$  style factors that have a common impact on funds in the same style. Then,  $x_{kt}$  is the value of style factor  $k$  at date  $t$ , and  $R_{it}$  is the return on assets to fund  $i$  on AUM held at date  $(t-1)$ .

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<sup>3</sup> By the “style” return, we mean the return on a generic “style” portfolio, based on the history of returns of all funds following this style over the entire sample period. This should not be confused with the average return of all funds in the style at any particular date, or the average return of each individual fund over the sample period.

Thus, the “alphas” contain everything about the individual funds’ returns that is not explained by the influence of style factors.<sup>4</sup> Econometrically, identifying alpha amounts to correctly identifying the style factors, or proxies thereof.

### **Decomposing “alpha” into talent, luck and size**

The value of a fund’s “alpha” reflects the manager’s own intrinsic ability or the quality of his/her strategy, but also may be influenced by the fund’s size, as well as the stochastic nature of returns. Success associated with a manager’s intrinsic properties (or the manager’s strategy) is identified with “talent.” Temporary success is “noise” or “luck.” To separate these factors, we decompose the term  $\alpha_{it}$ .

Define  $\mu_i$  as the fixed effect for hedge fund  $i$  (“talent”). Let  $q_{it-1}$  be the AUM for fund  $i$  invested at the end of date  $t-1$ . Then, we can decompose  $\alpha_{it}$  as follows:

$$\alpha_{it} = \mu_i + \theta \log q_{it-1} + \varepsilon_{it} \quad (2)$$

Here,  $\theta$  is the coefficient on individual fund size and variable  $\varepsilon_{it}$  is the transient component of returns, that may potentially be serially correlated.

Substituting (1) into (2), we obtain the following regression equation:

$$R_{it} = \mu_i + \sum_{k=1}^K \beta_k x_{kt} + \theta \log q_{it-1} + \varepsilon_{it} \quad (3)$$

Equation (3) has the structure of a dynamic panel regression, with the individual hedge fund as the unit of observation. The regression includes fixed effects for each fund, the AUM for each fund, and the style factors. To sum up, we are interested in estimating  $\mu_i$ ,  $\theta$  and  $\varepsilon_{it}$ .

We estimate (3) allowing coefficients  $\beta_k$  to vary across styles. We also run several alternative specifications, including allowing the size effect  $\theta$  to vary across styles too.

## **B. Data description**

Hedge fund data are drawn from the Center for International Securities and Derivatives Markets (CISDM) Hedge fund/CTA database. In 2005, the database contains about \$1.2 trillion worth of investments, compared to total industry investments at the time of about \$1.4 trillion.<sup>5</sup>

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<sup>4</sup> It is important to specify what we mean by “alpha” because it has implications for the estimation procedure. In particular, if we adopt definition (1), we should not run separate rolling regressions for each fund, because in that case, we would be measuring the individual fund’s over/underperformance vis-à-vis its own benchmark portfolio, that would also be allowed to change over time (over different rolling windows).

<sup>5</sup> Hedgeweek, “Hedge fund assets up 24 per cent to USD1.89trn in 2006”. Available at [http://www.hedgeweek.com/articles/detail.jsp?content\\_id=57851](http://www.hedgeweek.com/articles/detail.jsp?content_id=57851), last checked 2/13/2009.

We exclude *funds of funds* from most of our analysis. All funds with reported AUM currency not in US\$ are removed, including funds that do not report a currency. All funds without a reported style are also removed. Coverage is erratic until the early 1990s: hence we do not use data before 1994.<sup>6</sup> The resulting data contain 6186 hedge funds. In 2005 this includes 3406 funds, compared to about 6700 for the industry as a whole according to CISDM (2006). Thus, our data cover about half the hedge fund industry.

Defining size as AUM, equation (3) requires that we regress AUM at a given fund on the returns at that fund. However, *contemporaneous* AUM should not be regressed on returns, but should be *lagged*, as contemporaneous AUM may include the returns themselves. Moreover, returns on a given manager's strategy are unlikely to be immediate. Hence, we measure annual AUM as the AUM in January of the corresponding year, and measure the returns as the returns between February and the following January. Many of the style factors are unavailable except at an annual frequency, so this approach also allows us to avoid interpolating style-factor data.

The reason we expect to find a negative effect of size on the returns of individual funds is because the funds become large relative to available profit opportunities (e.g., arbitrage opportunities). In this case,  $q_{i,t}$  should be measured not as total AUM for the fund, but rather as AUM divided by some measure of opportunities.

We adopt four approaches to measuring opportunities. First, we allow the measures to depend on the style. For some, global market capitalization is appropriate – the sum of global equity and bonds. For others, emerging market cap is likely more suitable. For still others, global bond market or equity market cap is more appropriate. See Table 1. Each of these is measured in dollars, deflated using the GDP deflator. Second, we rescaled all funds using global market cap. Third, we rescaled using AUM in each style. Fourth, we used the unadjusted AUM data without rescaling. Results were robust to either approach.

There is a question as to whether the empirical counterpart of  $R_{it}$  should be net return, for example returns over and above the cost of capital. As mentioned earlier, the hedge fund industry is driven by absolute (not relative) returns. Moreover, as we show later, we need the return on assets (not the excess return) to map our econometric results into the return concept in our theoretical model. Hence, we define  $R_{it}$  as gross return, but also check whether our results are robust to defining  $R_{it}$  as net return instead, in which case we use LIBOR to measure the cost of capital.

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<sup>6</sup> We found that before 1994 it was not unusual for any given style in CISDM to experience an increase in AUM of over 100 percent, whereas after 1994 there were no such instances. We interpret this as evidence that data coverage stabilized after 1994.

CISDM provides 32 style codes. We aggregate them up to 14 styles, including Funds of Funds, as some of these 32 styles are very close to each other and have very few funds. See Table 1 for some summary statistics and a list of funds styles.

We use ten different style factors to account for style-wide or industry-wide returns. We use the five style factors suggested by Fung and Hsieh (2004) :

- The S&P 500 monthly return.
- The spread between the Russell 2000 and the S&P 500.
- The monthly change in the 10 year treasury yield.
- The monthly change in the Moody's Baa yield minus the 10 year treasury yield.
- The IFC (or MSCI) emerging market index.<sup>7</sup>

Fung and Hsieh (2004) arrive at this list by considering risk factors that affect the returns of traditional fund managers who may trade in the same markets as hedge funds. There are also five more risk-based factors developed in Fung and Hsieh (2001) which are kindly made available by the authors. They define a "primitive trend-following strategy" (PTFS) and compute the values of the following options.

- The return on a PTFS Bond lookback straddle
- The return on a PTFS Currency lookback straddle
- The return on a PTFS Commodity lookback straddle
- The return on a PTFS Short Term Interest Rate lookback straddle
- The return on a PTFS Stock Index lookback straddle

Thus, we have 10 style factors in total. These factors are available only back to 1994. Hence we use data from 1994 onwards.

For robustness, we also measured style factors using an agnostic approach, using dummy variables. For each style-date combination we introduced a dummy variable, to capture any time-varying style factors that might be omitted from the Fung and Hsieh (2001, 2004) list. Each dummy variable equals one for a particular style-date combination, and zero otherwise. We do this to check that estimates of  $\theta$  are robust to different approaches to modeling the factors. Each of the two approaches have different advantages, as discussed below.

Our introduction suggests that one of the style factors  $x_{kt}$  could be *the magnitude of the AUM invested in the style itself*, as congestion within styles may reduce returns. In our later theoretical section, we require a style effect (even if it is very small) for there to be an industry

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<sup>7</sup> This factor is recommended by them but not included in their original paper. See <http://faculty.fuqua.duke.edu/~dah7/HFRFData.htm>

equilibrium. As a matter of empirics, we do not have a direct measure of this variable from the data, because (a) hedge fund data do not cover the universe of funds, since reporting is voluntary; (b) certain financial intermediaries compete in the same markets (and with the same strategies) as hedge funds, for example private equity or the trading arms of investment banks (see Chan et al (2007)); and (c) even if we did have a good measure of the relevant style-level investments, there would be too few data points for identification.<sup>8</sup> Nonetheless, our two approaches to accounting for style factors are adequate for this purpose. In the case of the approach using style-date dummies, the dummies will account for all style-level factors, observed or otherwise, including style size. The approach using Fung and Hsieh (2001, 2004) factors captures style AUM implicitly, because style AUM is likely collinear with the factors themselves, since any persistence in the factors will imply that a positive innovation in any one of them will likely be linked to increased AUM.

### C. Some econometric concerns

We use the Fung and Hsieh (2001, 2004) factors as our benchmark. This is because, provided the Fung and Hsieh (2001, 2004) factors account for the most important common return factors, the alphas (and in particular the fund-specific fixed effects) should be consistently estimated even if the dataset is incomplete. With the dummy approach to measuring style factors, backfill bias (whereby recent years would have more incomplete data, and might contain only the best funds out of those that ever report) might be misidentified by the econometric strategy as an improvement in the quality of hedge funds over time. Nonetheless, as it turns out, our results are robust to either approach: in particular, our estimates of  $\theta$  and of the shape of the entrant distribution are similar, and there is no evidence that the shape of the entrant distribution varies systematically over time. We also study the effect of selection and backfill bias by repeating the estimation, omitting all funds under 2 years of age.

In our benchmark results we pool the data for all styles, although we allow the impact of each return factor to vary across styles. Then we characterize the distribution of entrant fixed effects over time in our preferred specification, and examine whether there is evidence of the entrant distribution varying over time.

