Journal of the Operational Research Society (2005) 56, 1265–1272
www.palgrave-journals.com/jors

Using hazard networks to determine risk reduction strategies

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In many cases, the interdependencies among the hazards in a system can be expressed by means of a *hazard network*, in which the nodes correspond to the hazards and the links indicate how they depend on one another. We formulate a set of mathematical optimization models which apply in such circumstances and can be used to determine the best protection or prevention strategy based on the estimated costs of mitigation. We illustrate how the model works in three different situations: (1) when total mitigation of any hazard eliminates all the hazards that follow it, (2) when a number of hazards must be totally mitigated to achieve the same effect, and (3) when partial mitigation helps to reduce the risks associated with subsequent hazards. The models developed and illustrated are readily scalable and should apply to a wide range of risk management problems.

Journal of the Operational Research Society (2005) **56,** 1265–1272. doi:10.1057/palgrave.jors.2601953
Published online 16 March 2005

Keywords: hazard mitigation; risk reduction; network models; risk management; optimization

Background

The notion of hazard networks, which we introduce here, was motivated by the discussion of risk networks in the project management literature, as evidenced by Chapman and Ward.[1] In earlier studies related to construction management, Ren[2] discussed the basic patterns of risk relationships in a system, as did Diekmann[3] with the aid of influence diagrams. More recently, risk networks were discussed by Rodrigues[4] in relation to analysing risk using system dynamics, and by Kuismanen *et al*[5] in conjunction with the concept of risk inter-relation management. The principal contribution of this research is intended to be the generalization of these concepts and the application of optimization methods to the related decision-making processes.

The hazard network concept

In general, operating a system means conducting a number of different interrelated activities. When risks are involved there will be associated hazards related to the conditions in which the activities are conducted (eg, at a construction site) or on the nature of the activities themselves (eg, the operation of machinery). The presence of these hazards can give rise directly to events with associated losses, and those events or losses can bring about new hazards (eg, a fire can weaken a structure, creating a collapse hazard). For simplicity, we assume that there is one and only one possible loss event E_j corresponding to each hazard H_j and that E_j either occurs or not. If the probability (or relative frequency of occurrence) of E_j is P_j and the consequence (loss or damage) is C_j, then the associated risk (or expected loss or damage) is $r_j = P_jC_j$. (More generally, when hazard H_j has multiple corresponding loss events E_{jn} ($n = 1, 2, \ldots N_j$) with associated probabilities P_{jn}, the single risk value r_j becomes the expected value $\Sigma_n P_{jn} C_{jn}$. This can be readily generalized to multiple types of consequences, provided they are additive). Strictly speaking, each P_j is conditional in nature because it is the probability that event E_j occurs given hazard H_j.

Now suppose that the occurrence of the event associated with one hazard can create another hazard, indicating an interdependency between the two hazards. For example, a worker who suffers an injury while operating a machine might lose control and create a hazard to other workers nearby. When a number of such situations exist in a system, we can represent the interactions involved by constructing a network model in which the nodes represent hazards and the arcs represent interdependencies between them. In this model, each node j corresponds to a hazard H_j and has an associated risk r_j, each arc (j, k) indicates that the occurrence of E_j (the event associated with hazard H_j) will create another hazard H_k, and the absence of an arc (j, k) means that the occurrence of E_j will not result in hazard H_k. We shall refer to this model as a *hazard network*. If any node in this model has no *incoming* arcs, it means that the associated hazard cannot be created by an event but exists instead

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because of the conditions under which the system is operated. Any node that has no *outgoing* arcs indicates that the associated event cannot create other hazards. When a node has multiple incoming arcs, the indication is that the associated hazard can be created by a combination of events. For simplicity, we assume that the hazard is created either by the intersection or the union of these events. In the former case, every one of the events must occur to create the hazard, while in the latter case, the occurrence of any of the events will create the hazard. A node that has multiple outgoing arcs indicates that the event associated with that hazard can, either alone or in conjunction with other events, create more than one other hazard.

