Technology and Contractions: Technical Appendix

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1 Estimation Formulation

We estimate a case of the following model

$$Y_{ict} = \delta_{i,c} + \delta_{i,t} + \delta_{c,t} + X_{ict}\beta + \varepsilon_{ict} \tag{7}$$

where i is industry, c is a country, t is a year. The coefficients $\delta_{i,c}$, $\delta_{i,t}$ and $\delta_{c,t}$ are regression coefficients on indicator variables for (i,c), (i,t) and (c,t) pairs respectively. We have that $c \in \{1, C\}$, $t \in \{1, T\}$ and $i \in \{1, N\}$. Also, the panel is unbalanced, so the number of observations is not $C \times T \times N$. C is the total number of countries, T year and N the total number of industries. X_{ict} is a vector of independent variables $[X_{ict1}\ X_{ict2}...]'$.

In order to estimate (7), we transform it so as to eliminate $\delta_{i,c}$, $\delta_{i,t}$ and $\delta_{c,t}$. First , we define the mean of Y_{ict} and X_{ict} by i, c, t. We use the "dot" notation for means. For example, $\overline{Y}_{ic.}$ is the mean of Y_{ict} averaging over different values of t. $\overline{Y}_{i.t}$ is the

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mean of Y_{ict} by c. $\overline{Y}_{...}$ is the mean by i, c and t. Thus,

$$\overline{Y}_{ic.} = \frac{1}{T_{ic}} \sum_{t=1}^{T_{ic}} Y_{ict}$$

$$\overline{Y}_{i.t} = \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} Y_{ict}$$

$$\overline{Y}_{.ct} = \frac{1}{N_{ct}} \sum_{i=1}^{N_{ct}} Y_{ict}$$

$$\overline{Y}_{i..} = \frac{1}{C_{it}} \frac{1}{T_{ic}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} Y_{ict}$$

$$\overline{Y}_{..t} = \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{i=1}^{N_{ct}} \sum_{c=1}^{C_{it}} Y_{ict}$$

$$\overline{Y}_{.c.} = \frac{1}{T_{ic}} \frac{1}{N_{ct}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} Y_{ict}$$

$$\overline{Y}_{...} = \frac{1}{T_{ic}} \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} Y_{ict}$$

Similarly ,

$$\overline{X}_{ic.} = \frac{1}{T_{ic}} \sum_{t=1}^{T_{ic}} X_{ict}$$

$$\overline{X}_{i.t} = \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} X_{ict}$$

$$\overline{X}_{.ct} = \frac{1}{N_{ct}} \sum_{i=1}^{N_{ct}} X_{ict}$$

$$\overline{X}_{i..} = \frac{1}{C_{it}} \frac{1}{T_{ic}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} X_{ict}$$

$$\overline{X}_{..t} = \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{i=1}^{N_{ct}} \sum_{c=1}^{C_{it}} X_{ict}$$

$$\overline{X}_{.c.} = \frac{1}{T_{ic}} \frac{1}{N_{ct}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} X_{ict}$$

$$\overline{X}_{...} = \frac{1}{T_{ic}} \frac{1}{N_{ct}} \frac{1}{C_{it}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} X_{ict}$$

Similar notation applies to $\delta_{i,c}$, $\delta_{i,t}$ and $\delta_{c,t}$.

First, we subtract the average over t, so that (7) becomes (notice the terms δ_{ic} are gone):

$$Y_{ict} - \overline{Y}_{ic.} = \left(X_{ict} - \overline{X}_{ic.}\right)' \beta + \left(\delta_{it} - \overline{\delta}_{i.}\right) + \left(\delta_{ct} - \overline{\delta}_{c.}\right) + \left(\varepsilon_{ict} - \overline{\varepsilon}_{ic.}\right) \tag{8}$$

Then de-mean (8) over c, yielding

$$\overline{Y}_{i.t} - \overline{Y}_{i..} = \left(\overline{X}_{i.t} - \overline{X}_{i..}\right)' \beta + \left(\delta_{it} - \overline{\delta}_{i.}\right) + \left(\overline{\delta}_{.t} - \overline{\delta}_{..}\right) + \left(\overline{\varepsilon}_{i.t} - \overline{\varepsilon}_{i..}\right) \tag{9}$$

Then subtract (9) from (8) (notice δ_{it} is gone) :

$$Y_{ict} - \overline{Y}_{ic.} - \overline{Y}_{i.t} + \overline{Y}_{i..} = \left(X_{ict} - \overline{X}_{ic.} - \overline{X}_{i.t} + \overline{X}_{i..}\right)' \beta + \left(\delta_{ct} - \overline{\delta}_{c.} - \overline{\delta}_{.t} + \overline{\delta}_{..}\right) + \left(\varepsilon_{ict} - \overline{\varepsilon}_{ic.} - \overline{\varepsilon}_{i.t} + \overline{\varepsilon}_{i..}\right) \tag{10}$$

