PPP 6085
Intermediate Microeconomics
Math Review Handout – Solutions

Graphing a Linear Equation

$Q = 500 - 50P$

Plug in the zeros

Where does the line intersect with the vertical axis? Where does the line intersect with the horizontal axis?

When $Q$ is 0, $P = ___$

$0 = 500 - 50P$

$50P = 500$

$P = 500/50 = 10$

When $P$ is 0, $Q = ___$

$Q = 500 - 50(0)$

$Q = 500$

![Graph of the linear equation](image)
Find the Slope

Slope is rise over run. Alternatively stated, slope is the change in P over the change in Q.

Using the vertical and horizontal intercepts above, we start at point (0, 10) and move along the line to point (500, 0). The rise is -10 and the run is 500. So the slope is:

\[-10/500 = -1/50 \text{ or } -0.02\]

There is another way to find the slope that you will need to know.

\[Q = 500 - 50P\]

Solve for P:

\[50P = 500 - Q\]
\[P = 10 - 0.02Q\]

Recall from algebra (or Khan Academy) that the slope-intercept form of a linear equation is \(y = mx + b\), where \(m\) is the slope and \(b\) is the \(y\)-intercept. How does the equation above relate to the slope-intercept form of a linear equation?

\[y = P\]
\[m = -0.02\]
\[x = Q\]
\[b = 10\]

\[Q = 10 + P\]
\[P = -10 + Q\]

P-intercept: -10 (There's no such thing as a negative price.)

Q-intercept: 10

Slope: 1

There is no such thing as a negative price or negative demand, so in this class we only use quadrant I on the graph. So how would you graph this? You can graph this because you have the slope. The slope is 1, so for every rise of 1 there is also a run of 1. If the rise is 45 then the run is 45. Starting at point (10, 0) draw the line at a 45 degree angle away from the \(P\) axis.
Notice that the slope is positive. That means this equation represents supply. Demand equations have a negative slope.

**Group Work**

\[ Q = \frac{1}{2} - \frac{3}{4}P \]

\[
\frac{3}{4}(P) = \frac{1}{2} - Q \\
P = (\frac{1}{2} \div \frac{3}{4}) - (Q \div \frac{3}{4}) \\
P = (\frac{1}{2} \times \frac{4}{3}) - \frac{4}{3}(Q) \\
P = \frac{2}{3} - \frac{4}{3}(Q) \\
P\text{-intercept: } \frac{2}{3} \\
Q\text{-intercept: } \frac{1}{2} \\
Slope: -\frac{4}{3}
\]

\[ 2000Q = 10,000 - 5000P \]

\[
5000P = 10,000 - 2000Q \\
P = 2 - 0.4Q
\]
P-intercept: 2
Q-intercept: 5,000
Slope: -0.4

**Solving a System of Two Linear Equations**

$Q_D = 50 - 10P_D$

$Q_S = 20 + 5P_S$

We want to know where these two lines meet; that is, where does $Q_D = Q_S$ and $P_D = P_S$?

$$Q_D = Q_S = 50 - 10P = 20 + 5P$$

Solve for $P$

$$50 - 10P = 20 + 5P$$

$$30 = 15P$$

$$P = 2$$

Plug $P$ into either the demand or the supply equation

$$Q_D = 50 - 10(2)$$

$$Q_D = 50 - 20 = 30$$

$25Q_D = 50 - P_D; Q_S = 5P_S$

There are two ways to solve it. You can isolate $Q_D$ and then set the equations equal to each other, as described below:

*Isolate $Q_D$:*

$$Q_D = 2 - 0.04P_D$$

*Set the equations equal to each other and solve:*

$$2 - 0.04P = 5P$$

$$5.04P = 2$$

$$P = 0.397 \text{ (last digit rounded up)}$$

*Or, you can simply plug $Q_S$ into the demand function:*

$$25(5P) = 50 - P$$

$$125P = 50 - P$$

$$126P = 50$$

$$P = 0.397 \text{ (last digit rounded up)}$$
Now, plug P into one of the original equations:

\[ Q_s = 5(0.397) = 1.985 \]

**Group Work**

\[
\frac{1}{4}Q_D = \frac{1}{2} - P_D; \quad Q_s = \frac{1}{4} + \frac{1}{2}P_s
\]

*Plug Q_s into demand equation*

\[
\frac{1}{4}(\frac{1}{4} + \frac{1}{2}P) = \frac{1}{2} - P
\]

\[
\frac{1}{16} + \frac{1}{4}P = \frac{1}{2} - P
\]

\[
\frac{5}{4}P = \frac{7}{16}
\]

\[
P = \frac{7}{16} \times \frac{4}{5} = \frac{7}{4} \times \frac{4}{5} = \frac{7}{20} = 0.35
\]

\[
8Q_D = 6 - 14P_D; \quad 3Q_s = 2 + 13P_s
\]

Isolate Q_D:

\[
Q_D = \frac{6}{8} - \frac{14}{8}P_D
\]

\[
Q_D = 0.75 - 1.75P_D
\]

*Plug Q_D into supply equation:*

\[
3(0.75 - 1.75P_D) = 2 + 13P_s
\]

\[
2.25 - 5.25P = 2 + 13P
\]

\[
0.25 = 18.25P
\]

\[
P = 0.014
\]

*Plug P into original demand equation*

\[
8Q_D = 6 - 14(0.014)
\]

\[
8Q_D = 5.804
\]

\[
Q_D = 0.7255
\]

**Exponents Overview**

\[
x^3 = x \times x \times x
\]

\[
x^{-3} = \frac{1}{x^3}
\]

\[
x^{(1/3)} = \sqrt[3]{x}
\]

\[
x^{(3/4)} = \frac{1}{\sqrt[4]{x^3}}
\]