The hazard network concept should prove to be particularly useful for analysing the risks associated with cascading events, where an initiating event can trigger other events by creating a succession of hazards. Unlike event trees, which only show which events can give rise to other events, depending on whether they happen or not, a hazard network shows all of the hazards in a system, identifying which ones are existing hazards and which ones can be created, individually or jointly, by the events associated with other hazards. The hazard network model thus contains more useful information for examining alternative approaches to risk reduction and determining optimal intervention strategies.

The distinction between hazard networks and Bayesian networks should also be explained. Bayesian networks provide a graphical means of displaying the probabilistic interdependencies among a large set of random variables. They help in understanding causal relationships and predicting the impacts of intervention. In a Bayesian network the nodes represent the random variables and the arcs show which ones are interdependent. The nodes that connect to any given node are referred to as the parents of that node. Probability distributions relate the conditional probability of each variable to each possible set of parents for the node corresponding to that variable. Thus, in a risk management context, the variables could be the outcomes of different loss events, related to one another by the conditional probability distribution of the loss outcome of each event given the loss outcomes of its parent events. Comparatively, in our hazard networks each node corresponds to a particular outcome (a hazard arises) rather than a random variable (the magnitude of a loss outcome). Additionally, each arc in a hazard network corresponds to a particular event that can cause one hazard to lead to another (with some probability) rather than the existence of an interdependence between two random variables (the loss outcomes of two different events); in both cases an arc represents interdependence but in one case it is between hazards and in the other it is not. These differences are consistent with the differences cited earlier between event trees and hazard networks, which is not surprising given the correspondence between Bayesian networks and decision or event trees. See Jensen[6] for a comprehensive discussion of Bayesian networks and King[7] on the use of Bayesian networks in causal modelling and intervention analysis for operational risk management.

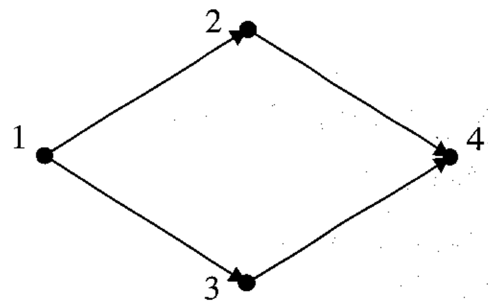

Figure 1 A simple hazard network.

To illustrate an elementary hazard network having a number of the features discussed above, suppose the first activity of concern is driving a car, which exposes the driver to an accident hazard (H_1) that can result in vehicle damage (E_1). This damage can create an electrical hazard (H_2) that can result in a short circuit (E_2), as well as a fuel hazard (H_3) that can result in a major leak (E_3). Together, the short circuit and fuel leak can produce an ignition hazard (H_4) that can result in a fire (E_4). The hazard network thus has four nodes numbered 1–4 connected by four directed arcs (1,2), (1,3), (2,4) and (3,4), as shown in Figure 1. Note that there is no standard way to show on a network diagram that the incoming arcs at node 4 correspond to a conjoint set of events (E_2 and E_3).

In the cases we consider below, we will distinguish between situations in which risk reduction measures are totally effective or only partially so, in which unions of events are included or not, and in which downstream effects of risk reductions extend beyond immediately succeeding hazards or not.

Preventive *versus* protective hazard mitigation

For a given hazard network, the total risk is the sum of the risks associated with the hazards. Each risk is reduced by mitigating the associated hazard, where the mitigation effort may be focused on probability reduction or consequence reduction. Hazard mitigation, which may be partial or total, can be preventive or protective in nature. We will use the term *preventive* hazard mitigation to refer to any effort that reduces the probability of a loss event. Likewise, we will use the term *protective* hazard mitigation to refer to any effort that reduces the consequence of a loss event. Preventive and protective measures are both achieved at a cost.

To illustrate, in the vehicle accident example the risk of a fire can be reduced either by designing the vehicle to contain any fuel leaks or by building the vehicle with flame retardant materials to keep any fires from spreading. The fuel leak containment measure is preventive in nature because it reduces the likelihood of a fire, while the flame retardant measure, which reduces the extent of fire damage, is protective in nature.

