Problem Sheet 4  
Due date: 10 February 2016 16:00

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.

Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format. I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework. News and .pdf-files of Problems also at home.gwu.edu/~hgrie/lectures/nupa-16I/nupa-16I.html.

1. The $\Delta(1232)$ Resonance in $\gamma p$ Scattering (4P):

The total cross section of photon scattering on protons shows for $\omega < 600$ MeV a resonance with spin $J = 3/2$ (plot in lab frame).

a) (3P) Take the figure to approximate the cross section around the resonance by the Breit-Wigner form and extract the position of the pole of the scattering matrix in the complex-energy plane, i.e. the invariant mass $M_\Delta$ and decay width $\Gamma$. In the cm frame, let’s take a quasi-nonrelativistic form, with $E_{\text{total}}^{\text{cm}} = \sqrt{s}$:

$$\sigma \propto \frac{1}{|k_{\text{cm}}|^2} \frac{1}{(E_{\text{total}}^{\text{cm}} - M_\Delta)^2 + \Gamma^2/4}$$

b) (1P) Why is $M_\Delta$ not just given by the peak position in the plot, plus the proton mass?

2. $e^+e^- \rightarrow \mu^+\mu^-$, Part II (10P): We continue the calculation of electron-positron annihilation into muons. In the last HW, you derived its matrix element for an electron (spin $s$, 4-momentum $k$) and positron (spin $r$; 4-momentum $p$) converted into a muon and anti-muon of spins $s'$ and $r'$ and momenta $k'$ and $p'$ via a virtual photon of 4-momentum $q$ as:

$$\mathcal{M}_{s' r' ; s r}(k, p; k', p') = -Ze^2 \bar{v}_r(p) \gamma^\mu u_s(k) \frac{1}{q^2 = (k + p)^2} \bar{U}_{s'}(k') \gamma^\mu V_{r'}(p') .$$

Upper-case spinors denote muons/anti-muons, and lower-case spinors $e^\pm$. Now finish the calculation:

a) (2P) To which Mandelstam variable is the 4-momentum $q$ of the virtual photon related? Argue that it is a time-like vector - unlike in $e\mu \rightarrow e\mu$, where it is spacelike.

Note: The following sub-problems are independent of a).

b) (4P) Calculate the spin-averaged matrix-element squared for massless particles, recalling our normalisations $\sum_s u_s(p)\bar{u}_s(p) = p \cdot \gamma + m$ and $\sum_s v_s(p)\bar{v}_s(p) = p \cdot \gamma - m$.

c) (3P) Rewrite it into Mandelstam variables and show the crossing symmetry to $e\mu \rightarrow e\mu$. The result is proportional to $\frac{t^2 + u^2}{s^2}$.

d) (1P) Derive the differential cross section in the cm frame (result given in the lecture).

Please turn over.
3. Inelastic Scattering (2P): Its most general hadron tensor was given in the lecture.
Show that $W_1(Q^2,x) = 0$ for a spin-zero target. Since targets without spin cannot directly couple to magnetic fields, that’s pretty good evidence that $W_1$ parametrises magnetic interactions.

4. $\beta$ Decay of Nuclei in the Liquid-Drop Model (4P): Write a short essay which compares the $\beta$ decay of an odd-$A$ nucleus against that of an even-$A$ nucleus. Provide qualitative arguments on the difference in typical decay rates or stability of the respective nucleus against that decay. You should base your arguments on the Bethe-Weizsäcker formula and include instructive sketches and examples. Yes, you may get inspired by a good textbook. This is a “Reading Assignment”.

5. Bethe-Weizsäcker Formula (6P): Let’s have a bit more qualitative fun. These problems are independent of each other.

   a) (2P) Imagine a world in which the fine structure constant of electromagnetic interactions would be different such that the most stable nucleus with $A = 186$ is not Wolfram but Plutonium (what a world: incandescent light bulbs made with Pu!). Lets assume that the strong interactions remain the same. How large would $\alpha$ have to be? Isaac Asimov wrote a novel about that...

   b) (1P) Compare the (classical) electromagnetic self-energy of a uniform sphere with total charge $Ze$ to the value of the parameter $a_C$ in the Bethe-Weizsäcker formula.

   c) (3P) The total binding energy of $^4$He is 28.3 MeV. Show that $\alpha$ decay becomes energetically allowed for all nuclei with $A \gtrsim 150$.

6. Virtual Particle Clouds and Charge Radii (4P): By the uncertainty principle, every particle is surrounded by a cloud of virtual particles. Charged hadrons can emit virtual photons, and these virtual photons in turn generate virtual electron-positron pairs. So hadrons are also surrounded by $e^+e^-$ pairs. Draw at least two Feynman diagrams which describe their coupling to the photon exchanged in electron scattering off hadrons. By estimating the size of this contribution to the hadron charge radius, show that such effects are much smaller than that from virtual mesons.