Injury-based protection with auditing under imperfect information

Philippe KOHLER* and Michael O. MOORE**

Revised version April 12, 2000.

Abstract.

We analyze optimal protection when a benevolent government must maintain non-negative domestic profits and when the domestic import-competing firm has private information about its costs. A costly audit mechanism can deter strategic manipulation of this private information. We show that a high-penalty/low-probability of investigation is optimal when the shadow price of the firm profit is low compared with the audit cost. A low-penalty/high-probability of investigation is optimal when there is a low investigation cost and a high shadow price of firm profit. In this latter case, the trade authority obtains truthful announcements by directly auditing the firm.

Keywords: Protection, administered protection, asymmetric information.

JEL classification: F 13, K 21, L41.
1. Introduction.

This paper focuses on the use of signals to limit distortionary protection when a domestic firm has private information about injury from foreign competition. We assume that a domestic authority is mandated to use trade policy to maintain domestic production while minimizing costs to domestic consumers. The presence of asymmetric information about domestic production costs means that the firm will have an incentive to overstate the harm it is suffering from imports. We analyze how the authority can use an incentive device to punish a firm if a costly audit determines that the firm has overstated its injury. An incentive-compatible mechanism is derived consisting of a probability of audit, a tariff and a penalty which will insure that the firm does not retain informational rents associated with the private information.

This basic problem of an authority assessing domestic injury arises in a number of critical trade policy contexts. The two most prominent examples in the GATT system are safeguard mechanisms and “unfair” trade remedies (i.e., antidumping and countervailing duty investigations). In both types of “administered protection” cases, a domestic trade authority must determine whether “injury” beyond some critical level has occurred before GATT-consistent protection can be imposed. Unfortunately, the implementation of these actions are based upon information provided to the authority by the affected domestic industry which has an obvious incentive to overstate the degree to which it suffers from foreign competition. Given that these types of “contingent protection” procedures are the single most important form of protection under the GATT system, potential misuse of private information is of great importance.

There are two strands of relevant literature. The first considers strategic behavior in the specific context of administered protection. The second concerns information asymmetries in more general trade policy outcomes.

The former strand has focused on strategic behavior between firms to exploit administered protection procedures. Prusa (1992) shows how antidumping cases can be manipulated by home and foreign firms to enforce collusion. Staiger and Wolak (1991) study how antidumping acts as a cartel-enforcing device in a non-cooperative infinitely-repeated game framework. In a two-period non-
cooperative game framework with uncertainty, Prusa (1994) shows that the home firm might feign first period injury in order to get the protection in the second period. This may induce foreign firms to raise their export prices and to lower their own domestic market price. As shown by Fischer (1992), firms will try to act strategically to increase the (endogenous) probability of protection. Leidy and Hoekman (1991) and Leidy (1994) extend this concept more broadly to contingent protection and call it “spurious” injury.


Our approach is based on both incentives and signals. We follow partially the approach of Moore and Kohler (1998) who analyze the same information asymmetries but use transfers to the domestic firm to elicit truth-telling. The present analysis takes a perhaps more realistic tack by considering how an authority can audit information provided by the firm to eliminate misrepresentation of injury. More accurately, we concentrate on a purely domestic aspect of information asymmetry, i.e., between the domestic government and import-competing industry. We ignore any strategic interaction with the foreign firm. In particular, we model strategic interaction when a government implements contingent protection procedures when there is private information about domestic costs.1 Since all imports create “injury” from competition, a critical challenge for a trade authority in the GATT system is to determine whether the imports cause injury beyond some critical level. The difficulty arises since the trade authority

---

1 Empirical work has been done on the use of privately-provided information in injury cases. Anderson (1993), Devault (1993) and Moore (1992) consider recent US antidumping injury decisions while Baldwin and Steagall (1994) compare safeguard, CVD and AD cases in the US. On the other hand, Eymann and Schuknecht (1996) evaluate EC decisions.
must rely on data provided by domestic firms which clearly have an incentive to overstate the degree of injury.

Moreover, we assume that the authority has two competing goals. It is constrained institutionally to provide sufficient protection to ensure that the domestic firm does not receive negative profits.\(^2\) However, we assume that domestic consumers’ welfare also plays a role in the authority’s decision so that “excessive” protection is to be avoided if possible. If the domestic firm has private information about its costs, then the provision of the appropriate level of protection is problematic. In order to resolve that problem, the authority uses the domestic firm’s announcement of its costs as a signal about the level of injury. By linking the domestic firm tariff request to an appropriate set of penalties for non-truthful announcements about its costs, the authority can elicit accurate information about production costs and thereby avoid auditing costs and discourage frivolous petitions.

These concerns are mirrored in actual practice of authorities administering injury-based GATT protection schemes. The US International Trade Commission, for example, always conducts audits for a subset of the domestic industry (usually one or two domestic firms) in a final injury decision. However, there are no provisions in US law or practice for any penalties---if a domestic firm has been found to have provided incorrect information, the record is simply corrected. The ITC commissioners can make “adverse inferences” about individual firms but since protection is provided to the industry as a whole, there is essentially no scope for individual firm’s to be punished.\(^3\) This creates clear incentives for firms to misrepresent their costs in antidumping, countervailing duty, and safeguard cases.

This paper is organized as follows. In section 2, we examine the complete information outcome as a benchmark. In section 3, we analyze the (constrained) optimal protection mechanism in a context of \textit{ex ante} asymmetric but \textit{ex post} symmetric information. We use the properties of the signaling game between the domestic firm and the trade authority to derive the optimal penalizing device that should

\(^2\) We consider only tariff-based support for the domestic firm. Clearly, other policies such as direct transfers or production subsidies would result in higher welfare.

\(^3\) For more details, see USITC (1998).
avoid strategic manipulation of the protection process. If the authority’s use of a punishment threat is credible, it is never used. In this case, the domestic firm chooses its protection request by considering the signaling effects of its choice. On one hand, a high tariff request generates a profit effect by increasing the benefits of protection. On the other hand, a high tariff request generates a probability effect by increasing the likelihood of an audit and a penalty if the tariff request is not justified.

Concluding remarks are contained in a final section.

2. Protection Rules Without Audits.

Consider a small open economy in which there exists two commodities: a numeraire and a consumption good. The consumption good is supplied by (identical) domestic and (identical) foreign firms within a perfectly competitive domestic market. Since all domestic firms are the same, we focus on the behavior of a single representative firm.

The demand for the consumption good, \( Q(\cdot) \), continuously differentiable with associated inverse demand function \( P(\cdot) \), arises from a quasi-linear utility function which allows us to ignore income effects in the analysis. The domestic production technology is summarized by the domestic cost parameter \( \theta \). That domestic production technology exhibits increasing marginal cost plus a fixed cost. The total quantity supplied on the domestic market is composed of the domestic supply \( q_h(\cdot) \) and the foreign supply \( q_f(\cdot) \). The former is an increasing function of the price (and the tariff) and the latter is a decreasing function since any domestic price rise is a consequence of an increased tariff, i.e., \( \partial q_f / \partial P < 0 \). Market clearing in the domestic market is given by \( Q(P) = q_h(P) + q_f(P) \).