Our purpose here is to develop a series of mathematical models for determining optimal risk reduction strategies for hazard networks, given a variety of assumptions for hazard mitigation. Below we specify three different cases for the model assumptions:

1. Each hazard can either be mitigated completely or not at all, using either a protective or preventive measure. A hazard is avoided when *any* of the events that can create it is kept from occurring by mitigating or avoiding any preceding hazard. This implies that only a single event or an intersection of multiple events can create a hazard. We will refer to these as *single-trigger* hazards.
2. Same as the first case, except that additional hazards are included which can only be avoided when *all* of the events that can create it are kept from occurring by mitigating or avoiding the preceding hazards. Only a union of multiple events can create each such hazard, which we will refer to as a *multiple-trigger* hazard.
3. Same as the first case, except that the hazards can be partially mitigated and the effects of any risk reduction are limited to immediately succeeding hazards.

Case 1—binary risk reduction with single-trigger hazards

This simple version of the risk reduction model helps the system manager decide which hazards will be mitigated using the resources under his or her control. We assume that the only resource involved is a hazard mitigation budget, but the extension to multiple resources is straightforward. Essentially, the budget will be allocated among the available hazard mitigation options. If there is more than one option per hazard, only the least expensive one will be included in the model. Without difficulty, the model can be altered to take the form of minimizing the total cost of achieving a specified reduction in the overall risk.

Introducing additional notation, we let c_j be the cost of mitigating hazard H_j and we denote the decision variables by y_j, where

$$y_j = 1 \quad \text{if } H_j \text{ is mitigated and 0 otherwise}$$

When a hazard H_j is mitigated we assume that every hazard that could be created (ie, triggered) by the event E_j—whether alone or in conjunction with other events—is avoided, as is every hazard that could be created in a similar way by the events associated with the avoided hazards, and so forth. To flag each mitigated or avoided hazard we introduce the indicator variables x_j, where

$$x_j = 0 \text{ if } H_j \text{ has been mitigated or avoided and 1 otherwise.}$$

Thus, if funds are spent to mitigate H_j then $y_j = 1$ and $x_j = 0$ as a direct result of the mitigation. Furthermore, as an indirect result of the mitigation (ie, due to hazard avoidance), $x_k = 0$ for every node k that succeeds node j in the hazard network.

To formulate the optimization model we require some additional notation:

N	the number of nodes in the hazard network
J	the set of nodes j in the network
r_j^{o}	the initial risk associated with H_j
$S(j)$	the set of nodes k that succeed node j in the network
$P(j)$	the set of nodes k that precede node j in the network
b	the hazard mitigation budget

In a network sense, node k succeeds node j if and only if it is on a directed path from node j to the final node in the network. Likewise, node k precedes node j if and only if it is on a directed path from the initial node in the network to node j. Assuming the risks are additive, the optimization model is then

$$\text{Min} \quad z = \sum_j r_j^{\mathrm{o}} x_j$$

$$\text{s.t.} \quad x_j + \sum_{k \in S(j)} x_k \leqslant N(1 - y_j) \text{ for all } j \in J \tag{1}$$

$$\sum_{k \in S(j)} y_k \leqslant N(1 - y_j) \quad \text{for all } j \in J \tag{2}$$

$$x_j \geqslant 1 - \left[y_j + \sum_{k \in P(j)} y_k \right] \quad \text{for all } j \in J \tag{3}$$

$$\sum_j c_j y_j \leqslant b \tag{4}$$

$$x_j, y_j = 0, 1 \quad \text{for all } j \in J$$

The objective function measures the total risk associated with the hazards that have not been mitigated or avoided (ie, the ones for which $x_j = 1$). When the decision is made to mitigate H_j, the right-hand side of the first constraint requires that hazard and every succeeding hazard to be mitigated or avoided. The second constraint requires that, as a further consequence of that decision, none of the succeeding hazards will be eligible for mitigation—otherwise, unnecessary expenditure could be incurred. Given a set of values for the decision variables, the third constraint prevents H_j from being mitigated or avoided unless the decision has been made to mitigate it or any of the hazards that precede it—otherwise, no-cost hazard avoidance would be possible. Finally, the fourth constraint ensures that the hazard mitigation budget is not exceeded.