Certain features of the hedge fund industry further inform our choice of estimation method.

Getmansky et al (2004) find evidence that returns are auto-correlated, which they interpret as indicative of hedge funds taking illiquid positions. We would expect some persistence in returns if there truly are fund fixed effects: nonetheless, the results of Getmansky et al (2004)

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<sup>8</sup> When we did include the observed style size in our regressions, we found a negative style size effect when pooling all styles, but statistical significance was sensitive to the specification. In style-by-style regressions there are only 11 style-size observations for each style, insufficient for identification of the style-size effect. It is worth noting that our *fund-level* size effects are not sensitive to the inclusion of the style-size variable.

suggest that our specification should distinguish between fixed effects and auto-correlated (but temporary) determinants of returns, which could otherwise lead to inconsistent estimates.

**To address this issue**, we assume first-order autocorrelation of the errors (“luck”), and use the method of Baltagi and Wu (1999) for estimating fixed effect dynamic panel models under these conditions.<sup>9</sup> The assumption is that errors display first order autocorrelation

$$\varepsilon_{i,t+1} = \rho\varepsilon_{it} + v_{it}$$

where  $v_{it}$  is a random variable with mean zero, finite variance drawn iid from some distribution.

There is also the possibility of selection bias. As discussed in Ackermann, McEnally and Ravenscraft (1999), Brown, Goetzmann and Park (2001), Malkiel and Saha (2005) and elsewhere, there are several sources of selection in the data. These include the fact that hedge funds that perform badly are likely to exit the data, whereas funds that do very well may leave the investible set and cease reporting.

The Baltagi and Wu (1999) method allows fixed-effect estimation of dynamic panel data with missing observations. Thus, the fact that hedge funds may enter and exit the database is of itself not a concern: our estimates of the fixed effects and of the size effects remain consistent. Moreover, since we are estimating fixed effects for each fund, rather than relying on the representativeness of our sample, selection bias should not be a serious concern so long as we adequately control for the style factors. As mentioned, we do so in two ways: using the Fung and Hsieh (2001, 2004) factors, which have been shown to account for a significant portion of time-series variation in hedge fund returns, and also using time-style dummies, in case there are any important omitted factors. However, we also study the effect of selection and backfill bias by repeating the estimation, omitting all funds under 2 years of age. Results are robust to this procedure.

As mentioned, a final concern is the fact that we do not have a measure of the industry style investments. This is another reason why we use two different approaches to measuring style factors. In the case of the dummy variable approach to accounting for style factors, style size is collinear with the dummies so it is well accounted for to the extent that our definitions of “styles” are reasonable. In the case of the Fung-Hsieh approach, we have an omitted variable bias problem. However, this is not severe if it is the case that style AUM is correlated with the style factors that are included. This is likely to be the case if factors that increase (decrease) style returns lead funds to increase (decrease) in number or on size.

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<sup>9</sup> Fixed effects, rather than random effects, are appropriate, given that size and the fixed effect are likely to be related. Indeed a Hausman test prefers a fixed effect specification.

## D. Results

### Size and returns

Pooling the data for all hedge funds (except Funds of Funds), we find evidence of individual negative size effects ( $\theta$ ). The value of coefficient  $\theta$  is between -0.06 and -0.08. This means that an increase in AUM of 10 percent lowers the expected return by about 25 basis points. These values are large and significant. See Table 2.

We conduct a variety of robustness tests. This includes changes in specification (for example, assuming that errors are not auto-correlated) and changes in the way we normalize returns relative to investment opportunities. We also delete all funds that exit the database within two years of incorporation, which amounts to intentionally increasing the severity of any possible survivorship bias. Results for  $\theta$  are similar regardless of the approach.

We also estimate equation (3) separately for each style, to allow  $\theta$  to vary across industries. We obtain a significant, negative estimate of  $\theta$  for almost all styles. Emerging Market funds exhibit the largest size effect ( $\theta$  is about -0.15). Long bias, short bias, Long/short equity and fixed income all have coefficients of  $\theta$  around -0.09 or -0.10, whereas other styles have smaller size effects. See Table 3.

It is interesting that directional styles have larger size effects than arbitrage-based styles. It could be that directional styles are trading on information, which is revealed when they trade to exploit the information, and which makes large trades less profitable, as it invites imitation. On the other hand, arbitrage-based styles may trade based on liquidity advantages, which are likely harder for other types of intermediary to exploit.

### Distribution of “talent”

The distribution of fund-specific fixed effects (“talent”) appears symmetrical, but has fat tails. In fact, the talent distribution matches well a Laplace distribution. See Figures 1-2. Skewness of talent is negative but not likely to be significant, given how well it matches a symmetrical Laplace distribution: see Table 4. Funds that have very strong or weak talent are those least likely to report to the database: as a result, the distribution of talent in the industry as a whole (including funds omitted from the database) is also likely to have fat tails.

Figure 3 displays the distribution of “talent” among entrants in each cohort 1994-2003 in the database.<sup>10</sup> Each subplot also displays the parameterized Laplace distribution that most closely matches the *pooled* talent distribution for all years displayed in Figure 1. Interestingly, the

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<sup>10</sup> Since at least two years of data are required to estimate each fund-specific fixed effect, and since return data were not available for all months in 2005, hedge funds born in 2004 and 2005 were not included in Figure 3.

talent distribution among entrants appears stable across cohorts – a feature that we use later in developing our theoretical model. This is consistent with Géhin and Vaissié (2006), who argue that there is no evidence of “alpha” declining over time.

### **Distribution of “luck”**

The distribution of luck also has fat tails, but is positively-skewed. See Figures 4-5. We find that it closely matches a Frechét distribution (a distribution often used in risk management to account for extreme events, see for example Lore and Borodovsky (2000)). Interestingly, the standard deviations of talent and of luck are of comparable magnitude (Table 4): luck and talent are of similar importance in accounting for the dispersion of returns. On the other hand, luck is not very persistent, with the annual autocorrelation coefficient somewhere close to 14 percent (see Table 1). Hence, while luck leads to significant variation in returns over time, the lack of persistence in luck means that talent should be the dominant factor of success in the hedge fund industry.<sup>11</sup>

To sum up, the following features stand out. First, there is a robust size effect, with a coefficient  $\theta$  of approximately -0.07. Second, the distribution of entrant talent is leptokurtic and stable over time. However, no obvious skewness is present. Third, the skewness of luck is positive, and luck also has fat tails.

Several other authors have found that excess kurtosis is present in hedge fund returns, for example Lhabitant and Learned (2002) and Getmansky et al (2004). We find that this is attributable to the distribution of luck and also of talent. On the other hand, the skewness of luck and of talent has opposite signs, consistent with the finding of Lhabitant and Learned (2002) that, while individual hedge funds have skewed returns, a diversified portfolio of hedge funds does not appear to display significant skewness. Our results indicate that the positive skewness of luck may to some extent be cancelled out by the negative skewness of the distribution of talent, although (as noted) this negative skewness is relatively weak.

## **IV. THEORETICAL MODEL**

### **A. Basic Structure**

A hedge fund is an investment vehicle, which raises funds from investors and places them in a strategy determined by the manager. Returns to the hedge fund depend on the manager’s talent, on luck (which may be persistent), and on the fund’s size, as measured by AUM. The size

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<sup>11</sup> Recall that Getmansky et al (2004) find that hedge fund returns are serially correlated, which they interpret as an indicator of illiquidity. Our lower value is attributable to the fact that, in our specification, (a) part of the autocorrelation is due to hedge fund fixed effects, and (b) we use annual data, so that the “remaining” serial correlation in higher-frequency data (interpretable in terms of liquidity) would be higher.

effect could be due to administrative costs that increase more than proportionately with AUM, as suggested by Ammann and Moerth (2005). Alternatively, for strategies based on liquidity or arbitrage, sufficiently large positions begin to erode the market conditions that the hedge fund strategy is designed to exploit (e.g. see Koutsougeras (2003)). The returns to the fund also depend on the AUM at similar funds, which we identify with the total AUM in the fund's style. For simplicity we develop the model assuming a single style. This enhances tractability, but does not affect the main quantitative results of the paper.

There is a single risk neutral investor,<sup>12</sup> who decides on the capital  $q$  to be allocated to the hedge funds. At any point in time, the investor may decide to close a hedge fund, if the expected profitability from the existing hedge fund is below the value of some outside investment option. The investor may also draw hedge funds from a distribution of potential fund managers, at a cost  $c_e$ .

## B. Hedge Funds

Time is discrete. In any period  $t$ , each hedge fund  $i$  is characterized by  $\alpha_{it}$ , which is an idiosyncratic determinant of returns unrelated to the returns of other funds. This includes a stochastic component  $\varepsilon_{it}$  (luck), and a deterministic component  $\mu_i$  (talent). Each is distributed on the real line.

Luck  $\varepsilon_{it}$  is a random variable that follows a Markov process given by a distribution function  $F(\varepsilon_{i,t+1} | \varepsilon_{it})$ . It is assumed that  $F$  has an ergodic distribution  $F^e$ . When a new fund is born at time  $t$ ,  $\varepsilon_{it}$  is drawn from a distribution  $A$ . The range of  $\varepsilon_{it}$  is assumed to be bounded.