Our model is built on three assumptions.

First, the foreign producers can sell in the domestic market below the domestic firm’s average cost. This could be a consequence of straightforward comparative advantage, or “unfair” pricing practices.

---

4 This section relies heavily on Kohler and Moore (1998).
such as a dumping or subsidies. In order to keep the model tractable, we do not model any of these aspects of foreign market behavior and assume that the foreign supply is sufficiently elastic to determine the domestic price (inclusive of any tariff).

Second, the domestic firm tries to limit competition with the foreign supplier by the use of an administered protection process. Before duties may be placed on the imports, the authority must determine whether the imports cause “injury” to the domestic firm. We define injury as negative profit so that the domestic firms faces material injury from the foreign firm if the domestic price falls below the minimum of the domestic average (total) cost of production. Hence, the level of injury is homothetic to the difference between the foreign price and the domestic minimum average cost and is contingent on the domestic cost parameter $\theta$. Duties will be imposed if material injury can be found by the administering authority.

Third, the duty actually imposed is that which will eliminate the injury to the domestic firm. This is consistent with WTO-sanctioned safeguard mechanisms as well as the European Union’s administration of antidumping law wherein the duty is bounded from above by the level needed to eliminate the injury. Often this is the level of price underselling but it can also mean prices below a constructed “target” price which equals average total cost of production plus a “reasonable” (accounting) profit.\footnote{For details see Vermulst and Maer (1996, p. 347). In sharp contrast, US and Canadian antidumping duties are always based on the difference between the foreign “fair value” and the import price and are therefore independent of domestic costs. Under those systems, domestic costs affect expected protection since they affect the probability of a successful petition but not the absolute level of protection.}

With complete information, the trade authority observes quantities supplied by the foreign and domestic firms ($q_f(\cdot)$ and $q_h(\cdot)$, respectively), the cost parameter of the domestic firm, $\theta$, and the price of the foreign firm in the domestic market, $P^*$. For simplicity, we assume that $P^*$ is fixed so that the foreign firm may adjust quantity supplied to the domestic market but not the price. After observing these variables, the trade authority implements an injury-based protection rule which sets the tariff (and implicitly the domestic price) in view of the parameters of domestic demand.

We let $C(q_h(P),\theta)$ be the domestic production total cost of a type $\theta$ firm. Domestic firm profit is
given by:

$$\Pi(\theta) = P \cdot q_h(P) - C(q_h(P),\theta)$$  \hspace{1cm} (1)$$

$$\int Q(v) \cdot d\nu$$ denotes the net surplus of the domestic consumers and the tariff revenue is given by

$$(P - P^*) \cdot q_f(P)$$ while the identity $$t = P - P^*$$ defines the domestic price through the tariff and the foreign firm price.

The trade authority’s objective function consists of maximizing the value of a social choice function $$W$$, which is the sum of domestic consumer surplus, domestic firm profit and tariff revenue. The trade authority reaches this goal by setting a tariff on the domestic market (and, implicitly, the optimal price) in order to maximize $$W$$ subject to the non-negative domestic firm profit constraint. Formally, the objective of the administering trade authority under complete information is given by program P1 where the selection of the optimal domestic price implicitly determines the optimal tariff:

$$\text{(P1)} \hspace{1cm} \begin{cases} \text{Max} \int_{\nu} Q(v) \cdot d\nu + P \cdot q_h(P) - C(q_h(P),\theta) + (P - P^*) \cdot q_f(P) \\ \text{s.t.} \hspace{0.5cm} P \cdot q_h(P) - C(q_h(P),\theta) = \Pi_d \end{cases} (\lambda)$$

where $$\lambda$$ is the Lagrangian multiplier on the profit constraint and $$\Pi_d$$ is the minimum profit level guaranteed by the authority because of domestic institutional requirements.

The solution of P1 is straightforward. Suppose that the institutional constraint is assumed to be $$\Pi_d = 0$$ at the maximum. (Higher levels of profit would meet the inequality constraint but would decrease consumer welfare.) Using Roy’s identity, the market clearing condition, and the fact that the domestic price equals marginal cost allows us to show that the complete information tariff $$t^{CI}(\theta)$$ equals the difference between the minimum of the average cost of domestic production $$AC_M(\cdot)$$ and the foreign supplier’s price in the domestic market $$P^*$$:

---

6 This objective is consistent with the safeguard process in the United States and European Union where the President and Council of Ministers evaluate “national” and “Community” interest before imposing duties. Similar considerations are present in European antidumping decisions, unlike the US where antidumping statute prohibits consideration of consumer interests.
The marginal welfare cost of the protection is given by:

\[ t^{CI}(\theta) = AC_M(q_h(P),\theta) - P^* = -\lambda \cdot q_h \cdot \left[ \frac{\partial q_f}{\partial P} \right]^{-1} \] (2)

Expression (2) shows that with complete information if \( \Pi_d = 0 \) (i.e., the domestic institutional constraint requires only that the firm just stays in business), the tariff should be set so that domestic price equals the minimum average cost of domestic production. We also see that the optimal complete information tariff under the institutional constraint will be lower the more sensitive foreign supply response is to changes in the domestic price.

Expression (3) gives the value of the shadow price \( \lambda \) of the domestic firm profit which can be interpreted as the social cost of the non-negative profit constraint. As one would expect, the shadow price will be larger, the higher is the domestic average cost of production and the lower is the foreign price.

It is useful to derive the ad valorem complete information tariff rate which is given by substituting (4) into (2):

\[ \frac{t^{CI}(\theta)}{P} = \lambda \cdot \frac{\eta}{\varepsilon} \] (5)

With expression (5), we see that for a given value of \( \lambda \), the (institutionally-constrained) optimal tariff is higher the larger is \( \eta \) and the smaller is \( \varepsilon \).
3. Amending the rules under asymmetric information.

Suppose now that $\theta$ is unobservable by the trade authority and consequently the actual level of injury caused by imports. We assume that the firm knows the objective function as well as the constraints faced by the authority. If the authority cannot verify the announcement of $\theta$, then the firm will have an obvious incentive to lie about its costs and obtain a tariff-distorted price above the minimum average cost of domestic production. Incentive-compatibility (i.e., the firm deciding that truthful revelations about its true costs are in its own interests) can be achieved only if the firm is:

- rewarded for truth-telling when information cannot be checked, or
- penalized when it is lying and private information can be verified.

Kohler and Moore (1998) deal with the former case. In this paper, we analyze the latter strategy by starting from a no-audit case as a benchmark.