Example

Figure 2 shows a hazard network with eight nodes and hazards H_1 to H_8. H_3, H_5, H_6, and H_8 are all single-trigger hazards. H_3 is avoided if H_1 or H_2 is mitigated because both events must occur to create H_3; similarly, H_5 is avoided if H_3 or H_4 is mitigated and H_8 is avoided if H_5, H_6 or H_7 is mitigated. To avoid H_6, only H_3 needs to be mitigated. For each node, Table 1 shows the magnitudes of the risks, the mitigation costs, and the set of predecessor and successor nodes. The hazard mitigation budget is 10.

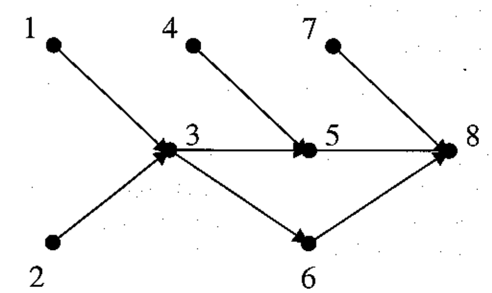

Figure 2 Hazard network for the Case 1–3 examples.

Table 1 Model data for the Case 1 and 2 examples

j	*Risk* r_j^o	*Cost* c_j	*Predecessors* $P(j)$	*Successors* $S(j)$
1	5	14	—	3,5,6,8
2	5	6	—	3,5,6,8
3	5	4	1,2	5,6,8
4	5	1	—	5,8
5	5	3	1,2,3,4	8
6	5	2	1,2,3	8
7	5	3	—	8
8	5	4	1,2,3,4,5,6,7,8	—

Table 2 Solutions to the Case 1 and 2 examples

j	x_j	y_j
(a) Case 1		
1	1	0
2	0	1
3	0	0
4	0	1
5	0	0
6	0	0
7	0	1
8	0	0
(b) Case 2		
1	1	0
2	0	1
3	1	0
4	0	1
5	0	0
6	0	1
7	0	1
8	0	0

Part (a) of Table 2 shows the optimal solution. Of the eight hazards, only H_1 is neither mitigated nor avoided; hence only risk 1 is not reduced, resulting in a value of 5 for the optimal residual risk z^*. Hazards H_2, H_4, and H_7 are mitigated at a total cost of 10, which causes the remaining hazards (H_3, H_5, H_6 and H_8) to be avoided.

Case 2—binary risk reduction with multiple-trigger hazards

We now extend the above model to admit the possibility of multiple-trigger hazards created by the union of multiple events. Under these circumstances, we include situations in which a hazard can be created unless every one of the immediately preceding hazards is mitigated or avoided.

In the Case 1 model, constraints (1), (2), and (3) were based on the fact that mitigating a hazard automatically avoided all of the downstream hazards, but in the Case 2 model, this is no longer the case. Now mitigating or avoiding a hazard H_j will not necessarily cause a subsequent hazard H_k to be avoided if H_k or some hazard between H_j and H_k may be a multi-trigger hazard. To incorporate this feature we introduce a new set of coefficients α_{jk} which we define as follows, where $A(j, k)$ is the set of all paths from j to k and J_M is the set of all nodes corresponding to multiple-trigger hazards:

$\lambda_{jk}=$ 1 if there exists at least one path in $A(j, k)$ that does not contain a node in J_M

$\lambda_{jk}=$ 0 if every path in $A(j, k)$ contains a node in J_M

The value of each λ_{jk} can be determined by first identifying every node in the network that belongs to J_M and then generating every path from j to k to find out which ones, if any, contain a node in J_M. The K-shortest paths algorithm as discussed by Eppstein[8] is an ideal tool for generating the paths in these circumstances.

The modified versions of the three constraint sets are then of the form

$$x_j + \sum_{k\in S(j)} \lambda_{jk}x_k \leqslant N(1-y_j) \quad \text{for all } j\in J \qquad (1')$$

$$\sum_{k\in S(j)} \lambda_{jk}y_k \leqslant N(1-y_j) \quad \text{for all } j\in J \qquad (2')$$

$$x_j \geqslant 1 - \left(y_j + \sum_{k\in P(j)} \lambda_{kj}y_k\right) \quad \text{for all } j\in J \qquad (3')$$

These new constraints have the following implications: in constraint (1′) H_k is not avoided by mitigating H_j if every path from j to k contains a multiple-trigger node; in constraint (2′), the need to prevent H_k is not precluded by preventing H_j if every path from j to k contains a multiple-trigger node; and in constraint (3′), H_j is not avoided when H_k is prevented if every path from k to j contains a multiple-trigger node.