Each period a volume  $\omega$  of potential fund managers enters the industry, each one with a value of talent drawn from a distribution  $\zeta$ . The volume of entrants is exogenous. Not all fund managers may be active in equilibrium, however. Which hedge funds are *active* is determined in equilibrium. There is a cost to the investor  $c_e$  of activating a new hedge fund, which represents the cost of due diligence. This cost is precisely the cost of due diligence, which reveals the underlying talent to the investor. Firms with an expected value below this cost will not be activated. Define  $\mu_i^*$  as the level of talent below which the entrant distribution is censored.

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<sup>12</sup> Hedge funds are defined by the fact that they cater to a small number of sophisticated investors and, alternatively, we could assume that there is one hedge fund per investor. Since investors are risk neutral, this is without loss of generality. For example, hedge funds that fall under Sections 3(c)1 of the Investment Company Act of 1940 must have 100 or fewer investors. In the absence of strategic interaction among investors, the representative investor assumption is without loss of generality.

There is also a per-period cost  $\kappa$  that the investor must pay to keep the fund active. This is the cost of ongoing due diligence.

Liang and Park (2009) distinguish between two different sources of exit. First, funds may exit because of exogenous reasons unrelated to firm profitability. This occurs in the model with probability  $\delta_e$ . We interpret this as a change in the manager's career concerns, as the manager's retirement, or as closure due to legal problems. Second, funds may exit due to the lack of profitability of their strategy. We allow this to happen in the model in two ways. In any period the fund manager may liquidate the position and shut down the fund if the fund is unprofitable. In addition, the fund's strategy may cease to become profitable exogenously with probability  $\delta_s$ . This is interpreted as the obsolescence of the fund manager's strategy (for example, due to its revelation and imitation by other types of financial intermediaries, who may engage in similar strategies: see Chan et al (2007)).<sup>13</sup> Thus, fund managers exit the industry exogenously each period with probability  $\delta = \delta_e + \delta_s$ . Allowing  $\delta > 0$  ensures that all hedge funds close in finite time with probability one. Thus,  $\delta$  represents a lower bound on the exit rate of funds in the industry.

Suppose that, at the end of period  $t-1$ , the hedge fund invests capital  $q_{i,t-1}$ . The quantity  $q_{i,t-1}$  is equal to the fund's assets under management (AUM). Then, the AUM at the beginning of the following period (which is equal to total revenues from the investment  $q_{i,t-1}$ ) is equal to:

$$f(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) \quad (4)$$

The AUM depends on AUM in the position  $q_{i,t-1}$ , and also on  $Q_{i,t-1}$  which is the total AUM in all funds in that particular style. Finally, there are decreasing returns to scale, so that  $f_1 > 0$ ,  $f_{11} < 0$ . This captures the notion that fund returns are non-scalable. We also assume that  $f(0, Q_{i,t-1}, \mu_i, \varepsilon_{it}) = 0$ , so that a hedge fund requires positive AUM in order to generate returns. When a hedge fund is born,  $q_{i,t-1} = 0$ , and the hedge fund must raise its initial capital.

We assume that that  $f_2 < 0$ , so that higher assets under management in the style lower the returns to all hedge funds in that style. We further assume that  $f(q, 0, \mu_i, \varepsilon_{it}) = \infty$  and  $f(q, \infty, \mu_i, \varepsilon_{it}) = 0$ . These assumptions will help with the existence proof.

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<sup>13</sup> An interpretation of this is that with probability  $1-\delta$  the stochastic process for  $\varepsilon$  follows a continuous distribution, and with probability  $\delta$  it drops to a low productivity state ( $\varepsilon = -\infty$ ) from which it does not exit. As noted, Getmansky et al (2004) find that a link between deteriorating performance and exit appears within 12 months of the exit itself. Since we are using annual data, and since we identify a period in the model with one year, this indicates that (in terms of the model) a continuous productivity distribution on its own may not account for the entirety of profit-related exits – something that we use our calibration procedure to capture.

Finally, returns also depend upon talent  $\mu_i$  and on luck  $\varepsilon_{it}$ , so that  $f_3 > 0$  and  $f_4 > 0$ .

The opportunity cost of funds equals the marginal utility of consumption, which we normalize to one. There could be alternative (scalable) investment opportunities for the investor but, with risk neutrality, they are indifferent between these opportunities and current consumption. An interpretation of this with risk-averse investors is that hedge funds make up a small portion of the investors' portfolios, so that the impact of the change in hedge fund returns on their marginal utility of consumption is negligible.

The hedge fund maximizes expected discounted distributions to the investor  $d_{i,t}$ . We can represent the hedge fund manager's problem recursively.<sup>14</sup> Consider at the beginning of period  $t$  a hedge fund with assets under management  $q_{i,t-1}$ , talent  $\mu_i$ , and a realization of luck  $\varepsilon_{it}$ , in an industry with total AUM  $Q_{i,t-1}$ . Define  $V(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it})$  as the value of the expected discounted profits of investing in such a hedge fund.

Furthermore, define  $V_c(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it})$  to be the value of investing in the fund assuming that it continues in operation, and let  $V_e(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it})$  be the value to the investor of investing in a fund assuming it is not going to continue. Then, the value of operating a hedge fund equals the maximum of continuing or closing the hedge fund.

$$V(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) = \max \left\{ V_e(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}), \delta V_e(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) + (1 - \delta) V_c(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) \right\} \quad (5)$$

Then, the continuation value of the fund is:

$$V_c(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) = d_{i,t} + \max_{q_{i,t}} \left\{ \frac{1}{1+r} E_\varepsilon V(q_{i,t}, Q_{i,t}, \mu_i, \varepsilon_{i,t+1}) \right\} \quad (6)$$

*s.t.*

$$d_{i,t} + pq_{i,t} - \kappa \leq f(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it})$$

Where the expectation is taken with respect to the distribution of future luck conditional on current luck (distribution  $F$ ). The value to the investor of closing the fund is simply

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<sup>14</sup> The existence of a unique recursive representation of the hedge fund manager's problem follows from standard results in dynamic programming: see Stokey et al (1989).

$$V_e(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) = f(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) \quad (7)$$

This specification assumes that there is a one-period delay required to open a new hedge fund: this assumption is realistic but not necessary. There is a non-negativity constraint on  $q_{i,t}$ , but not on  $d_{i,t}$ .

Equations (5) – (7) should be interpreted as follows. Having made an investment of  $q_{i,t-1}$  in the previous period, at the beginning of period  $t$  the returns  $f$  are realized. At this point the investor decides whether or not to liquidate part of the portfolio or raise further capital. The expression  $d_{i,t}$ , if positive, implies that the fund is taking profits. If negative, it implies that the fund is raising capital. Then, the fund decides whether or not to exit: naturally if it exits then  $d_{i,t} = f$ . In addition, with probability  $\delta$  the fund may close for exogenous reasons.

This problem can be rewritten by substituting out for distributions  $d_{i,t}$  using the fund's budget constraint. maximizes the following value function:

$$V(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) = f(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) + (1 - \delta) \max \left\{ 0, \max_{q_{it}} \left[ -q_{it} - \kappa + \frac{1}{1+r} E_\varepsilon V(q_{it}, Q_{it}, \mu_i, \varepsilon_{i,t+1}) \right] \right\} \quad (8)$$

where the expectation  $E$  is taken with respect to the realization of luck. Define  $M$  as the transition function for  $\xi_t$ : it is *endogenous*, but this endogeneity is set aside for the time being.

This completes the description of the hedge fund's decision problem. In order to describe the dynamics of the hedge fund industry requires specifying the distribution of different kinds of hedge funds, and describing how this distribution changes over time as hedge funds enter, exit, and experience different realizations of "luck." Thus, the model requires defining this distribution and characterizing its evolution over time. The corresponding discussion is somewhat technical and is relegated to the Appendix.

### C. Equilibrium

The model distinguishes between the distribution of active fund managers and the distribution of potential fund managers. The distribution of potential fund managers is exogenous, and can be defined using Hopenhayn and Prescott (1992). The distribution of active fund managers is related, but is truncated. The truncation rule is complex: there is a talent threshold above which firms do not enter, but after entering they may exit quickly as a result of bad luck – whereupon they may try again unless they exit. Thus, much of the flow of entrants and exiters will be due to the same agents, who start and close lots of funds that are short lived.

Recall that at the beginning of any period a hedge fund takes as given the vector  $x_{it} = (q_{it-1}, \mu_i, \varepsilon_{it})$ . Let  $X = \mathfrak{R}^+ \times \mathfrak{R}^2$  be the set of possible values of the vector  $x_{it}$ . Recall that returns (and the economy's state) also depend on  $Q_{i,t-1}$ , which is drawn from the real numbers. Let  $\xi_t : X \rightarrow \mathfrak{R}^+$  be the measure over types of funds at the beginning of date  $t$ .

Definition 1: An *equilibrium* is a sequence  $\{\xi_t, Q_t, \mu_t^*\}_{t=0}^\infty$  and decision rules such that the sequence  $\{\xi_t, Q_t, \mu_t^*\}_{t=0}^\infty$  results from optimal behavior, and the behavior is optimal given the sequence  $\{\xi_t, Q_t, \mu_t^*\}_{t=0}^\infty$ .

Since there is no aggregate uncertainty in the model, and since the volume of entrants  $\omega$  is constant over time, the model environment is stationary, and we can define a stationary equilibrium:

Definition 2: A *stationary equilibrium* is a pair  $\xi^*, Q^*, \mu^*$  such that  $\xi_t = \xi^*, Q_t = Q^*$  and  $\int V(0, Q^*, \mu^*, \varepsilon_{it}) dA(\varepsilon_{it}) = c_e$  for all  $t \geq 0$ .