3.1 The no-audit case.

Suppose that the trade authority does not have at its disposal any means of verifying the injury announcement by the domestic firm. A good starting point is to ask if a single tariff protection scheme can bring the firm not to lie about its cost type under incomplete information. In order to answer this question, consider the simple protection mechanism that links the firm tariff request to its cost type:

$$t(\cdot; \theta) \rightarrow t(\theta)$$  \hspace{1cm} (6)

Now assume that the authority forms expectations about costs by using a prior cumulative distribution function $G$ defined over the range $\Theta \equiv [\theta^-, \theta^+]$, with $G(\theta^-) = 0, G(\theta^+) = 1$ and with differentiable density $g(\theta)$ such that $g(\theta) > 0$ for all $\theta \in \Theta$. We assume that $G(\theta) / g(\theta)$ has the monotone hazard rate property, i.e., $G/g$ is non-decreasing in $\theta$. The function $G$ is known to all agents. The trade authority will then seek to maximize the expected value of $W$ only by setting $t(\theta)$ subject to the same domestic firm profit constraints.

\footnote{This assumption is commonly used in the mechanism design literature. See, for example, Fudenberg and Tirole (1991), p.267.}
In this spirit, the decisions take place within the following three stage game denoted by $\Omega$:

- **Stage 1: Foreign suppliers’ decision and domestic cost realization.** Foreign suppliers offer output at $P^*$ on the domestic market. Simultaneously, a cost parameter $\theta$ is drawn from the distribution $G(\cdot)$ but is known only to the domestic firm. The domestic firm petitions the trade authority to impose a tariff and announces its intention to ask for protection.

- **Stage 2: Domestic trade authority contract decision.** The trade authority proposes a “contract” to the domestic firm. This contract specifies a tariff as a function of the announced injury level which is an increasing function of the cost parameter $\theta$.

- **Stage 3: Firm tariff choice.** The rule derived in stage 2 requires that the domestic firm choose the level of protection, $t(\theta)$ by announcing $\theta$. The domestic firm selects the tariff from the set proposed by the authority in order to eliminate injury from foreign imports. Output is finally sold on the domestic market at the tariff-distorted price.

From a market point of view, the domestic firm behaves as a price taker in all stages of the game. The market price is based on the foreign firm’s price plus the tariff imposed by the trade authority in stage 3. However, from an informational point of view, the domestic firm is the price maker at stage 3. The post-tariff price is based on the message sent by the domestic firm to the trade authority about its production costs. The duty decision is therefore conditional on this privately-held information.

The types of contracts offered by the authority as it maximizes the expected value of national welfare with no auditing emerge from the maximization problem of the domestic trade authority given by program P2a:

$$
\text{(P2a)} \quad \left\{ \begin{array}{l}
\text{Max}_{t(\theta)} \int_{P}^{\infty} Q(v) \cdot dv + P \cdot q_h(P) - C(q_h(P), \theta) + \left( P - P^* \right) \cdot q_f(P) \cdot dG(\theta) \\
\text{s.t.} \quad P \cdot q_h(P) - C(q_h(P), \theta) = \Pi_d
\end{array} \right.
$$

Assume now that the authority, knowing the informational asymmetry problem, seeks to restrict
itself only to truthtelling announcements of costs. Suppose that $\theta$ is the true cost parameter and $\tilde{\theta}$ is the announced cost parameter. Formally, the authority restricts itself to firm strategies that verify the incentive compatible condition:

$$\Pi(\theta, \theta) \geq \Pi(\tilde{\theta}, \theta) \quad \forall (\tilde{\theta}, \theta) \in \Theta^2$$  \hspace{1cm} (7)

Standard manipulation of (7) gives (8), the first order condition for incentive compatibility:

$$\frac{d\Pi(\theta)}{d\theta} = -C_0 \leq 0$$  \hspace{1cm} (8)

Expression (8) requires that the single tariff be a decreasing function of the firm cost type. Hence the trade authority can bind the profit of the least efficient firm (i.e., the type $\theta^+$ firm) at zero without forcing any other type of firm out of business. Formally, we have:

$$\Pi(\theta^+) = 0 \land \Pi(\theta) > 0 \quad \forall \theta \in [\theta^-, \theta^+]$$  \hspace{1cm} (9)

Program P2b below exhibits the authority’s objective modifications through the incorporation of informational requirements (8) and (9):

\begin{equation}
\begin{aligned}
\text{(P2b)} \quad & \left\{ \begin{array}{l}
\max_{(P, \theta)} \int_{L(\theta)} \left[ \int_{Q(v)} Q(v) \cdot d\mathcal{N} + P \cdot q_h(P) - C(q_h(P), \theta) + \left( P - P^+ \right) \cdot q_f(P) \right] \cdot dG(\theta) \\
\text{s.t.} \quad P \cdot q_h(P) - C(q_h(P), \theta) = \Pi_d \\
\quad \frac{d\Pi(\theta)}{d\theta} = -C_0 (q_h(P), \theta) \\
\quad \Pi(\theta^+) = 0
\end{array} \right.
\end{aligned}
\end{equation}

Once again, the maximization of the objective function with respect to the domestic price implicitly determines the tariff. If the objective function $W$ is transformed through integration by parts and the use of the envelope theorem, the full asymmetric information tariff, $t^{AI}(\theta)$ which solves P2b is given by (10):

---

8 See Kohler and Moore (1998).
Furthermore, it can be shown that the asymmetric information tariff (without auditing) will exceed the full information tariff:

\[ t^{AI}(\theta) - t^{CI}(\theta) = -\lambda \cdot \frac{G(\theta)}{g(\theta)} \cdot \frac{\partial C_d}{\partial q_h} \cdot \frac{\partial q_h}{\partial q_f} > 0 \]  

This difference in tariffs can be readily interpreted. Suppose that the authority offers a tariff to the type \( \theta^- \) firm. In this case \( t^{AI}(\theta^-) = t^{CI}(\theta^-) \) because \( G(\theta^-) = 0 \), and no informational distortion appears in the tariff protection. Suppose now that the authority offers a tariff to the type \( \theta \) firm with \( \theta > \theta^- \), in this case \( t^{AI}(\theta) > t^{CI}(\theta) \) but only if \( \lambda \neq 0 \). Thus, an informational distortion causes the authority to increase the tariff under incomplete information only if the non-negative firm profit constraint matters. The term \( G(\theta)/g(\theta) \) allows us to characterize the informational nature of the distortion. As \( \theta \) increases, the rate \( G(\theta)/g(\theta) \) also increases. Therefore, the probability that \( t^{AI}(\theta) \) will be picked by a lying firm grows also, implying a growing informational distortion. As long as the authority is constrained by the institutional constraint of the firm non-negative profit condition (reflected by a positive value of \( \lambda \)), it has to incur this informational distortion in its tariff decision. Expression (11) shows that the more sensitive marginal production cost is to a change in \( \theta \), the higher will be the distortion under incomplete information.

