Problem Sheet 1

Due date: 9th September 2008 **12:00 noon**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at *http://home.gwu.edu/˜hgrie/lectures/math-methods08/math-methods08.html*.

1. Animal Physics (**10P**):

   a) (**2P**) A Canada goose (mass about 8 kg) has a cruising speed of about 20 m s^{-1}. An ostrich is the fastest running bird, with speeds up to 75 km h^{-1}. Determine the cruise speed which an ostrich would have (mass up to 150 kg) in flight – assuming they would have a comparable wing-form. Explain why ostriches are not very likely to fly.

   b) (**2P**) A desert animal has to cover great distances between water holes. It stores water in the volume of its body, but loses water by evaporation through its surface. Show that it is advantageous to be big in a desert.

   c) (**2P**) Mammals in the arctic are also pretty big. Explain this by comparing their heat production (proportional to the number of cells the animal has) to their heat loss.

   d) (**4P**) Why does a horse run uphill slower than a dog, although both have the same running speed on level ground?

      Hint: Compare the energy needed to lift an animal by a height h to the power it can generate in its muscles. Muscular power is directly proportional to the section of the muscle, not to its volume.

2. Planck's Units (**10P**): Planck's (reduced) quantum $\hbar$, the speed of light c and the gravitational constant G_N contain only the three basic SI units metre, second and kilogramme. In the (super)-natural system of units, they are all set to unity. Now, we will derive "Planck's units", namely those length, time and mass scales which can be obtained by combining only these three constants.

   Warning: You get the points for a readable and concise *derivation*, <u>not</u> for stating the result! I know you could look up the numbers anywhere.

   a) (**2P**) Out of the three constants (and no additional numerical factors), build a quantity with units of time and calculate its value in SI units. This is the *Planck-time* $T_{\rm Pl} = \sqrt{G_n \hbar / c^5} \approx 10^{-44}$ s.

   b) (**2P**) The same for the fundamental length-scale, i.e. the *Planck-length* $L_{\rm Pl}$.

   c) (**2P**) Dito for the fundamental mass-scale, i.e. the *Planck-mass* $M_{\rm Pl}$.

   d) (**2P**) And what about the fundamental temperature scale, i.e. the *Planck-temperature* $T_{\rm Pl}$?

   e) (**2P**) What's your height and mass in Planck units? Would you use them in everyday life?

Please turn over.

Some Essential Numbers which you should know by heart when woken up at night: Because formulae are nice, but we also want to explain natural phenomena quantitatively.
Units are arbitrary: Some calculate in metres, grams and Celsius, while others prefer feet, pounds and Fahrenheit. Still, some systems facilitate conversions and calculations more than others. A very efficient system is the **Natural System of Units** in which as many fundamental constants as possible have as simple a value as possible. It is particularly popular in Nuclear and High-Energy Physics.

Set the speed of light and Planck's quantum (more accurately: $\hbar = h/(2\pi)$) to $c = \hbar = 1$. This expresses velocities in units of c, and actions and angular momenta in units of $\hbar$. There is then only one fundamental unit, namely either an energy- or a length-scale. Time-scales have the same units as length-scales. When one sets on top of that Boltzmann's constant $k_\mathrm{B} = 1$, energy and temperature have the same units. This is particularly nice because one only has to memorise a handful of really relevant numbers for quick estimates. In this system, "matching units" between different parts of an equation becomes very easy: $E = m$ instead of $E = mc^2$, etc. This example also shows how to "restore" the SI answer from an answer in "natural units": Throw in enough powers of $c\,[\mathrm{m/s}]$ and $\hbar\,[\mathrm{kg\,m^2/s}]$ till you match the proper SI units. In the example, you have to convert $\mathrm{kg\,m^2/s^2}$ into kg, i.e. add two powers of m/s, which shows you have to multiply with c^2.
[As a last step, one could set Newton's gravitational constant $G_N = 1$, which eliminates *any* dimension-ful unit – but that is done only by String Theorists.]

Here some numbers which appear regularly. The list is not exhaustive, not meant to be relevant and may not be useful. Your mileage may vary. It's better to know too much than not enough. You should check these values and understand how they come about.
$\approx$: number rounded for easier memorising – it suffices to know the first significant digit.
$\widehat{=}$, $\widehat{\approx}$: correspondences are correct only in the natural system of units.

Quantity	**Value**
speed of light in vacuum	$c \approx 3\times 10^{8}$ m s^{-1}
energy electron gains when accelerated by 1 V	1 electron Volt (eV) $\approx 1.6\times 10^{-19}$ Joule $= 1\frac{q}{[\mathrm{C}]}$ J
typ. length-scale in Atomic Physics (size of H-atom, atom-spacing in solid)	1 Ångstrom (Å) $= 10^{-10}$ m $= 0.1$ nm
typ. length-scale in Sub-Atomic Physics (size of proton, neutron)	1 fermi (femtometre, fm) $= 10^{-15}$ m
conversion factor energy-length	$\hbar c \,\widehat{=}\, 1 \approx 200$ MeV fm $\approx 2\times 10^{-7}$ eV m ≈ 2 keV Å
$\Longrightarrow$ conversion factor distance-time (nat. units) (time for light to travel a typ. distance-scale)	1 fm $\widehat{\approx}\ \frac{1}{3}\times 10^{-23}$ s; 1 Å $\widehat{\approx}\ \frac{1}{3}\times 10^{-18}$ s
conversion factor energy-temperature: $E = k_\mathrm{B} T$	1 eV $\widehat{\approx}$ 11 600 Kelvin
fine-structure constant/measure of el. strength	$\alpha = \left.\frac{q^2}{\hbar c}\right\vert_{\text{Gauß}} = \left.\frac{q^2}{4\pi\varepsilon_0\hbar c}\right\vert_{\text{SI}} \approx \frac{1}{137}$ (no units!)
electron mass	$m_\mathrm{e} \approx 511$ keV
nucleon (proton, neutron) mass	$m_\mathrm{N} \approx 940$ MeV $\approx 1800\, m_\mathrm{e}$
energy levels of the Hydrogen-atom	$E_n = -\frac{\alpha^2 m_e c^2}{2n^2} \approx -\frac{13.6\ \mathrm{eV}}{n^2}$, $n = 1, 2, 3, \dots$
Bohr-radius of the Hydrogen-atom	$a_\mathrm{B} = \frac{\hbar}{\alpha m_e c} \approx 0.5$ Å
typ. energy & wave-length of visible light	$E_\mathrm{vis} \approx [2;3]\,\mathrm{eV}$, $\lambda_\mathrm{vis} \approx [400;700]\,\mathrm{nm} \approx [4000;7000]$ Å
Avogadro's number of particles per mol	$N_\mathrm{A} \approx 6\times 10^{23}$ mol^{-1}
volume of 1 mol ideal gas at STP (1 bar, 273 K)	22.4 litres (l) mol^{-1}