In such an equilibrium, individual hedge funds may grow and shrink, enter or exit, but the entire industry will be characterized by stable distributions of size and of returns that do not change over time. We focus on such an equilibrium because we are interested in characterizing the long-run behavior of the hedge fund industry, including its long run response to changes in the policy environment.

To reproduce the stationary features of the hedge fund industry data and to guarantee existence of a stationary equilibrium, we require the following assumptions:

Assumption 1: (Persistence)  $F(\varepsilon_{i,t+1} | \varepsilon_{it})$  is decreasing in  $\varepsilon_{it}$ .

Assumption 2: (Mean reversion) There is some  $\varepsilon^*$  such that, for all  $\varepsilon_{it} > \varepsilon^*$ ,

$$\int_{-\infty}^{\infty} \varepsilon_{i,t+1} dF(\varepsilon_{i,t+1} | \varepsilon_{it}) < \varepsilon_{it}. \quad (9)$$

Assumption 2 ensures that expected returns are bounded. Along with Assumption 1 and with the ergodicity of  $F$ , Assumption 2 means that all funds may close someday with positive probability.

**Proposition 1:** *There exists a unique stationary equilibrium.*

**Proof:** See Appendix. ■

## V. SIMULATED INDUSTRY DYNAMICS

### A. Calibration

In the remainder of the paper, we will adopt the functional form

$$f(q_{i,t-1}, Q_{i,t-1}, \mu_i, \varepsilon_{it}) = A e^{\mu_i + \varepsilon_{it}} q_{i,t-1}^\phi h(Q_{i,t-1}) \quad (10)$$

where  $h$  is a decreasing function. Defining the log rate of return on assets at a given fund as  $R_{it} = \log[f(\cdot)/q_{i,t-1}]$ , for this functional form we have that

$$R_{it} = \log A + \mu_i + (\phi - 1) \log q_{i,t-1} + \log h(Q_{i,t-1}) + \varepsilon_{it} \quad (11)$$

This expression maps directly into equation (3), assuming that style factors including style size  $Q_{i,t-1}$  are stationary.

In this case, optimal AUM satisfies:

$$q_{it}^* = \left( \frac{\phi A}{1+r} h(Q_{i,t-1}) \int e^{\mu_i + \varepsilon_{it+1}} dF(\varepsilon_{i,t+1} | \varepsilon_{it}) \right)^{\frac{1}{1-\phi}} \quad (12)$$

The realized return at a given hedge fund is

$$R_{i,t} = \log(1+r) - \log \phi + \log \frac{e^{\varepsilon_{it+1}}}{\int e^{\tilde{\varepsilon}_{i,t+1}} dF(\tilde{\varepsilon}_{i,t+1} | \varepsilon_{it})} \quad (13)$$

where  $\varepsilon_{i,t+1}$  is the realization of luck and  $\tilde{\varepsilon}_{i,t+1}$  indicates luck as a variable of integration.

Interestingly, realized equilibrium returns depend on luck but **not on manager talent**.

Moreover, they do not depend on the style size  $Q_{i,t-1}$  either.

Realized returns may vary across hedge funds: however, the expected value of equation (13) is also the average realized return across the industry as a whole. Evaluating (13), we have simply

$$E[R_{i,t+1}] = \log(1+r) - \log \phi \quad (14)$$

Thus, using the functional form (11) implicit in typical regression analysis, equilibrium hedge fund returns depend only on the discount factor, and on the decreasing returns parameter  $\phi$ .

Notice that, if we consider the existence of many styles and that the discount rate is constant across the economy, (14) predicts a *negative* correlation across styles between the rate of return

and the magnitude of  $\phi$ . Using Table 3 to compute style level vales of this parameter and Table 1 to measure style-level returns, we find rank correlations of -0.56 and -0.39 when we condition on FH factors and dummy factors respectively, or -0.70 and -0.60 if we remove an outlier (Short Bias).

This functional form illustrates one of the advantages of our empirical and theoretical strategy for identifying manager talent through fund-specific fixed effects. Often manager talent is identified with the manager’s ability to systematically exceed the market return. For example, using this definition, Foster, Stine and Young (2008) develop an empirical test and are unable to find strong evidence that differences in manager ability (thus defined) exist – see also Fung et al (2004). Similarly, Brown, Goetzmann and Ibbotson (1999) find at best weak evidence of performance persistence in annual returns. However, when there are decreasing returns to hedge fund size, our model indicates that *there may be no differences at all* in the expected returns of hedge funds of different sizes, even if there are significant differences in manager talent. Instead, more talented managers optimally operate larger hedge funds.

Assuming that  $f(q_{it-1}, \mu_i, \varepsilon_{it}) = Ae^{\mu_i + \varepsilon_{it}} q_{i,t-1}^\phi Q_{i,t-1}^\psi$ , log returns at a given hedge fund can be written

$$\log R_{it} = \log A + \mu_i + (\phi - 1) \log q_{i,t-1} + \psi \log Q_{i,t-1} + \varepsilon_{it} \quad (15)$$

where  $\varepsilon_{it}$  captures both idiosyncratic and style uncertainty. This equation equals the empirical specification if there is not time variation in style factors,<sup>15</sup> with the relationship between the estimated coefficient on fund AUM ( $\theta$ ) in Section III and the decreasing returns parameter in the model ( $\phi$ ) satisfying the relationship  $\theta = \phi - 1$ . We do not have a measure of  $\psi$ , as a result we perform a series of simulations with different values of this parameter for robustness. However, we require  $\psi < 0$  for equilibrium in the model to be well defined.

We calibrate the distributions of  $\varepsilon_{it}$  and of  $\mu_i$  to take account of the “fat tails” indicated by our earlier empirical work. We assume that  $\varepsilon_{it}$  follows a Frechét distribution, and that  $\mu_i$  is drawn from a Laplace distribution. These are distributions that where earlier shown to match the data closely: the Laplace distribution is symmetrical but has fat tails, and the Frechét distribution is right-skewed and also has fat tails. The Laplace distribution has two parameters: a sample mean  $\bar{\mu}$  and a shape parameter  $\sigma_\mu$ . The probability distribution function (PDF) of the distribution is:

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<sup>15</sup> This is consistent with our focus on the long run impact of policy in a stationary environment without aggregate uncertainty.

$$f(\mu_i) = \frac{1}{2\sigma_\mu} e^{-\frac{|\mu_i - \bar{\mu}|}{\sigma_\mu}} \quad (17)$$

The Frechét distribution has a shape parameter  $\sigma$ . The PDF of the distribution is

$$f(\varepsilon_{t+1} | \varepsilon_t) = \frac{1}{\sigma} e^{-\frac{(\varepsilon_{t+1} - \rho\varepsilon_t)}{\sigma} + e^{-\frac{(\varepsilon_{t+1} - \rho\varepsilon_t)}{\sigma}}} \quad (18)$$

where  $\rho$  is a persistence parameter. Finally, the distribution of entrants is drawn from a Frechét distribution with mean equal to the ergodic mean of  $F$  and standard error  $\sigma_\varepsilon$ .

Thus, the parameters of the model are  $A, \phi, \psi, \bar{\mu}, \rho, \kappa, \delta, r, \sigma, \sigma_\varepsilon, \sigma_\mu$ . We calibrate them as follows. Values are reported in Table 5.

1.  $\phi$ : from the regressions,  $\phi = \theta + 1$ . We assume a value of  $\theta = -0.073$ , which in terms of magnitude is on the low end of the estimates.
2.  $\rho$ : from the regressions,  $\rho = 0.14$ .
3.  $\bar{\mu}$ : this is collinear with  $\log A$ : we set it to zero without loss of generality.
4.  $\psi$ : as discussed earlier, we do not have estimates of this parameter. Hence, we report results for a range of parameters. However, as we shall see, several results do not hinge on the value of  $\psi$ . We start with a low value of -0.001 (so that the spillover term  $Q_{i,t-1}^\psi$  lowers the productivity of hedge funds by about 2.5%), and raise it to -0.1 (so that the spillover term  $Q_{i,t-1}^\psi$  lowers the productivity of hedge funds by about 94%), viewing this latter value as being unrealistically large. Nonetheless, we find that results are insensitive to the value of this parameter.
5.  $A$ : we use this parameter match to the average fund size, which is \$190 million in our data.
4.  $\sigma_\mu$ : matches the standard deviation of returns in the dataset.
5.  $\sigma$ : Set so that the ratio  $\sigma_\varepsilon / \sigma_\mu$  matches that estimated earlier.
6.  $\sigma_\varepsilon$ : In the data we find that the variance of entrant epsilons is a bit lower than in the general population but we expect the true variance to be somewhat larger due to reporting bias. Hence we set  $\sigma_\varepsilon = \sigma$ , so that the environment faced by entrants is similar. This makes little difference to the results as luck is not persistent.
7.  $r$ : Using equation (11), we set the discount rate so that the average return on hedge funds equals 9.6% as in Table 1.
8.  $\delta$ : Getmansky (2005) argues that the average hazard rate of hedge funds is 7.1%. Liang and Park (2009) find that about a third of exits occur for reasons related to profitability, and that the

remainder exit for other exogenous reasons. Thus, we set  $\delta_e = 7.1\% - 2.4\% = 4.7\%$ , and set probability  $\delta_e$  so that the overall exit rate equals 7.1%.