The final solution to the game is obtained by backward induction. It is clear that if the domestic firm knows the protection scheme in (10), it will always announce the highest possible level of injury. In this framework, the excessive tariff (i.e., beyond the level which would keep the firm in operation) arises from the fact that from an informational point of view, the authority is a follower and the domestic firm a leader in this game. This protection scheme under asymmetric information precludes the trade authority from behaving as a leader (by taking into account the lying strategies of the firm in its own set of information)
strategies). The simple tariff scheme keeps the domestic firm as a leader. Formally, the second order condition for incentive compatibility is not satisfied for any firm type except the highest cost type firm. The simple tariff protection rule has thus to be amended.

### 3.2 Optimal protection with auditing

The contracts offered by the authority to the domestic firm must thus provide sufficient incentives to insure that the firm truthfully reports its costs. The authority has to find another set of contracts (including the possibility of auditing and subsequent penalties) that provide the highest level of domestic welfare while maintaining firm participation in the mechanism. As in the no-audit case, because the interaction between players takes place in an incomplete information context, the game is Bayesian. Incentive compatible and individually rational strategies of the domestic firm must be analyzed before seeing how these strategies are taken into account by the trade authority in its optimization problem. In this game between the domestic firm and the domestic trade authority, the authority cannot modify the lying strategy of the firm by using a tariff alone. However, a punishment device such as an auditing system which acts a credible threat can be designed in order to deter the plaintiff from lying.

The authority can use two possible auditing strategies. First, the authority can always audit the firm and punish the firm by imposing a penalty if there is a lie. This case is straightforward: the firm will rationally never lie since it knows that it will always be audited. However, this means that society would always bear the social costs of auditing. A second strategy is that the authority can randomly audit the firm. This strategy has the advantage that the auditing cost can sometimes be avoided because the trade authority might find it optimal not to investigate in all cases. In this case, the authority must consider both the determination of a penalty (if a lie is detected) and a probability of a costly investigation. The problem faced by the authority is then the trade-off between incurring the audit cost and social cost of lying and the consequent excessive tariff.
Assume now that the audit occurs with probability $\mu$ so that $E_\mu \Pi(\cdot)$ is the firm’s expected profits when there is an *ex post* control possibility. The standard incentive compatibility and participation constraints from above must be modified and are given by (12) and (13):

\[
E_\mu \Pi(\theta, \theta) \geq E_\mu \Pi(\tilde{\theta}, \theta) \quad \forall (\tilde{\theta}, \theta) \in \Theta^2 \quad (IC) \tag{12}
\]

\[
E_\mu \Pi(\theta, \theta) \geq 0 \quad \forall \theta \in \left[\theta^-, \theta^+\right] \quad (PC) \tag{13}
\]

If expression (12) holds then the domestic firm expects that profits will be at least as much by telling the truth (i.e., announcing $\tilde{\theta} = \theta$) as it does by lying. If expression (13) holds then the domestic firm cannot do worse than obtain zero profits; otherwise it would not be rational to participate in the contract. Because the non-negative profit constraint has to be verified for any possible injured domestic firm, expression (13) can only be binding for the highest cost type firm ($\theta = \theta^+$).

Because we assume that the trade authority investigates with probability $\mu$ at cost $z$, game $\Omega$ has to be reformulated into $\Omega'$. Stage 1 from the no-audit case remains unchanged. However, in $\Omega'$, we now have:

- **Stage 2’**: Domestic trade authority contract decision (with potential audit and penalty).

  The trade authority proposes a contract to the domestic firm which specifies a tariff as a function of the announced injury. The authority also announces that there is some probability that the firm will be audited after the realization of the tariff. The penalty schedule $(K(\tilde{\theta}, \theta))$ depends on the type $\theta$ domestic firm when it announces a type $\tilde{\theta}$. The probability of investigation $\mu(\cdot)$ is a function of the tariff chosen implicitly by the firm.

- **Stage 3’**: Firm tariff choice. The domestic firm selects the contract from the set proposed by the authority by announcing its cost parameter.

- **Stage 4’**: Domestic trade authority audit decision. Once the domestic firm chooses a tariff by announcing its cost parameter, the trade authority must choose whether to audit the firm. The audit decision consists of a investigation decision $I$ ($I = \text{investigation}$) according to the
probability $\mu(I^t)$. If an investigation occurs, the trade authority observes the true level of injury and imposes a penalty $K(\hat{\theta}, \theta)$.

Payoffs to the firm and the authority are given by $\Pi(\cdot, t(\cdot), I(t(\cdot)))$ and $W(\cdot, t(\cdot), I(t(\cdot)))$, respectively. The penalty function is increasing and convex in the magnitude of the lie:

$$K(\theta, \theta) = 0, \frac{dK(\theta, \theta)}{d\theta} > 0, \frac{d^2 K(\theta, \theta)}{d\theta^2} > 0 \quad (14)$$

According to the Revelation Principle $^9$, we can restrict attention to truthful direct mechanisms by which the trade authority elicits direct truthful statements about $\theta$. These mechanisms will consist of a tariff, a penalty and an investigation probability. The tariff schedule is determined at stage 2' by maximizing expected national welfare subject to individual rationality and incentive compatibility, while holding the probability of investigation and penalty fixed. Conditions for the optimal penalty and investigation probability are also made at this time. Once again, expected national welfare is maximized with respect to $\mu$ while holding prices fixed. At stage 3’, the firm knows the audit probability and penalty when choosing its injury announcement. Thus, it weighs the trade-off between increasing the probability of an audit and increasing profits through a higher tariff. Since the trade authority makes its investigation choice after the firm has picked its tariff request, the investigation decision takes place within a signaling game. At stage 4’, the injury investigation decision will be optimal if there exists only a separating equilibrium of this signaling game in which the domestic firm picks the tariff consistent with its true injury level. If all of these conditions hold, then expected national welfare will be the highest level possible given the presence of private information about costs.

The trade authority strategy space consists of a set of contracts offered to the domestic firm. These contracts are chosen to maximize expected welfare $W$:

- subject to the incentive-compatibility and individual rationality constraints;
- through a protection choice based on a tariff determination;

$^9$ See Myerson (1979).
- matched by an audit rule that consists of a probability of investigation and a penalty determination.

- **Incentive compatibility and participation constraint.**

  Incentive compatibility requires that the trade authority has to consider the following *ex ante* expected profit (inclusive of the expected penalty $K(\cdot,\cdot)$) for type $\theta$ domestic firm when announcing $\tilde{\theta}$ \(^{10}\):

  \[
  E_\mu \Pi(\tilde{\theta},\theta) = (1 - \mu) \cdot \left[ P(\tilde{\theta}) \cdot q_h \left( P(\tilde{\theta}) \right) - C(q_h \left( P(\tilde{\theta}) \right), \theta) \right] - \mu \cdot K(\tilde{\theta},\theta)
  \]  

  (15)

  In (15), the expected profit under the lie is a weighted average of:

  - the firm’s profit if it lies and is not audited;
  - the firm penalty if it lies and is audited.

