1 Estimation Formulation

We estimate a case of the following model

\[ Y_{ict} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + X_{ict}\beta + \varepsilon_{ict} \]  

(7)

where \( i \) is industry, \( c \) is a country, \( t \) is a year. The coefficients \( \delta_{i,c} \), \( \delta_{i,t} \) and \( \delta_{c,t} \) are regression coefficients on indicator variables for \((i, c)\), \((i, t)\) and \((c, t)\) pairs respectively. We have that \( c \in \{1, C\} \), \( t \in \{1, T\} \) and \( i \in \{1, N\} \). Also, the panel is unbalanced, so the number of observations is not \( C \times T \times N \). \( C \) is the total number of countries, \( T \) year and \( N \) the total number of industries. \( X_{ict} \) is a vector of independent variables \([X_{ict1} \ X_{ict2} \ldots]'\).

In order to estimate (7), we transform it so as to eliminate \( \delta_{i,c} \), \( \delta_{i,t} \) and \( \delta_{c,t} \). First, we define the mean of \( Y_{ict} \) and \( X_{ict} \) by \( i, c, t \). We use the "dot" notation for means. For example, \( \bar{Y}_{ic} \) is the mean of \( Y_{ict} \) averaging over different values of \( t \). \( \bar{X}_{ic} \) is the
mean of $Y_{ict}$ by $c$. $\overline{Y}_{...}$ is the mean by $i, c$ and $t$. Thus,

\[
\overline{Y}_{ic\cdot} = \frac{1}{T_{ic}} \sum_{t=1}^{T_{ic}} Y_{ict}
\]

\[
\overline{Y}_{i\cdot t} = \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} Y_{ict}
\]

\[
\overline{Y}_{.ct} = \frac{1}{N_{ct}} \sum_{i=1}^{N_{ct}} Y_{ict}
\]

\[
\overline{Y}_{i\cdot \cdot} = \frac{1}{C_{it}} \frac{1}{T_{ic}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} Y_{ict}
\]

\[
\overline{Y}_{\cdot t\cdot} = \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{i=1}^{N_{ct}} \sum_{c=1}^{C_{it}} Y_{ict}
\]

\[
\overline{Y}_{\cdot \cdot\cdot} = \frac{1}{N_{ct}} \frac{1}{T_{ic}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} Y_{ict}
\]

\[
\overline{Y}_{\cdot \cdot t} = \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} Y_{ict}
\]

\[
\overline{Y}_{\cdot \cdot \cdot} = \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} Y_{ict}
\]
Similarly,

\[
\bar{X}_{ic} = \frac{1}{T_{ic}} \sum_{t=1}^{T_{ic}} X_{ict}
\]

\[
\bar{X}_{i,t} = \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} X_{ict}
\]

\[
\bar{X}_{c,t} = \frac{1}{N_{ct}} \sum_{i=1}^{N_{ct}} X_{ict}
\]

\[
\bar{X}_{i..} = \frac{1}{C_{it}} \frac{1}{T_{ic}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} X_{ict}
\]

\[
\bar{X}_{.,t} = \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{i=1}^{N_{ct}} \sum_{c=1}^{C_{it}} X_{ict}
\]

\[
\bar{X}_{.,c} = \frac{1}{T_{ic}} \frac{1}{N_{ct}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} X_{ict}
\]

\[
\bar{X}_{..,t} = \frac{1}{T_{ic}} \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} X_{ict}
\]

Similar notation applies to \( \delta_{i,c} \), \( \delta_{i,t} \) and \( \delta_{c,t} \).

First, we subtract the average over \( t \), so that (7) becomes (notice the terms \( \delta_{ic} \) are gone):

\[
Y_{ict} - \bar{Y}_{ic} = (X_{ict} - \bar{X}_{ic})' \beta + (\delta_{it} - \bar{\delta}_t) + (\delta_{ct} - \bar{\delta}_c) + (\varepsilon_{ict} - \bar{\varepsilon}_{ic}) \quad (8)
\]

Then de-mean (8) over \( c \), yielding

\[
\bar{Y}_{i,t} - \bar{Y}_{i..} = (\bar{X}_{i,t} - \bar{X}_{i..})' \beta + (\delta_{it} - \bar{\delta}_t) + (\delta_{.t} - \bar{\delta}_..) + (\varepsilon_{i,t} - \bar{\varepsilon}_{i..}) \quad (9)
\]