9.  $c_e$ : This parameter equals the initial cost of due diligence. We follow Brown et al (2008b) and set it to equal \$50,000.<sup>16</sup>

10.  $\kappa$ : This is the cost of continuing a fund. This is likely to be small compared to the initial cost. We set it to equal an estimate of the cost of ongoing due diligence to the investor. Brown et al (2008b) indicate that, at the Princeton University Investment Company, that this is about a proportion 7/40 of the initial due diligence cost.<sup>17</sup> We follow this as a rough indicator of the order of magnitude: in any case, the overall point is that ongoing costs are considerably lower than startup costs. As we shall see, this implies that the fixed cost  $c_e$  and the discounted expected cost of  $\kappa$  affect which funds enter, but that the cost  $\kappa$  has very little impact on fund dynamics beyond entry.

11.  $\omega$ : This parameter is the mass of entrants. We set it so that the total assets under management in the industry equals \$1.2 trillion in equilibrium.

As a result of the calibration, by construction, the model matches exactly the extent of decreasing returns measured earlier, as well as the average return on assets and the distribution of talent and luck in the data. In addition, as in the data, the distribution of entrants is stable over time. This is what makes the model appropriate for the policy analysis that follows.

Interestingly, endogenous exit is a negligible share of exits in the model. The reason for this is that endogenous exit occurs if a fund was sufficiently talented to enter, and overcome the startup cost, but receives a draw of luck so low that it is not worth paying the continuation cost. Continuation costs are small relative to the profitability of a fund that can overcome the entry hurdle and, moreover, luck displays very little persistence, so that a bad current draw says little about the fund's future prospects.

A closely related issue is the concept of the optimal fund size. The first order condition of the fund's problem with respect to  $q_{i,t}$  is:

$$1 = \frac{1}{1+r} \int f_1(q_{it}, Q_{it}, \mu_i, \varepsilon_{i,t+1}) dF(\varepsilon_{t+1} | \varepsilon_t)$$

Notice that, in the special case in which the shocks  $\varepsilon_{it}$  are iid (so that  $F(\varepsilon_{t+1} | \varepsilon_t) = F(\varepsilon_{t+1})$ ), the optimal fund size depends only on the manager's talent and not on the current realization of luck. Thus, if "luck" is not persistent, the model accounts for the observation that hedge funds often close down to new investment after some point. Even if there were some persistence,

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<sup>16</sup> Strachman (2007) also reports the cost of starting a hedge fund to be about \$50,000.

<sup>17</sup> See [www.house.gov/apps/list/hearing/financialsvcs\\_dem/ht031307.shtml](http://www.house.gov/apps/list/hearing/financialsvcs_dem/ht031307.shtml).

realizations of luck need not result in large capital inflows and outflows at a given hedge fund: the increase in the value of the assets after a positive shock could still result in negligible *new* capital being added because the gain in the value of the assets themselves would do the trick: the profits from a “lucky” realization might simply be re-invested. See Tille and Van Wincoop (2008) for a discussion of this mechanism in the context of international capital flows.

## B. Experiments

We now use the model to perform several policy experiments, changing different aspects of the hedge fund’s environment to study the impact on the size and profitability of the industry. We focus on experiments that can be interpreted as changes in model parameters, while preserving the structure of the model.

Before proceeding further, equation (14) indicates that for our functional form the only way to change the return to hedge funds is to change the extent of decreasing returns. If  $\phi$  is closer to unity, then expected returns are lower – e.g if decreasing returns are being due to illiquid investments or arbitrage opportunities, greater investment liquidity or fewer arbitrage opportunities lower HF returns. However, it is difficult to identify (and to quantify) policies that could be interpreted as affecting  $\phi$ . This would require a specific model of sources of decreasing returns, and the appropriate model would likely differ across fund styles (e.g. arbitrage vs. liquidity). Our objective is to study the impact of policies that maintain the structure of the model and which are applicable to hedge funds regardless of the specific style.

As a result, the experiments we consider relate to policies that might change  $A$ , the fixed effects  $\mu$ , or the costs of opening and maintaining hedge funds. We compare the model steady state under the benchmark parameterization with a steady state featuring the values of all parameters except for those that are affected by policies. We do not model the benefits (or costs) that this might have on returns by changing the severity of principal-agent problems, or other factors from which the model abstracts. Rather, we provide a picture of the effects of regulatory burdens on long-run profitability. It is worth noting, however, that the Brown et al (2008a) study of the imposition of certain mandatory disclosure requirements on hedge funds in 2006 suggests that these requirements did not provide “material” new information to investors, suggesting that this abstraction is empirically reasonable. We also do not provide an analysis of returns as they relate to systemic risk, as this would require an extension with aggregate uncertainty and possibly in which the hedge fund industry is embedded in a full model of the financial sector or of the macro-economy. This is beyond the scope of the paper and we leave it for future work.

### Changes in the cost of due diligence

It is likely from the calibration that changes in the cost of due diligence have a small effect on the hedge fund industry. Both  $c_e$  and  $\kappa$  are too small relative to the returns at most of the hedge

funds to have any impact on the industry overall. To get a sense of this, the average expected cost of due diligence over the lifetime of a hedge fund is about \$173,000, whereas the expected return to the average entrant is about \$166 million. In our experiment, we double the entry cost  $c_e$  from \$50,000 to \$100,000. We assume that this also leads to a doubling in the continuation cost  $\kappa$ . See Table 6.

In fact, the number of funds drops by about 2%. However, industry assets under management are almost unchanged. This is because the funds that no longer exist in a stationary equilibrium with new parameters are all very small. As a result, the average hedge fund AUM rises also by about 2%. Industry profits are also unchanged, since the equilibrium rate of return depends on  $\phi$  (which is constant) and the industry AUM are negligibly affected.

When we raise the entry cost by a factor of five (to \$250,000), the entry cost lowers the number of funds by about 10%, which leads to a 1.5% drop in the industry AUM (and total industry profits). Thus, a large change in the cost of due diligence (or other fixed costs of entry or continuation) can have a large influence on the number of funds, but the impact on the overall size of the industry in terms of AUM is an order of magnitude lower, since the funds affected are the smallest. This experiment underlines the importance of modeling size and talent jointly, since the allocation of resources across managers with different talent is central to the impact of policy on the industry as a whole.

### Changes in leverage ratios

Disclosure and regulation may increase the holding costs of HF positions. We discuss how to formulate the model in terms of holding costs, and then discuss how changes in leverage ratios (and in the cost of leverage) can be interpreted in terms of holding costs in a manner that preserves the functional forms of the calibrated economy.

So far, we have assumed that the return function is of the form

$$A_{it} q_{i,t-1}^\phi \tag{19}$$

where  $A_{it} \equiv A e^{\mu_i + \varepsilon_{it}}$  includes the industry productivity term  $A$  as well as “talent” and “luck”.

Consider an alternative whereby the capital  $q$  can be raised at a cost  $c$  per dollar. Then, only a share  $(1-c)$  of each dollar raised is actually invested. Then the return is  $A_{it} \left( (1-c) q_{i,t-1} \right)^\phi$ , or:

$$A_{it} \left( (1-c) q_{i,t-1} \right)^\phi = A_{it} (1-c)^\phi q_{i,t-1}^\phi \equiv \tilde{A}_{it} q_{i,t-1}^\phi \tag{20}$$

which preserves the functional form. Hence, a model with holding costs can be reformulated in terms of our basic model, where  $\tilde{A}_{it} = A_{it} (1-c)^\phi$ .

There is a lot of current concern regarding the likely impact of changes in regulation. A significant change that is often mentioned is the imposition of limits on leverage. We discuss how to think of the structure of the model in terms of leverage.

In practice, leverage ratios are determined by best practice, so we think of  $x$  as being exogenous. The experiment we contemplate is an imposition by law of a certain leverage ratio, at a value below the industry current best practice.

Again, we have assumed that the return function is of the form

$$A_{it} q_{i,t-1}^{\phi} \quad (21)$$

Consider an alternative whereby the capital  $q$  can be expanded by up to a factor  $x$ , using leverage. Any dollar raised in leverage is raised at cost  $c$ .

Capital  $q$  thus corresponds to a net position of magnitude  $q+qx(1-c)$ . Suppose that the return from such a position is of the same functional form,

$$A_{it} \left[ q_{i,t-1} (1+x(1-c)) \right]^{\phi} \quad (22)$$

Rearranging, we have that

$$A_{it} \left[ q_{i,t-1} (1+x(1-c)) \right]^{\phi} = A_{it} q_{i,t-1}^{\phi} (1+x(1-c))^{\phi} \equiv \tilde{A}_{it} q_{i,t-1}^{\phi} \quad (23)$$

where  $\tilde{A}_{it} = A_{it} [1+x(1-c)]^{\phi}$ . Thus, the model can be interpreted in terms of leverage ratios and costs of leverage, and returns can be affected by either a change in the allowable leverage ratio  $x$ , or in the cost of raising funds for leverage  $i$ .

Notice that changes in allowable leverage ratios and in the cost of raising leveraged funds both affect hedge funds essentially by reducing the “productivity” of capital. As a result, we would expect either of these changes to reduce not only the number of viable hedge funds, but also the size of the surviving hedge funds.