  If the firm is audited, the market price is set at the firm minimum average cost so revenue equals production cost. But since a penalty is imposed, this means that penalizing a lie can result in negative profits if the domestic firm is caught cheating.

  Following Baron and Myerson (1982), local incentive-compatibility\(^ {11}\) requires that the firm profit be a non-increasing function of the firm cost type:

  \[
  \frac{dE_\mu \Pi(\theta)}{d\theta} = -(1 - \mu) \cdot C_\theta - \mu \cdot K_\theta \leq 0
  \]  

  (16)

  where $\Pi(\theta) = \Pi(\theta,\theta)$, $C_\theta > 0$ and $K_\theta < 0$\(^ {12}\). In other words, incentive-compatibility requires that profit must fall as $\theta$ increases through the combined effects of the expected cost differential if the authority does not audit and expected changes in the penalty if an investigation occurs. Integrating (16) over $[\theta, \tilde{\theta}]$ with $\tilde{\theta} \geq \theta$, we obtain the *ex ante* expected informational rent of a type $\theta$ firm announcing $\tilde{\theta}$:

---

\(^{10}\) See appendix 1 for details of expressions (15), (16) and (17).

\(^{11}\) As in the Baron and Besanko (1984) audit model, we restrict ourselves to a first-order incentive condition approach for the sake of simplicity.

\(^{12}\) See (A.7) and (A.8) for details.
\[ E_\mu \Pi(\theta) = (1 - \mu) \int_0^\theta C_\theta \cdot d\theta + \mu \int_0^\theta K_\theta \cdot d\theta + \Pi(\tilde{\theta}) \]  

(17)

This expected rent is composed of three elements: the informational rent weighted by the probability that there will be no audit, the penalty weighted by the probability of an audit and the normal profit of the mimicked firm. Without audit, the general informational rent from lying about costs is given by (18):

\[ \Gamma(\tilde{\theta}, \theta) = \int_0^\theta C_\theta \cdot d\theta + \Pi(\tilde{\theta}) \]  

(18)

(18) depends on one hand, on the difference between the real firm’s cost type \( \theta \) (which is, from the authority viewpoint, formally drawn from the probability distribution \( G \)) and its announcement \( \tilde{\theta} \) and on the other hand on how sensitive production costs are to changes in the cost type \( \theta \): the more that costs rise with an increase in \( \theta \), the more the firm can take advantage of untruthful announcements about its costs.

The participation constraint which reflects institutional requirements must also be incorporated into the authority’s decision. In order to discourage the least efficient firm from lying, the individual rationality constraint must be binding at \( \theta^+ \), i.e., \( \Pi(\theta^+) = 0 \).

- **Tariff determination.**

When proposing the tariff to the domestic firm, the trade authority seeks to maximize the expected value of the sum of consumer surplus, profit, tariff revenue adjusted for expected audit costs and subject to the incentive compatibility constraint (16) and to the participation constraint (13). Taking into account expression (16) and \( \Pi(\theta^+) = 0 \), the optimal trade authority choice of the tariff emerges from the solution of program P3:

\[
\begin{align*}
\text{Max} & \quad \int_0^\theta \left[ \int_0^{\theta} Q(\nu) \cdot d\nu + (1 + \lambda) \cdot \left( P(\theta) \cdot q_h(P(\theta)) - C(q_h(P(\theta)), \theta) \right) + \left( P(\theta) - P^* \right) \cdot q_j(P(\theta)) \right] \\
& \quad \cdot dG(\theta) \\
\text{S.t.} & \quad -\lambda \cdot \left[ (1 - \mu) \cdot C_0 + \mu \cdot K_0 \cdot \frac{G(\theta)}{g(\theta)} - z \cdot \mu \right]
\end{align*}
\]  

(P3)
The optimal tariff schedule when the firm can be audited is:

\[
\frac{t^{\text{audit}}(\theta)}{p(\theta)} = \lambda \cdot \left[ \frac{\eta}{e} - (1 - \mu) \cdot \frac{1}{p(\theta)} \cdot \frac{G(\theta)}{g(\theta)} \cdot \frac{\partial C_0}{\partial q_h} \cdot \frac{\partial q_h}{\partial q_f} \right]
\]  

(19)

In (19), we can see that this tariff policy is identical to the complete information tariff \((t^{CI})\) given by (5) when the authority always audits \((\mu = 1)\) and equals the asymmetric solution \((t^{AI})\) without a penalty scheme when \(\mu = 0\).

**Penalty determination and investigation probability.**

The trade authority must also determine a penalty and investigation probability schedule. When the audit might actually take place (stage \(4^*\)), the firm has already picked the tariff in the set proposed by the trade authority. Consequently, when determining the penalty and probability, the price is considered “fixed”.

Conditions for the value of the optimal penalty and investigation probability schedules are determined by the solution of program P4:

\[
\text{(P4) } \begin{array}{l}
\text{Max} \\
\mu \in [0, 1]
\end{array}
\ \left\{ \begin{array}{l}
\int_0^\infty Q(\nu) \cdot d\nu + (1 + \lambda) \cdot \left[ P \cdot q_h(P) - C(\theta) \right] + \left( P - P^* \right) \cdot q_f(P) - \mu \cdot z \\
- \lambda \cdot (1 - \mu) C_0 + \mu \cdot K_0 \cdot \frac{G(\theta)}{g(\theta)}
\end{array} \right\} \cdot dG(\theta)
\]

The first-order conditions to this problem yields:\(^{13}\)

**Proposition 1:**

1.1 The optimal penalty \(K(\tilde{\theta}, \theta)\) imposed on the type \(\theta\) firm when it announces \(\tilde{\theta} > \theta\) is given by:

\[
K(\tilde{\theta}, \theta) = \frac{z}{\lambda} \cdot \log \left( \frac{G(\tilde{\theta})}{G(\theta)} \right) - \int_\theta^{\tilde{\theta}} C_0 \cdot d\theta
\]

(20)

---

\(^{13}\) See appendix 2 for details.
1.2 The optimal probability of investigation $\mu$ must satisfy:

$$-\lambda \cdot \frac{d}{d\mu} \left[ \frac{d E_u \Pi(\theta)}{d\theta} \right] \cdot G(\theta) = z \cdot g(\theta)$$  \hspace{1cm} (21)

Expression (20) shows that the optimal penalty is zero when the firm announces its true costs. The first term on the RHS of (20) also demonstrates the direct effect of lying on the penalty. The greater is the difference between the announced and true costs $(\tilde{\theta} - \theta)$, the larger will be $\log \left( G(\tilde{\theta}) / G(\theta) \right)$ and the resulting penalty. However, the relative costs of the audit and protection $(z/\lambda)$ will affect the final value of the penalty. If an audit is costly relative to the shadow price of protection (i.e., $z$ is large compared to $\lambda$), then the penalty should be high to discourage lying. However, as will be shown below, a costly audit will decrease the probability of the audit.