Constraint (4) is unchanged, but two new constraints are needed. Defining the set $\pi(j)$ to be the set of immediate predecessors to node j, where $j \in J_M$, and letting $n[\pi(j)]$ be the number of nodes in that set, the constraints are

$$n[\pi(j)]x_j \geqslant \sum_{k \in \pi(j)} x_k \quad \text{for all } j \in J_M \tag{5}$$

$$\sum_{k \in S(j)} \lambda_{jk} x_k \leqslant N x_j \quad \text{for all } j \in J_M \tag{6}$$

Constraint (5) requires that if any of the immediate predecessors of a multiple-trigger hazard H_j is neither mitigated nor avoided then H_j cannot be avoided. Constraint (6) requires that, when a multi-trigger hazard H_j is mitigated or avoided, every successor H_k will be mitigated if all the paths from j to k contain a multi-trigger node.

Example

This example is the same one used for Case 1, except that H_3 is now a multiple-trigger hazard, which means that H_3 is not avoided unless H_1 and H_2 are both mitigated. We have $J_M = \{3\}$, $\pi(3) = \{1,2\}$, and $n[\pi(3)] = 2$. The non-empty sets of all paths $A(j, k)$ are displayed in Table 2. The corresponding values of λ_{jk} are zero unless node 3 is absent from a path, in which case $\lambda_{jk} = 1$.

Part (b) of Table 2 contains the optimal solution. The total residual risk is minimized at $z^* = 10$ and the total mitigation cost is 8. The selected strategy is to mitigate hazards 2, 4, 6, and 7. The residual risks are then associated with hazards H_1 and H_3, where H_1 was left unmitigated and H_3, a multi-trigger hazard, could only have been avoided by mitigating both H_1 and H_2.

Continuous risk reduction with single-trigger hazards and no downstream impacts

Unlike the first two cases, in which hazard mitigation was either 0 or 100%, in this case we allow for the possibility of partial mitigation. Before, whether the focus of the mitigation was of a preventive (probability-related) or protective (consequence-related) nature, the impacts were the same: total elimination of the target hazard, total avoidance of any immediately succeeding single-trigger hazards, and—in the absence of any intervening multi-trigger hazards—total avoidance of all downstream single-trigger hazards.

Now, however, we need to differentiate between taking preventive and protective steps, because we are making the following basic assumptions, given that a direct link exists between two hazards H_j and H_k in a hazard network:

1. A less-than-total *preventive* measure that reduces the probability component of risk r_j (ie, the probability of E_j) does not reduce risk r_k because the consequence of E_j is unchanged.
2. A less-than-total *protective* measure taken to reduce the consequence component of risk r_j (ie, the consequence of E_j) does reduce r_k because the event E_j that creates H_k becomes less severe, which makes the next event E_k less likely or less severe, or both.

Based on this understanding, we now proceed to formulate an optimization model for determining the best combination of non-binary prevention and protection decisions in a hazard network, given a total risk reduction budget.

Let w_j represent the fractional reduction in the consequence C_j associated with risk r_j, where $0 \leqslant w_j \leqslant 1$. As w_j increases from 0 to 1, we assume two things happen: (1) r_j declines linearly from r_j^o to 0, and (2) a fractional reduction in risk is triggered for every node that immediately succeeds node j. We assume further that each such reduction does not begin until a certain lower threshold value of w_j is reached (which could be zero) and terminates when a certain upper threshold value of w_j is reached (which could be 1), and that the reduction is linear in between.