To perform experiments with these variables, we take values of  $x$  and  $c$  from the data, and set  $\tilde{A}_{it}$  so that  $A_{it}$  equals the original calibrated value. CISDM reports leverage values for most funds in the database: the median value is 1.1 and the variation is very small: see Figure 6. While there are no laws dictating leverage ratios at the time of writing, leverage ratios are limited by “best practice” industry guidelines. As for  $c$ , we identify it with one minus LIBOR in discounted terms. Over the past 10 years, LIBOR has averaged about 4%. The discount rate is about 2.5%. Hence we set  $c=1-0.025$ , where 0.025 is approximately the geometric difference between 4% and 2.5%.

We ask what happens when we change  $x$  from 1.1 to 1 and to 0.9. We also check what happens if we raise the cost of leverage by 1%. We interpret this as an increase in the funding costs of

leverage, for example due to a change in the willingness to lend in the market (for example, consider the spike in LIBOR in late 2008). See Tables 7 and 8.

First, it should be clear that ex-ante returns do not change: equation (14) implies that equilibrium returns are insensitive to all parameters except the discount factor and the extent of decreasing returns. Ex-post returns do not change noticeably either, because the ongoing due diligence cost is so small relative to industry returns. AUM across the industry declines dramatically as the lack of ability to leverage significantly erodes how far a given investment can generate returns. Since the equilibrium rate of return is unaffected, profits of the industry are reduced commensurately.

The set of active hedge funds declines negligibly. It may seem surprising that the set of active hedge funds is so insensitive to policy change when profits can be sensitive. The productivity of a firm with talent two standard deviations above the mean is almost double the productivity of a firm two standard deviations below the mean. Since decreasing returns due to the parameter  $\phi$  are not steep, a small difference in productivity translates in to a significant difference in profits: the expected lifetime profits of a firm with talent two standard deviations above the mean is about \$1.37 billion, whereas those of a firm two standard deviations below the mean are about \$1.32 million. As a result, even a very large drop in profits across the hedge fund industry has little impact on the number of funds, because the range of the productivity distribution whose expected lifetime profits are driven below  $c_e$  by the change is small.

There are additional possible second-round effects of policy from which we abstract, but which could magnify the impact of any regulatory changes. One important simplifying assumption of our analysis is that the flow of entrants to the hedge fund industry is constant over time. This assumption is consistent with the fact that there are low barriers to entry into the hedge fund industry, indicating and that the talent distribution is stable, and that the flow of entrants is determined by other factors. Relaxing this assumption requires not just a model of the hedge fund industry but of the financial sector as a whole, or of the financial labor market, something that is beyond the scope of this paper.

Any policy or change in the environment lowering hedge fund profits could lead to a decreased flow of entrants in the long run. This would not affect equilibrium returns in any of the contemplated scenarios, but would imply a lower number of funds, industry AUM and total industry profits in the long run. In this sense, our results offer a lower bound on the impact of certain parameter changes on the size of the hedge fund industry. At the same time, these differences are likely to be very small compared to the effects computed in this paper, unless it involves a significant change in the number of the largest and most profitable hedge funds. We think this unlikely given the high rates of compensation of the managers at such funds.

## VI. CONCLUSION

The hedge fund literature has emphasized the importance of “alpha”, fund-specific determinants of returns that cannot be attributed to industry-wide or style-wide factors. Using data on a significant portion of the hedge fund universe, we decompose “alpha” into a fund-fixed effect (talent), a size effect, and a persistent shock (luck). We characterize the distribution of talent and luck, and develop a theoretical model of the hedge fund industry to learn how these features of the hedge fund industry might affect its response to policy changes and to changes in the cost of funds. We find that increasing the fixed costs of regulation does not significantly affect the returns to the hedge fund industry or its size in terms of assets, although they may significantly affect the number of funds. By contrast, changes in leverage ratios can have a very large impact on the industry AUM, while having little impact on the number of funds.<sup>18</sup>

We see several avenues for future work. On the empirical side, we think that better data on the costs of operating hedge funds and style-specific analysis might provide robustness to our results. Given that our results and those of other authors indicate the presence of size effects, we think it would be worth exploring whether this is a feature of financial institutions in general. On the theoretical side, more precise micro-foundations for the size effects – perhaps specific to each style – might be useful for better understanding the underlying sources of decreasing returns, which in our model are what allow managers of different degrees of talent to survive and receive funds. This would require a generalization of functional forms, likely weakening the tight link between model and data, and might also require opening the “black box” of the hedge fund return function. We view this tight link as being central to the contributions of the paper, and hence leave any extensions that weaken it for future work. At the same time, opening the “black box” in a model with an explicit information structure might also be useful for assessing the impact of changes in reporting requirements, or transparency, that might reveal the underlying strategies of hedge funds. Finally, an extension with aggregate uncertainty might allow the discussion of systemic risk in the context of the hedge fund industry.

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<sup>18</sup> As discussed in the introduction, scarcity of manager talent and decreasing returns are closely-linked concepts. Our simulations emphasize this link by, for example, showing how the impact of policy depends on whether it primarily affects the number or the size of HFs.

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## A. Appendix

In this section, we provide some auxiliary results for the discussion in the text.

### Definition and evolution of the measure of hedge fund types

The following discussion follows Hopenhayn (1992) and Hopenhayn and Prescott (1992).

Recall that at the beginning of any period a hedge fund takes as given the vector  $x_{it} = (q_{it-1}, \mu_i, \varepsilon_{it})$ . Let  $X = \mathfrak{R}^+ \times \mathfrak{R}^2$  be the set of possible values of the vector  $x_{it}$ . Let the bounded positive function  $\xi_t : X \rightarrow \mathfrak{R}^+$  be the measure over types of funds. Thus,  $\xi_t$  is a measure over possible values of  $x_{it}$ , which is determined in equilibrium based on the investor's choice of managers and the history of the economy. At date  $t$ , it is taken as given.

Let  $\Xi$  be the space of integrable distributions over  $X$ , so that  $\xi_t \in \Xi$ . The distribution  $\xi_t$  changes over time according to a transition function  $M : \Xi \rightarrow \Xi$ , which satisfies:

$$\begin{aligned} \xi_{t+1}(\chi) = & \int_{(q^*(\mu_i, \varepsilon_t), \mu_i, \varepsilon_{t+1}) \in \chi} \int_{\varepsilon_t \geq \varepsilon(\mu_i)} dF(\varepsilon_{t+1} | \varepsilon_t) d\xi_t \\ & + \omega \int_{(q^*(\mu_i, \varepsilon_t), \mu_i, \varepsilon_{t+1}) \in \chi} \int_{\varepsilon_t \geq \varepsilon(\mu_i)} dA(\varepsilon_{t+1}) d\psi(\mu_i) \end{aligned} \quad (\text{A})$$

where  $\chi$  is any Borel subset of  $X$ , the set of possible hedge funds,  $\varepsilon(\mu_i)$  is a cut-off value of luck below which a fund with a given level of talent  $\mu_i$  closes, and  $q^*(\mu_i, \varepsilon_{it})$  is the optimal choice of AUM at a fund with talent  $\mu_i$  and luck  $\varepsilon_{it}$ . Equation (A), together with Assumptions 1 and 2, implies that the functional  $M$  satisfies the premises of Theorems 1 and 2 from Hopenhayn and Prescott (1992), so  $\xi^*$  exists and is unique given the decision rules.

### Proof of Proposition 1

Consider an industry with total AUM equal to  $Q$ . For any given fund, this yields an optimal investment  $q_{it} = q(Q, x)$  where  $x$  is the fund's type. Notice that the function  $q$  is decreasing in  $Q$ .

Now define the function  $\tilde{Q}(Q) = \int q(Q, x) d\xi^*$ . The distribution  $\xi^*$  exists and is unique as a

result of standard recursive methods, as described above. The function  $\tilde{Q}$  is monotonic for a given value of  $\omega$ , because a higher  $Q$  leads funds to shrink and also shrinks the distribution of active fund managers. To see this, using a value of  $Q$ , we can compute the value of entry

$V^*(\mu; Q) \equiv \int V(0, Q, \mu, \varepsilon) dA$ . Active funds will be those such that  $V^*(\mu; Q) \geq c_e$ . Since

shrinking  $Q$  increases fund size and also fund value (so it increases the set of active funds), and since it is straightforward to show that  $\lim_{x \rightarrow 0} \tilde{Q}(x) = \infty$  and  $\lim_{x \rightarrow \infty} \tilde{Q}(x) = 0$ , there exists a unique

fixed point  $Q^* = \tilde{Q}(Q^*)$ . Hence the equilibrium exists. ■

**Table 1: Hedge Fund Styles**

This table presents the list of hedge fund styles used in this paper, along with the number of funds of each type and the average annual percentage return. We also indicate the measure of style opportunity we use later in order to scale AUM relative to investment opportunities. Hedge fund data are from CISDM (1994-2005).