Surprisingly, the penalty is a decreasing function of the shadow price $\lambda$ of the domestic firm profit constraint. This can be explained by the fact that the penalty is a device to deter the firm from lying and not to increase welfare directly. In other words, a very inefficient domestic firm (i.e., a firm with a large $\lambda$) should receive a relatively low penalty, \textit{ceteris paribus}.

The second term on the RHS of (20) is the informational rent of the lying firm when it is not caught cheating. The penalty is a decreasing function of the informational rent—the larger the rent the lower the penalty. When the rent is low, then the penalty is high and the deterring effect of the penalty works. However, if the rent is high, then the firm has an incentive to cheat. Can the penalty alone discourage the firm from lying when the rent is high? Expression (20) indicates that it cannot. Another means (the optimum probability of an investigation) must be found by the administering authority to induce truth-telling by the low-cost firm.

Expression (21) is derived using the first-order conditions from P4 along with (16), and characterizes the optimal probability of investigation consistent with welfare maximization, incentive compatibility, and firm participation. The LHS of (21) shows how expected rents from lying (evaluated at
its social cost $\lambda$) will fall when the probability of investigation is marginally increased. The RHS of (21) shows the expected cost of the investigation. Thus, the probability of an audit should be set such that (expected) investigation benefits (i.e., decrease of the informational rent) equal (expected) investigation costs.

Expressions (20) and (21) also show two effects of the authority’s decision on the firm’s behavior: a penalty effect and an investigation effect. When $\lambda$ is low, the firm's rent does not have a large effect on the value of the welfare function. In this case, the value of the penalty is high so that the penalty effect dominates and can be used to deter the firm from lying in order to save the investigation cost.

However, when $\lambda$ is high, the firm's rent has a large effect on the welfare function. Allowing the firm to retain a positive rent is costly from a welfare point of view compared with the investigation cost. In this case, the investigation effect dominates and is used to deter the firm from lying. In other words, a high $\lambda$ results in a high probability of an audit.

- Optimal auditing strategy.

Finally, we must characterize conditions for a separating equilibrium (that is when the equilibrium strategy of the firm is truth-telling) that insures efficiency of the whole game.

For the sake of simplicity, we assume that, within the signaling game, the firm cost-type can take only two values: $\theta$ and $\tilde{\theta}$ with $\tilde{\theta} > \theta$. The probability that the actual cost type does not exceed $\theta$ is given by $G(\theta)$ and the probability it exceeds $\theta$ (i.e., equals $\tilde{\theta}$) is given by: $1 - G(\theta)$. For each $t(\cdot)$ the investigation decision $I^*(t(\cdot))$ must maximize the trade authority's objective $W$, given the conditional probability of investigation $\mu(I^*|t)$. The firm's choice of $t(\cdot)$ must maximize its profit given the trade authority investigation strategy. The optimal firm tariff decision $t(\cdot)$ decision solves:

$$t(\cdot) \in \arg \max_{\cdot} \mathbb{E}_t \{ \Pi(\cdot,t(\cdot),I^*(t(\cdot))) \}$$

(22)
There are two possible pure strategy Perfect Bayesian Equilibria of this signaling game:\(^{14}\):

- a pooling equilibrium on \( t(\tilde{\theta}) \), i.e., both a \( \tilde{\theta} \) -type firm and a \( \theta \) -type firm picking \( t(\tilde{\theta}) \);
- a separating equilibrium with a \( \tilde{\theta} \) -type firm picking \( t(\tilde{\theta}) \) and \( \theta \) -type firm picking \( t(\theta) \).

From an efficiency point of view, the pooling equilibrium is not optimal since the domestic firm is lying. The separating equilibrium is optimal because it ensures that the firm will adopt a truth-telling strategy.

The separating equilibrium will be sub-game perfect if both the trade authority's and firm’s payoff exceeds the payoff under the pooling equilibrium. When the firm has made its tariff choice, the investigation decision is made with a probability \( \mu(t|\theta) \). The firm tariff choice is made according to that probability and to the level of the level of the possible penalty. The firm will choose the truth-telling strategy if the expected payoff under the pooling equilibrium is below the expected payoff under the separating equilibrium, that is when:

\[
(1 - \mu(t|\theta)) \int_{\tilde{\theta}}^{\theta} C_\theta(q_\theta(P(\theta),\theta)) \cdot d\theta + \mu(t|\theta) \int_{\tilde{\theta}}^{\theta} K_\theta \cdot d\theta \leq 0
\]

where the LHS of (23) is the expected payoff with lying under the pooling equilibrium and the RHS is the expected payoff under truth-telling (and equal to zero because the firm has accurately reported it's minimum average cost). This expression allows us to characterize, through proposition 2, the incentive efficiency conditions for the protection mechanism to be sub-game perfect.

\(^{14}\) Indeed there are theoretically four equilibria but two of them are excluded, i.e. the pooling and separating equilibria in which the type \( \tilde{\theta} \) firm chooses irrationally \( t(\theta) \).
**Proposition 2.** The optimal probability of investigation is given by:

\[
\mu(l) = \begin{cases} 
\frac{\int_0^\theta C_\theta \cdot d\theta}{z \cdot \log \left( \frac{G(\theta)}{G(\theta)} \right)} & \text{if } 0 \leq \frac{\int_0^\theta C_\theta \cdot d\theta}{z \cdot \log \left( \frac{G(\theta)}{G(\theta)} \right)} < 1 \\
1 & \text{otherwise.}
\end{cases}
\]

(24)

Proof: See appendix 3.

In (24), the condition for the investigation probability to be consistent with the penalty function shows an inverse relation between the two elements of the trade authority strategy (investigation decision probability and penalty level) when the domestic firm has chosen the tariff. We can see that the investigation probability is also affected by the shadow price of the domestic firm profit. For a sufficiently high \( \lambda \), i.e., when the foreign price is far below the minimum average cost level, the investigation occurs with certainty because the lie is very costly from a welfare point of view. Moreover, the investigation probability is a decreasing function of the investigation cost. If, from an overall welfare point of view, it is costly to investigate, the trade authority has to find a trade-off between incurring the lying cost and the investigation cost. The optimal trade authority strategy is given by the Perfect Bayesian Equilibrium of the signaling game and is given by proposition 3:
**Proposition 3:** The optimal injury-based protection rule, consisting of a set containing a tariff, a probability of investigation and a penalty, is defined by the mechanism:

\[
t, \mu, K : (\hat{\theta}, \theta) \in \Theta^2 \rightarrow \left[ t(\theta), \mu(I | t), K(\hat{\theta}, \theta) \right]
\]