Now we de-mean (10) over i :

$$\overline{Y}_{.ct} - \overline{Y}_{.c.} - \overline{Y}_{..t} + \overline{Y}_{...} = \left(\overline{X}_{.ct} - \overline{X}_{.c.} - \overline{X}_{..t} + \overline{X}_{...}\right)' \beta + \left(\delta_{ct} - \overline{\delta}_{c.} - \overline{\delta}_{.t} + \overline{\delta}_{..}\right) + \left(\varepsilon_{.ct} - \overline{\varepsilon}_{.c.} - \overline{\varepsilon}_{..t} + \overline{\varepsilon}_{...}\right) \tag{11}$$

Then subtract (11) from (10)(notice δ_{ct} is gone):

$$\begin{aligned} & Y_{ict} - \overline{Y}_{ic.} - \overline{Y}_{i.t} + \overline{Y}_{i..} - \overline{Y}_{.ct} + \overline{Y}_{.c.} + \overline{Y}_{..t} - \overline{Y}_{...} \\ & = \left(X_{ict} - \overline{X}_{ic.} - \overline{X}_{i.t} + \overline{X}_{i..} - \overline{X}_{.ct} + \overline{X}_{.c.} + \overline{X}_{..t} - \overline{X}_{...}\right)' \theta \\ & + \left(\varepsilon_{ict} - \overline{\varepsilon}_{ic.} - \overline{\varepsilon}_{i.t} + \overline{\varepsilon}_{i..} - \varepsilon_{.ct} + \overline{\varepsilon}_{.c.} + \overline{\varepsilon}_{..t} - \overline{\varepsilon}_{...}\right) \end{aligned} \tag{12}$$

Thus, we can rewrite (12) *in the following form, and estimate the following equation:*

$$\begin{aligned} \widetilde{Y_{ict}} &= \widetilde{X'}_{ict}\beta + \widetilde{\varepsilon}_{ict} \\ \text{where } \widetilde{Y_{ict}} &= Y_{ict} - \overline{Y}_{ic.} - \overline{Y}_{i.t} + \overline{Y}_{i..} - \overline{Y}_{.ct} + \overline{Y}_{.c.} + \overline{Y}_{..t} - \overline{Y}_{...} \\ \widetilde{X_{ict}} &= X_{ict} - \overline{X}_{ic.} - \overline{X}_{i.t} + \overline{X}_{i..} - \overline{X}_{.ct} + \overline{X}_{.c.} + \overline{X}_{..t} - \overline{X}_{...} \\ \widetilde{\varepsilon_{ict}} &= \varepsilon_{ict} - \overline{\varepsilon}_{ic.} - \overline{\varepsilon}_{i.t} + \overline{\varepsilon}_{i..} - \varepsilon_{.ct} + \overline{\varepsilon}_{.c.} + \overline{\varepsilon}_{..t} - \overline{\varepsilon}_{...} \end{aligned} \tag{13}$$

We can estimate β using:

$$\widehat{\beta} = \left(\widetilde{X'}_{ict}\widetilde{X_{ict}}\right)^{-1} \widetilde{X_{ict}}\widetilde{Y_{ict}}$$

and the standard errors using:

$$\begin{aligned} & \left(\#^{-1}\widetilde{X'}_{ict}\widetilde{X_{ict}}\right)^{-1} \frac{1}{\sqrt{\#}}\widetilde{X'}_{ict}\widetilde{\varepsilon_{ict}} \\ & = \left(\#^{-1}\widetilde{X'}_{ict}\widetilde{X_{ict}}\right)^{-1} \frac{1}{\sqrt{\#}} \sum_{c=1}^{C_{it}} \sum_{t=1}^{T_{ic}} \sum_{i=1}^{N_{ct}} \widetilde{X_{ict}}\widetilde{\varepsilon_{ict}} \end{aligned}$$

where # is the total number of observations.

In our paper, we estimate the transformed form (13) instead of (7). In our estimation,X_{ict} is a vector of $[Recession_{c,t} \times X_i \; Controls_{i,c,t}]'$. However, we do not know the distribution of $\widetilde{\varepsilon_{ict}}$. We tested various distributions for $\widetilde{\varepsilon_{ict}}$, including bootstrap, cluster, controlling heterogeneity. We find that our results are robust to various distributions of $\widetilde{\varepsilon_{ict}}$. In the paper, we report the bootstrap error only.