	Style	Opportunity index	Funds	Average return
1	Short bias	Equity	44	3.0
2	Convertible Arbitrage	All	183	8.7
3	Fixed income	Bond	117	9.9
4	Emerging Markets	EM	316	12.7
5	Equity Market Neutral	Equity	179	7.6
6	Event Driven	All	367	10.6
7	Fixed income arbitrage	Bond	134	4.9
8	Global Macro	All	234	8.7
9	Long/short equity	Equity	1278	12.4
10	CTA	All	935	9.7
11	Funds of funds	Total HF AUM	1224	6.8
12	Relative value	All	98	8.7
13	Long bias	Equity	47	11.5
14	CPO	All	1030	6.6
	TOTAL	-	6186	9.1
	TOTAL (excluding FoFs)	-	4962	9.6

**Table 2: Hedge fund returns, pooled results**

This table presents the results of the panel regression estimation of equation (3), with fixed effects and allowing for serial correlation. Standard errors (in brackets) are corrected for heteroskedasticity using the Huber-White procedure. Fixed effects are not reported. Funds of funds are omitted. Specification A has different factor coefficients by style. B has common factors for all styles. C has different factors by style, and excludes funds under 2 years. D is like (A) but adjusts for opportunities using the style AUM. E is like (A) but does not allow for serial correlation. F is like (A) but the dependent variable is excess returns (returns minus LIBOR). G is like (A) but does not adjust for opportunity nor for inflation.

Style factors	Variable	logAUM ( $\theta$ )	$\rho$	Obs	Groups	R <sup>2</sup> - within
Fung-Hsieh	A	-.073*** (.003)	.10	17709	4227	.763
	B	-.076*** (.003)	.07	17709	4227	.732
	C	-.077*** (.004)	.10	11878	2922	.765
	D	-.066*** (.003)	.10	17709	4227	.761
	E	-.062*** (.002)	-	22671	4962	.728
	F	-.069*** (.003)	.11	17709	4227	.795
	G	-.070*** (.002)	.06	17709	4227	.221
Time-style dummies	A	-.075*** (.003)	.14	17709	4227	.760
	B	-.077*** (.003)	.11	17709	4227	.725
	C	-.080*** (.004)	.14	11878	2922	.759
	D	-.074*** (.003)	.14	17709	4227	.760
	E	-.057*** (.002)	-	22671	4962	.717
	F	-.070*** (.003)	.16	17709	4227	.795
	G	-.066*** (.002)	.06	17709	4227	.224

**Table 3: Hedge fund returns, style-specific results**

This table presents the results of estimating the size coefficient  $\theta$  by the panel regression estimation of equation (3), with fixed effects and allowing for serial correlation. The regression conditions on a matrix of style-time dummies. Standard errors (in brackets) are corrected for heteroskedasticity using the Huber-White procedure. Fixed effects are not reported.

Style	Size coefficient $\theta$		Obs	Funds
	Fung-Hsieh factors	Dummy factors		
Short bias	-.089*** (.027)	-.107*** (.026)	167	32
Convertible Arbitrage	-.022 (.014)	-.036*** (.013)	703	166
Fixed income	-.111*** (.020)	-.094*** (.021)	376	87
Emerging Markets	-.160*** (.017)	-.151*** (.017)	1015	258
Equity Market Neutral	-.042*** (.015)	-.057*** (.014)	484	137
Event Driven	-.050*** (.009)	-.047*** (.010)	1431	321
Fixed income arbitrage	-.041 (.026)	-.073*** (.027)	355	114
Global Macro	-.065*** (.015)	-.068*** (.017)	678	181
Long/short equity	-.097*** (.007)	-.100*** (.008)	4249	1062
CTA	-.072*** (.006)	-.076*** (.006)	3784	819
FoF, Multi	-.046*** (.005)	-.045*** (.005)	4239	992
Relative value	.014 (.012)	.010 (.013)	369	85
Long bias	-.107** (.046)	-.109** (.047)	150	43
CPO	-.051*** (.006)	-.053*** (.006)	3950	923
TOTAL	-	-	21950	5220

**Table 4: Moments of the distributions of talent and luck**

Second, third and fourth moments computed from the empirical distributions of talent ( $\mu$ ) and luck ( $\varepsilon$ ). Talent is the fund-specific fixed effect and luck is the regression residual, computed using the Baltagi and Wu (1999) estimator for fixed-effect dynamic panels with serially correlated disturbances and missing observations.

Variable	Factors	St. Dev.	Skewness	Kurtosis
Talent	Fung and Hsieh (2001, 2004)	0.2378	-0.501	6.45
	Time-style indicators	0.2839	-0.504	6.69
Luck	Fung and Hsieh (2001, 2004)	0.2139	2.11	10.95
	Time-style indicators	0.2201	1.93	9.51

**Table 5: Parameters used in calibration**

Parameters of the model economy are matched to data from different sources and to our econometric estimates. Details may be found in the text.

$A$	$\psi$	$\phi$	$\rho$	$\sigma_{\mu}$	$\sigma$	$\sigma_{\varepsilon}$	$\beta$	$\kappa$	$c_e$	$\delta_e$	$\delta_s$
3.153	-.001	.927	.14	.1751	.1431	.1431	.984	8750	50000	.047	.024

**Table 6: Experiments - raising the cost of entry**

The table reports the impact of an increase in the cost of entry by a factor of 2 or 5.

Change in entry costs	$\psi$	# of funds	Total AUM	Average AUM
x 2	-0.001	-2.32	-0.00%	+2.37
	-0.1	-1.90	-0.08%	+1.80
x 5	-0.001	-10.5	-1.52%	+9.14
	-0.1	-8.59	-0.17	+9.25

**Table 7: Experiments - lowering the leverage ratio**

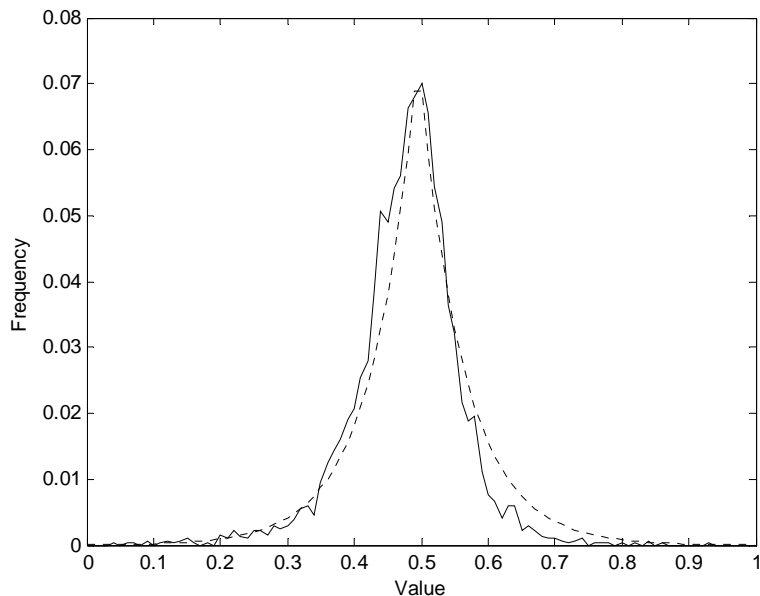
The table reports the impact of a decrease in the leverage ratio from 1.1 to 1 and to 0.5

Change in leverage	$\psi$	# of funds	Total AUM	Average AUM
1.1 to 1	-0.001	-0.00%	-45.8%	-45.8%
	-0.1	-0.00%	-45.8%	-45.8%
1.1 to 0.9	-0.001	-0.00%	-71.5%	-71.5%
	-0.1	-0.00%	-71.5%	-71.5%

**Table 8: Experiments - raising the cost of leverage**

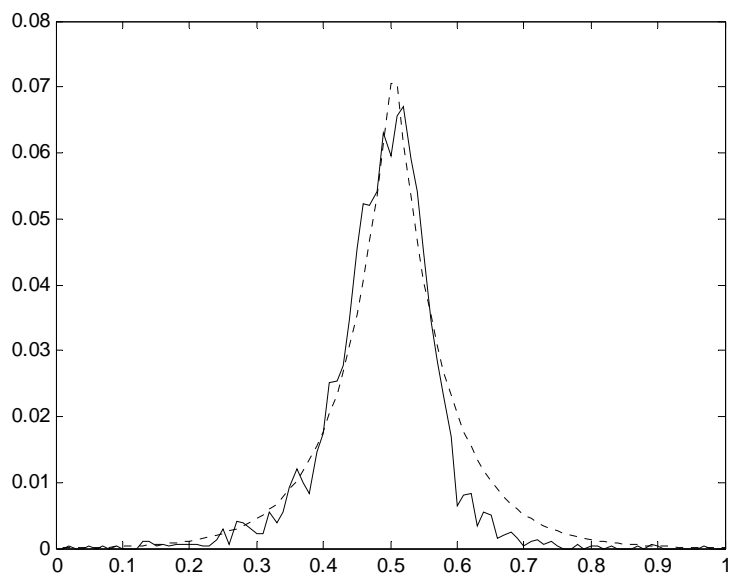
The table reports the impact of an increase in the cost of leverage of 1% or 2%.