where:

\[
\frac{t_{\text{audit}}(\theta)}{P(\theta)} = P(\theta) - P^* = \lambda \cdot \left[ \frac{\eta}{\varepsilon} \left( 1 - \mu \right) \cdot \frac{1}{p(\theta)} \cdot \frac{G(\theta)}{g(\theta)} \cdot \frac{\partial C_0}{\partial q_h} \cdot \frac{\partial q_f}{\partial f} \right]
\]

\[
K(\hat{\theta}, \theta) = \frac{z}{\lambda} \cdot \log\left( \frac{G(\hat{\theta})}{G(\theta)} \right) - \int_0^\theta C_0 \cdot d\theta
\]

\[
\begin{align*}
\mu(I | t) &= \int_0^\theta C_0 \cdot d\theta & \text{if } 0 \leq \int_0^\theta C_0 \cdot d\theta < 1 \\
\mu(I | t) &= \frac{z}{\lambda} \cdot \log\left( \frac{G(\hat{\theta})}{G(\theta)} \right) & \text{otherwise}
\end{align*}
\]

Proposition 3 shows us that the protection process which relies on privately-provided domestic information about costs should be devised in one of the following two ways:

- A combination of a high penalty and low probability of investigation is optimal when the **penalty effect** dominates, i.e., when the shadow price of the firm profit is low compared with the audit cost. In this case, the trade authority obtains truthful announcements through the deterrence effect of a large \( K(\cdot, \cdot) \).

- A combination of a low penalty and high probability of investigation is optimal when the **investigation effect** dominates, i.e., when there is a low cost of investigation and a high shadow price of the firm profit. In this case, the trade authority obtains truthful announcements by directly auditing the firm.

This latter option result is particularly important when a institutional context might deny the trade
authority to ability to penalize the domestic firm. In that context, an investigation is always necessary to reach a (constrained) optimal injury-based protection tariff.


Injury-based contingent protection lies at the heart of the GATT system. Governments have agreed to reduce trade barriers only if they are allowed to impose trade restrictions if a domestic industry is injured from foreign competition. Injury determinations, based on economic reports provided by the “injured” firms themselves, are used in all of the prominent GATT-consistent import regimes---antidumping, countervailing duty, and safeguards.

We have pointed out that there are inherent incentives problems associated with this system. Those who stand to gain from tariffs are the ones who provide the data used to determine whether the restrictions will be imposed. We have investigated therefore how a benevolent government might amend protection procedures in order to discourage domestic firms from manipulating injury information.

If there is asymmetric information with no possibility of auditing injury announcements, the domestic firm will always announce a higher-than-actual level of injury in order to obtain the higher-than-necessary protection. However, we have found that if an authority will couple an injury investigation possibility with a penalty scheme within a signaling game framework, any informational rent can be extracted from the domestic firm.

The perfect Bayesian equilibrium of the signaling game shows two properties of the optimal mechanism when the authority is forced to maintain non-negative domestic profits. On one hand, if the welfare cost of the firm’s informational rents is high compared with the investigation cost, then the trade authority uses the controlling effect of the investigation process to avoid information manipulation. On the other hand, if the welfare cost of the informational firm rents is low compared with the investigation cost, then the trade authority uses the deterrence effect of the investigation process in order to avoid information manipulation.

The problem of eliciting accurate information from self-interested parties has been confronted by
governments. In the US for example, antidumping and countervailing duty regulations allow the Department of Commerce to punish foreign firms which misrepresent data. Indeed, US officials are required by law to verify all data provided by foreign firms; foreign firm’s failure to cooperate can lead to serious penalties (i.e., much higher antidumping rates). Such procedures are explicitly seen as providing incentives to foreign firms to provide truthful information. Thus, US officials have incorporated at least some of the spirit of the solutions analyzed in this paper into their treatment of foreign firms in contingent protection cases. As noted in the introduction, US domestic firms providing information about injury face much less onerous consequences—an audit conducted for only one or two domestic firms but essentially never results in any penalty to the firm. In other words, the current system of providing information about injury is very likely incentive-incompatible.

In sum, this study does provide some insights into how a benevolent government might limit the abuse of private information in injury determinations. However, this work presumes that governments would be willing to punish domestic firm’s found to have misused private information. This would take considerable political courage given the strong support protection-seeking industries often have. Thus, the study of how to technically amend contingent trade protection provisions like antidumping protection cannot be disentangled from the study of the political genesis of such provisions. One could argue therefore that reforming contingent trade protection is primarily a political question rather than a technical problem. Nevertheless, if these procedures still exist, one should at least endeavor to implement them in the least-costly way possible.
References:


Appendix 1: Determination of the incentive compatibility first order condition and expected profit.

Let $\mu$ be the probability that the authority investigates the domestic firm’s cost parameter announcement, and $K(\bar{\theta}, \theta)$ the penalty incurred by the lying firm when there is an investigation (the announcement is $\bar{\theta}$ and the investigation yields $\theta$).

The expected profit $E_\mu \Pi(\bar{\theta}, \theta)$ under the lie is:

$$E_\mu \Pi(\bar{\theta}, \theta) = (1 - \mu) \cdot (P(\bar{\theta}) \cdot q_h(\bar{\theta}) - C(q_h(\bar{\theta}), \theta)) - \mu \cdot K(\bar{\theta}, \theta)$$  \hspace{1cm} (A.1)

By adding and subtracting $E_\mu \Pi(\bar{\theta}, \bar{\theta})$, this can be rewritten as:

$$E_\mu \Pi(\bar{\theta}, \theta) = E_\mu \Pi(\bar{\theta}, \bar{\theta}) + (1 - \mu) \cdot \left[ C(\bar{\theta}, \bar{\theta}) - C(\bar{\theta}, \theta) \right] + \mu \cdot \left( K(\bar{\theta}, \bar{\theta}) - K(\bar{\theta}, \theta) \right)$$  \hspace{1cm} (A.2)

where $C(\theta, \theta) \equiv C(q_h(P(\theta), \theta)$.