Then subtract (9) from (8) (notice \( \delta_{it} \) is gone) :

\[
Y_{ict} - \bar{Y}_{ic} - \bar{Y}_{i,t} + \bar{Y}_{i..} = (X_{ict} - \bar{X}_{ic} - \bar{X}_{i,t} + \bar{X}_{i..})' \beta + (\delta_{ct} - \bar{\delta}_c - \bar{\delta}_t + \bar{\delta}_..) + (\varepsilon_{ict} - \bar{\varepsilon}_{ic} - \bar{\varepsilon}_{i,t} + \bar{\varepsilon}_{i..}) \quad (10)
\]

Now we de-mean (10) over \( i \) :

\[
\bar{Y}_{c,t} - \bar{Y}_{c..} - \bar{Y}_{.,t} + \bar{Y}_{..} = (\bar{X}_{c,t} - \bar{X}_{c..} - \bar{X}_{.,t} + \bar{X}_{..})' \beta + (\delta_{ct} - \bar{\delta}_c - \bar{\delta}_t + \bar{\delta}_..) + (\varepsilon_{c,t} - \bar{\varepsilon}_{c..} - \bar{\varepsilon}_{.,t} + \bar{\varepsilon}_{..}) \quad (11)
\]
Then subtract (11) from (10) (notice \( \delta_{it} \) is gone):

\[
Y_{ict} - \bar{Y}_{ic} - \bar{Y}_{i.t} + \bar{Y}_{i..} - \bar{Y}_{.ct} + \bar{Y}_{..c} + \bar{Y}_{..t} - \bar{Y}_{..}
= (X_{ict} - \bar{X}_{ic} - \bar{X}_{i.t} + \bar{X}_{i..} - \bar{X}_{.ct} + \bar{X}_{..c} + \bar{X}_{..t} - \bar{X}_{..})' \theta
+ (\varepsilon_{ict} - \bar{\varepsilon}_{ic} - \bar{\varepsilon}_{i.t} + \bar{\varepsilon}_{i..} - \bar{\varepsilon}_{.ct} + \bar{\varepsilon}_{..c} + \bar{\varepsilon}_{..t} - \bar{\varepsilon}_{..})
\]

Thus, we can rewrite (12) in the following form, and estimate the following equation:

\[
\tilde{Y}_{ict} = \tilde{X}'_{ict} \beta + \tilde{\varepsilon}_{ict}
\]

where \( \tilde{Y}_{ict} = Y_{ict} - \bar{Y}_{ic} - \bar{Y}_{i.t} + \bar{Y}_{i..} - \bar{Y}_{.ct} + \bar{Y}_{..c} + \bar{Y}_{..t} - \bar{Y}_{..} \),
\( \tilde{X}_{ict} = X_{ict} - \bar{X}_{ic} - \bar{X}_{i.t} + \bar{X}_{i..} - \bar{X}_{.ct} + \bar{X}_{..c} + \bar{X}_{..t} - \bar{X}_{..} \),
\( \tilde{\varepsilon}_{ict} = \varepsilon_{ict} - \bar{\varepsilon}_{ic} - \bar{\varepsilon}_{i.t} + \bar{\varepsilon}_{i..} - \bar{\varepsilon}_{.ct} + \bar{\varepsilon}_{..c} + \bar{\varepsilon}_{..t} - \bar{\varepsilon}_{..} \).

We can estimate \( \beta \) using:

\[
\hat{\beta} = \left( \tilde{X}'_{ict} \tilde{X}_{ict} \right)^{-1} \tilde{X}_{ict} \tilde{Y}_{ict}
\]

and the standard errors using:

\[
\left( \#^{-1} \tilde{X}'_{ict} \tilde{X}_{ict} \right)^{-1} \frac{1}{\sqrt{\#}} \tilde{X}'_{ict} \tilde{\varepsilon}_{ict}
= \left( \#^{-1} \tilde{X}'_{ict} \tilde{X}_{ict} \right)^{-1} \frac{1}{\sqrt{\#}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} \tilde{X}_{ict} \tilde{\varepsilon}_{ict}
\]

where \( \# \) is the total number of observations.

In our paper, we estimate the transformed form (13) instead of (7). In our estimation, \( X_{ict} \) is a vector of \([\text{Recession}_{c.t} \times X_i \ Controls_{i,c.t}]'\). However, we do not know the distribution of \( \tilde{\varepsilon}_{ict} \). We tested various distributions for \( \tilde{\varepsilon}_{ict} \), including bootstrap, cluster, controlling heterogeneity. We find that our results are robust to various distributions of \( \tilde{\varepsilon}_{ict} \). In the paper, we report the bootstrap error only.