Note that this version of the continuous risk reduction model is based on two simplifying assumptions: (1) unlike Case 2, there are no multiple-trigger hazards, and (2) unlike Cases 1 and 2, there are no downstream impacts beyond the nodes that immediately succeed the nodes where risk reductions involving protective measures are made (ie, where hazards are mitigated to some degree). The second assumption is most realistic in situations where the only effect that a protective risk reduction measure at any node j has on an immediately succeeding node k is to partially reduce the probability component of the risk r_k. Under those circumstances the effect is strictly preventive in nature, and according to the differentiation articulated above, such an effect will not generate any further risk reduction downstream.

Continuing now with the model formulation, we introduce the following notation: let r_k^o be the initial value of r_k and let α_{jk} and β_{jk} be the lower and upper threshold values of w_j that relate to the reduction in r_k, where node k is an immediate successor of node j. Then Δr_k, the reduction in r_k due to an increase in w_j, is as follows for every $k \in \text{IS}(j)$, where IS(j) the set of nodes that immediately succeed node j:

i Δr_k remains zero until C_j is reduced to C_{jk}^a, at which point $w_j = \alpha_{jk} = 1 - C_{jk}^a / C_j$

ii Δr_k then increases at a constant rate until C_j is reduced further to C_{jk}^b, at which point $w_j = \beta_{jk} = 1 - C_{jk}^b / C_j$; and $\Delta r_k = r_k^o - r_{jk}^\prime$

iii Δr_k then remains constant as C_j is reduced further to zero, at which point $w_j = 1$.

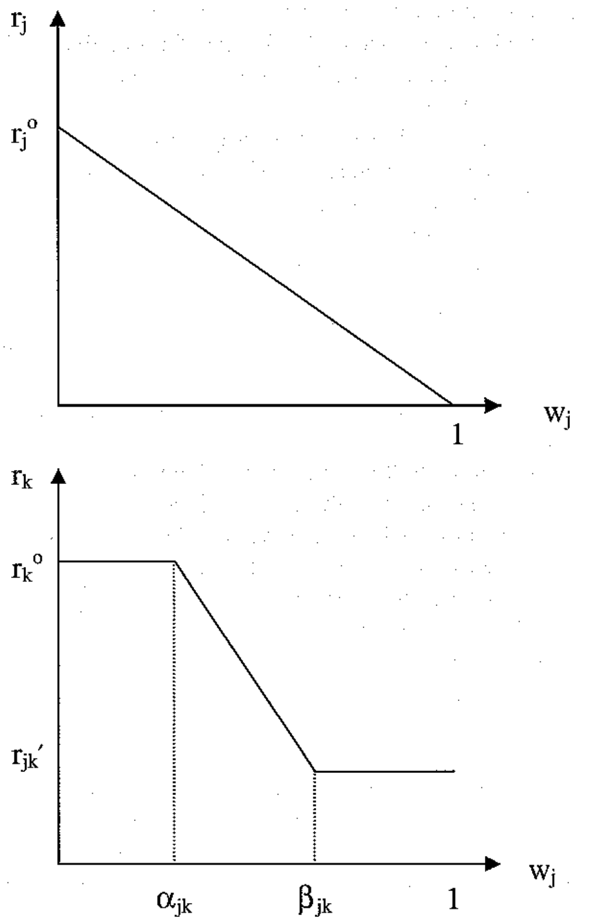

Figure 3 Dependence of r_j and r_k on w_j when $k \in \mathrm{IS}(j)$.

If we let u_{jk} be the fractional reduction in r_k attributable to w_j, then this reduction pattern translates to the following values for u_{jk}:

i $u_{jk}=0$ if $w_j \leqslant \alpha_{jk}$
ii $u_{jk}=\delta_{jk}(w_j-\alpha_{jk})$ if $\alpha_{jk}<w_j<\beta_{jk}$, where δ_{jk} is rate of change in u_{jk} from α_{jk} to β_{jk}
iii $u_{jk}=\gamma_{jk}$ if $w_j \geqslant \beta_{jk}$, where $\gamma_{jk}=\delta_{jk}(\beta_{jk}-\alpha_{jk})=1-r'_{jk}/r^{o}_{k}$.