The injury-based protection rule is incentive compatible if:

$$E_\mu \Pi(\theta, \theta) \geq E_\mu \Pi(\bar{\theta}, \theta) = E_\mu \Pi(\bar{\theta}, \bar{\theta}) + (1 - \mu) \cdot \left[ C(\bar{\theta}, \bar{\theta}) - C(\bar{\theta}, \theta) \right] + \mu \cdot \left( K(\bar{\theta}, \bar{\theta}) - K(\bar{\theta}, \theta) \right)$$  \hspace{1cm} (A.3)

Rearranging terms into (A.3) yields (A.4):

$$E_\mu \Pi(\theta, \theta) - E_\mu \Pi(\bar{\theta}, \bar{\theta}) \geq (1 - \mu) \cdot \left[ C(\bar{\theta}, \bar{\theta}) - C(\bar{\theta}, \theta) \right] + \mu \cdot \left( K(\bar{\theta}, \bar{\theta}) - K(\bar{\theta}, \theta) \right)$$  \hspace{1cm} (A.4)

By reversing the roles of $\theta$ and $\bar{\theta}$ in (A.4) and by taking the limit when $\bar{\theta} \to \theta$ we get (A.5), the first condition for the protection rule to be incentive compatible:

$$\frac{dE_\mu \Pi(\theta)}{d\theta} = -(1 - \mu) \cdot C_0 - \mu \cdot K_0 \leq 0$$  \hspace{1cm} (A.5)

where:

$$\Pi(\theta) = \Pi(\theta, \theta)$$  \hspace{1cm} (A.6)

$$C_0 = \lim_{\bar{\theta} \to \theta} \frac{\left[ C(\bar{\theta}, \bar{\theta}) - C(\bar{\theta}, \theta) \right]}{\bar{\theta} - \theta} > 0$$  \hspace{1cm} (A.7)

$$K_0 = \lim_{\bar{\theta} \to \theta} \frac{\left[ K(\bar{\theta}, \bar{\theta}) - K(\bar{\theta}, \theta) \right]}{\bar{\theta} - \theta} < 0$$  \hspace{1cm} (A.8)

By integrating (A.5) over $[\theta, \bar{\theta}]$ with $\bar{\theta} > \theta$, we get:

$$E_\mu \Pi(\theta) = (1 - \mu) \cdot \int_{\theta}^{\bar{\theta}} C_0 \cdot d\theta + \mu \cdot \int_{\theta}^{\bar{\theta}} K_0 \cdot d\theta + \Pi(\bar{\theta})$$  \hspace{1cm} (A.9)

Assume now that $\bar{\theta} = \theta^*$. By setting $\Pi(\theta^*) = 0$ and using integration by parts over $[\theta^*, \theta^+]$ on the expectation (with respect to $g$) of (A.9), we get the expected value of profit:
\[
\int_{\theta^-}^{\theta^+} E_\mu \Pi(\theta) \cdot g(\theta) \cdot d\theta = \int_{\theta^-}^{\theta^+} [(1 - \mu) \cdot \mathcal{C}_\theta(q_a(P)) + \mu \cdot K_\theta] \cdot G(\theta) \, d\theta 
\] (A.10)
• Appendix 2: Proof of Proposition 1.

The first step consists of finding the penalty by maximization of expected welfare with respect to the probability of investigation while holding the price fixed. Differentiating $W$ in P4 with respect to $\mu$ yields:

$$\frac{\partial W}{\partial \mu} = 0 \iff -\lambda \cdot (C_0 + K_0) \cdot \frac{G(\theta)}{g(\theta)} = z \iff K_0 = C_0 - \frac{z}{\lambda} \cdot \frac{g(\theta)}{G(\theta)}$$  \hspace{1cm} (A.11)

By integrating (A.11) over $[\theta, \tilde{\theta}]$, we get:

$$\int_{\theta}^{\tilde{\theta}} K_0 \cdot d\theta = \int_{\theta}^{\tilde{\theta}} C_0 \cdot d\theta - \frac{z}{\lambda} \int_{\theta}^{\tilde{\theta}} \frac{g(\theta)}{G(\theta)} \cdot d\theta$$  \hspace{1cm} (A.12)

Rearranging terms in (A.12), we get:

$$K(\tilde{\theta}, \tilde{\theta}) - K(\theta, \theta) = \int_{\theta}^{\tilde{\theta}} C_0 \cdot d\theta - \frac{z}{\lambda} \cdot \int_{\theta}^{\tilde{\theta}} \frac{G(\theta)}{G(\theta)} \cdot d\theta$$  \hspace{1cm} (A.13)

Taking into account that $K(\tilde{\theta}, \tilde{\theta}) = 0$, we get the expression of the penalty:

$$K(\tilde{\theta}, \theta) = \frac{z}{\lambda} \cdot \int_{\theta}^{\tilde{\theta}} \frac{G(\theta)}{G(\theta)} \cdot d\theta$$  \hspace{1cm} (A.14)

Conditions for an optimal investigation probability are obtained from the same first order condition. We have:

$$\frac{\partial W}{\partial \mu} = 0 \iff -\lambda \cdot (C_0 + K_0) \cdot \frac{G(\theta)}{g(\theta)} = z$$  \hspace{1cm} (A.15)

Using (A.5), this can be rewritten as:

$$-\lambda \cdot \frac{dE_\mu}{d\mu} \left[ \frac{dE_\mu}{d\theta} \Pi(\theta) \right] \cdot G(\theta) = z \cdot g(\theta)$$  \hspace{1cm} (A.16)
Appendix 3: Proof of Proposition 2.

We are looking for the probability which yields higher expected profits under the truth than under the lie with a chance of auditing and penalty. The expected firm profit under the lie and under the truth are, respectively:

\[ \mu(t|\mu) \cdot \left[ -K(\theta, \delta) \right] + (1 - \mu(t|\mu)) \cdot \int_0^\theta C_0 \cdot d\theta \]  
(A.17)

and

\[ \mu(t|\mu) \cdot \left[ 0 \right] + (1 - \mu(t|\mu)) \cdot \left[ 0 \right] = 0 \]  
(A.18)


The firm will choose the truth telling strategy if:

\[ \mu(t|\mu) \cdot \left[ 0 \right] + (1 - \mu(t|\mu)) \cdot \left[ 0 \right] \geq \mu(t|\mu) \cdot \left[ -K(\theta, \delta) \right] + (1 - \mu(t|\mu)) \cdot \int_0^\theta C_0 \cdot d\theta \]  
(A.19)

This yields:

\[ \mu(t|\mu) \geq \frac{\int_0^\theta C_0 \cdot d\theta}{\int_0^\theta C_0 \cdot d\theta + K(\theta, \delta)} \]  
(A.20)

Which implies:

\[ \mu(t|\mu) \geq \frac{\int_0^\theta C_0 \cdot d\theta}{\frac{z}{\lambda} \cdot \log \left( \frac{G(\hat{\theta})}{G(\theta)} \right) - \int_0^\theta C_0 \cdot d\theta + \int_0^\theta C_0 \cdot d\theta} \]  
(A.21)

This yields the optimal probability of investigation:

\[ \mu(t|\mu) = \begin{cases} \int_0^\theta C_0 \cdot d\theta & \text{if } 0 \leq \frac{\int_0^\theta C_0 \cdot d\theta}{\frac{z}{\lambda} \cdot \log \left( \frac{G(\hat{\theta})}{G(\theta)} \right)} < 1 \\ \frac{z}{\lambda} \cdot \log \left( \frac{G(\hat{\theta})}{G(\theta)} \right) & \text{otherwise} \end{cases} \]  
(A.22)

because \( \mu(t|\mu) \) must belong to \([0,1]\).