Change in leverage cost	$\psi$	# of funds	Total AUM	Average AUM
1% increase	-0.001	-0.00%	-6.53%	-6.53%
	-0.1	-0.00%	-6.66%	-6.66%
2% increase	-0.001	-0.00%	-12.7%	-12.7%
	-0.1	-0.00%	-12.7%	-12.7%



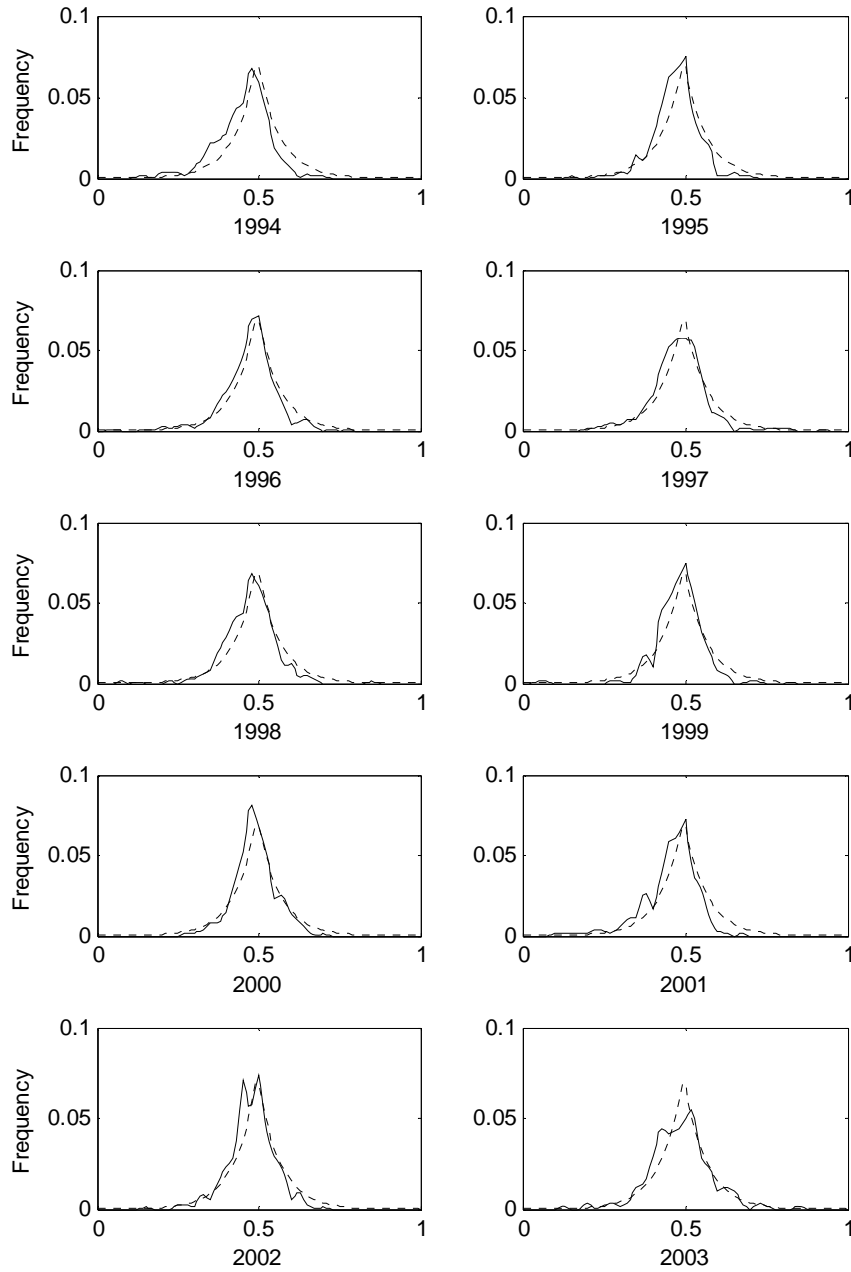
**Figure 1: Manager fixed effects, Fung and Hsieh (2001, 2004) factors.**

Fixed effects are normalized to lie between zero and one. Style returns are controlled for using the Fung-Hsieh (2001, 2004) factors. The full line reflects the empirical distribution, whereas the dotted line is a fitted Laplace distribution



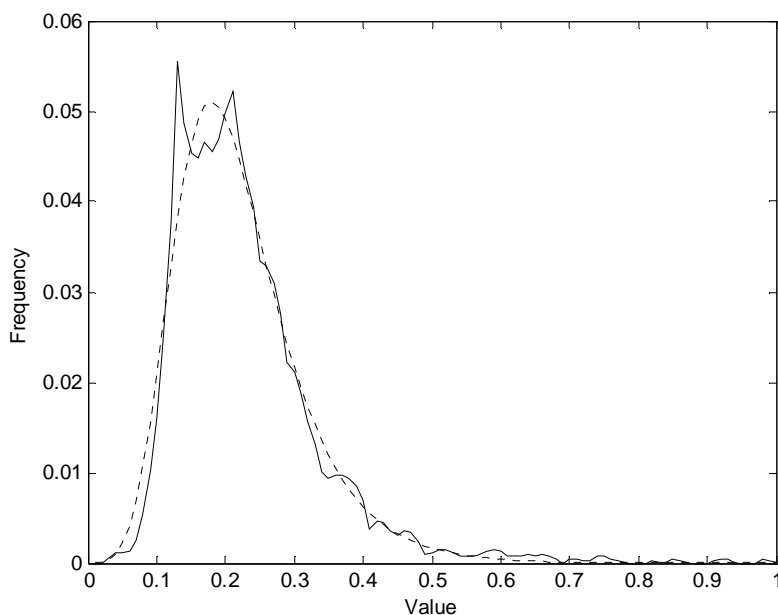
**Figure 2: Manager fixed effects, Dummy factors.**

Fixed effects are normalized to lie between zero and one. Style returns are controlled for using the dummy factors. The full line reflects the empirical distribution, whereas the dotted line is a fitted Laplace distribution.



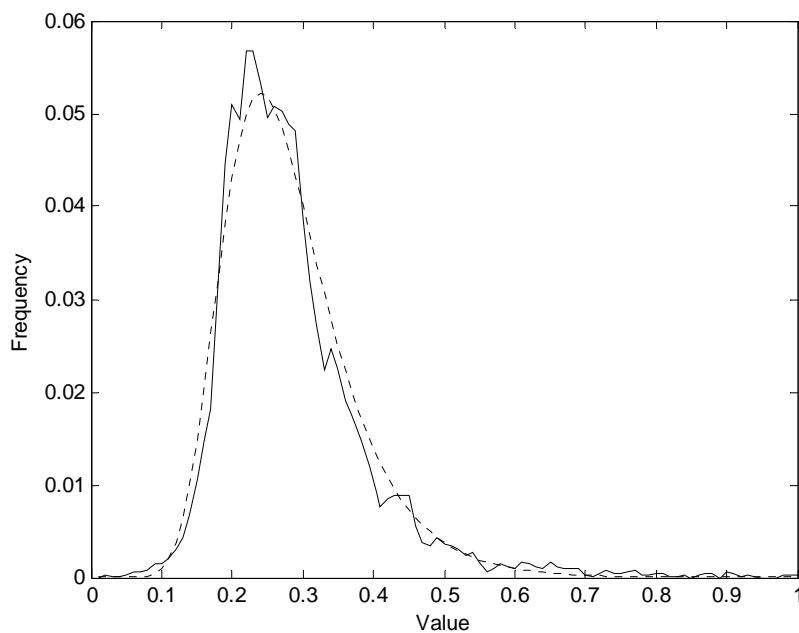
**Figure 3: Manager fixed effects for entrants each year, Fung and Hsieh (2001, 2004) factors.**

Fixed effects are estimated based on the full panel of hedge funds. Fixed effects are normalized to lie between zero and one. Style returns are controlled for using the Fung and Hsieh (2001, 2004) factors. The full line reflects the empirical distribution, whereas the dotted line is a fitted Laplace distribution. The fitted distribution is the same as that used as in Figure 1: it is the Laplace distribution that most closely matches the distribution of manager fixed effect in the full sample, including *all* years. It has been rescaled to fit the *number* of HFs in each panel, but the shape is preserved.



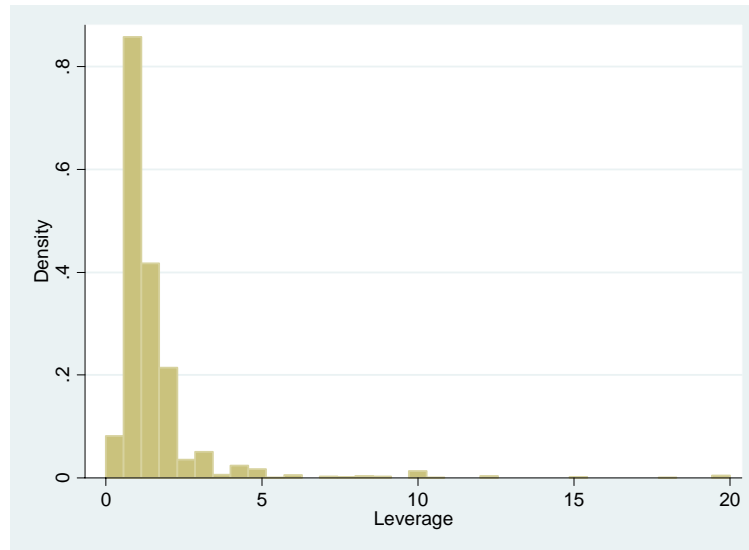
**Figure 4: Distribution of Luck, Fung and Hsieh (2001, 2004) factors.**

Fixed effects are normalized to lie between zero and one. Style returns are controlled for using the Fung -Hsieh (2001, 2004) factors. The full line reflects the empirical distribution, whereas the dotted line is a fitted Frechét distribution.



**Figure 5: Distribution of Luck, Dummy factors.**

Fixed effects are normalized to lie between zero and one. Style returns are controlled for using the dummy factors. The full line reflects the empirical distribution, whereas the dotted line is a fitted Frechét distribution.



**Figure 6: Distribution of leverage ratios, CISDM database.**

The leverage ratio is leverage as a share of fund capital (AUM). Thus, a leverage ratio of one implies that the hedge fund is investing borrowed funds equal to its AUM.