Figure 3 shows graphically the nature of the assumed relationships between (a) r_j and w_j and (b) r_k and w_j, and Figure 4 shows the relationship between u_{jk} and w_j. To incorporate the function shown in Figure 4 in the optimization model, we replace the formulas for u_{jk} by the constraints below, which require a new set of 0-1 variables z_{jk}

$z_{jk} \leqslant w_j/\alpha_{jk}$
$z_{jk} \geqslant w_j/(M\beta_{jk})$, where M is a suitably large number
$u_{jk}=z_{jk}\gamma_{jk}$
$u_{jk} \leqslant \delta_{jk}(w_j-\alpha_{jk})$

When $w_j<\alpha_{jk}$, $z_{jk}=0$ and u_{jk} is 0, and when $w_j>\beta_{jk}$, $z_{jk}=1$ and $u_{jk}=\gamma_{jk}$. Otherwise, $u_{jk}=\delta_{jk}(w_j-\alpha_j)$.

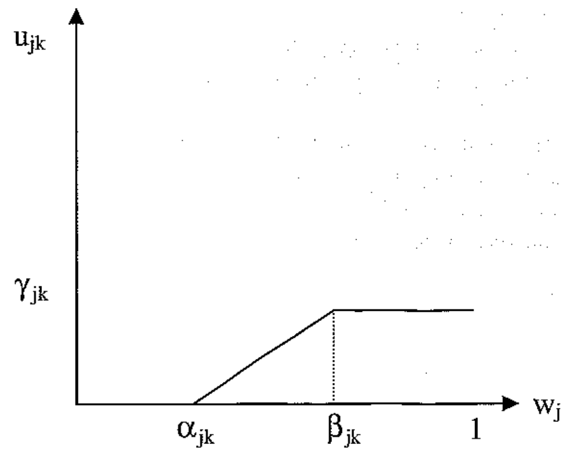

Figure 4 u_{jk} as a function of w_j when $k \in \mathrm{IS}(j)$.

If we let IP(k) be the set of nodes j that immediately precede node k, the risk r_k will be reduced by a factor equal to the product of all the terms $1-u_{jk}$, or $\Pi_{j\in \mathrm{IP}(k)}(1-u_{jk})$. This risk will also be reduced by two other factors $1-v_k$ and $1-w_k$, reflecting the reductions in P_k and C_k, respectively. The value of v_k, as with w_k, can range from 0 to 1 and is assumed to have a linear effect on r_k.

Based on this discussion, the model for minimizing the total risk in the network is as follows, where d_j and e_j are the costs of a one-percent reduction in P_j and C_j, respectively, and b is the budget for total prevention and protection expenditure

$$\text{Min} \quad z=\sum_{k\in N}[r^{o}_{k}(1-v_k)(1-w_k)\Pi_{j\in \mathrm{IP}(k)}(1-u_{jk})]$$

$$\begin{aligned} \text{s.t.} \quad & z_{jk} \leqslant w_j/\alpha_{jk} \quad \text{for all } j\in N, k\in \mathrm{IS}(j) \\ & z_{jk} \geqslant w_j/(M\beta_{jk}) \quad \text{for all } j\in N, k\in \mathrm{IS}(j) \\ & u_{jk}=z_{jk}\gamma_{jk} \quad \text{for all } j\in N, k\in \mathrm{IS}(j) \\ & u_{jk} \leqslant \delta_{jk}(w_j-\alpha_{jk}) \quad \text{for all } j\in N, k\in \mathrm{IS}(j) \\ & \sum_{j\in J}(d_j v_j + e_j w_j) \leqslant b/100 \\ & 0 \leqslant u_{jk} \leqslant 1 \quad \text{for all } j\in N, k\in \mathrm{IS}(j) \\ & 0 \leqslant v_j \leqslant 1 \quad \text{for all } j\in N \\ & 0 \leqslant w_j \leqslant 1 \quad \text{for all } j\in N \\ & z_{jk}=0,1 \quad \text{for all } j\in N, k\in \mathrm{IS}(j) \end{aligned}$$

Example

Again, we use the network depicted in Figure 2 to illustrate the application of the optimization model, except that we revert to the single-trigger interpretation from Case 1 and we alter the initial risk values as shown in Table 3. Table 3 also shows the other data used for this example. The budget limit is set at a value of 15. Whereas the optimization models for the first two cases were linear integer zero-one programming problems, the model in this case has the more complicated form of a mixed-integer zero-one polynomial programming

Table 3 Model data for the Case 3 example

	Probability	*Consequence*	*Risk*	*Costs*		*Node sets*	
j	*Pj*	C_j	r_j^o	d_j	e_j	*IP(j)*	*IS(j)*
1	0.1	50	5	2	6	—	3
2	0.4	40	16	6	5	—	3
3	0.3	60	18	4	8	1,2	5,6
4	0.05	100	5	2	2	—	5
5	0.6	20	12	3	3	3,4	8
6	0.5	30	15	4	7	3	8
7	0.05	40	2	3	3	—	8
8	0.2	30	6	4	4	5,6,7	—

Risk reduction shape parameters

(*j*, *k*)	C^a_{jk}	C^b_{jk}	α_{jk}	β_{jk}	r'_{jk}	γ_{jk}	δ_{jk}
1,3	30	14	0.40	0.72	12	0.33	1.04
2,3	20	10	0.50	0.75	10	0.44	1.78
3,5	40	10	0.33	0.83	12	0.00	0.00
3,6	40	10	0.33	0.83	11	0.27	0.53
4,5	40	25	0.60	0.75	12	0.00	0.00
5,8	15	5	0.25	0.75	5	0.17	0.33
6,8	25	5	0.17	0.83	5	0.17	0.25
7,8	25	15	0.38	0.63	5	0.17	0.67

problem. Solution algorithms have been developed specifically for the purpose of solving such problems, as discussed in the literature (Sherali and Tuncbilek[9]). For the sake of convenience we used the nonlinear GRG option provided in the Premium Solver add-in software package.

The optimal solution reduced the total risk from an initial value of 79 to a final value of 20.52. The only non-zero values in the optimal solution were $w_2 = v_5 = v_6 = 1.00$, $u_{23} = 0.44$, $v_3 = 0.75$, and $z_{23} = 1$. The interpretation of $w_2 = 1.00$ is that the consequence associated with the risk at node 2 is reduced to 0. This reduction triggers a 44% reduction in the risk at node 3, which is compounded by a 75% reduction in the probability associated with the risk at node 3. In addition, the probabilities associated with the risks at nodes 5 and 6 are reduced to zero. The total cost of these reductions is $5(1) + 4(.75) + 3(1) + 4(1) = 15$.

Conclusions

The research discussed here employs the concept of a hazard network to represent the relationships among the hazards and risks in a system and introduces a set of optimization models to determine the best intervention strategy. The concept is intentionally general in nature and should have a wide range of applicability to different kinds of system safety investigations. Two of the three optimization models are conventional integer zero-one programming problems for which solution algorithms are readily available, while the third one has a polynomial structure in the objective function, which puts it in a less common class of problems which has received less attention but has been an active area of research in recent years. For the small size of the network we considered, the Premium Solver software package proved to be more than adequate, but to solve more realistic problems, more specific tools may be required. Any problem of realistic size that contains multiple triggers will also require a tool for solving the *K*-shortest path algorithm, which would not be difficult to find. Despite the small size of the example, the models we presented are readily scalable to much larger problems.

As formulated, the proposed model includes a budget constraint, which makes it is useful for exploring the tradeoff between the minimum achievable risk and the funds available for risk reduction, assuming that the risk value cannot be monetized. Starting with a low value for *b* and solving the optimization problem for each successively higher range of *b* the entire tradeoff curve can be generated. Alternatively, when the risk can be expressed in monetary terms, we can add the left hand side of the budget constraint (ie, the cost of risk reduction) to the objective and eliminate the constraint. To do so is to use net risk reduction benefit for the objective function, because minimizing the sum of residual risk plus risk reduction cost is equivalent to maximizing the difference between risk reduction (initial risk less residual risk) and risk reduction cost. Assuming that the r_k^o values in the example are in fact monetary, and that the cost $\sum_{j \in J}(d_j v_j + e_j w_j)$ is included in the objective function, the optimal solution shows that the residual risk declines from 20.52 to 9.59, at a risk reduction cost of 24.59, which is of course higher than the original budget limit of 15.

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Received June 2004;
accepted December 2004 after one revision