

# PHYS 209: Mathematical Methods of Theoretical Physics

## Syllabus

PHYS 209 (CRN 53526), section 10

**Teacher:** Dr. Harald W. Griesshammer, Samson Hall 317, 202-994-3849, hgrie@gwu.edu.

**Lectures:** *Monday/Wednesday 12:20 to 14:00 in Samson Hall 311* (100 min each, for 4 credits).

**No lectures:** *27 September – 5 October*. Thus, there will be a makeup lecture on 9 December 2008.

**Surgery hours:** *Wednesdays at t.b.a. in Samson Hall 209* to discuss the problem sheets and for questions, discussions and suggestions; duration: till all questions are answered.

**More office hours** by appointment weekdays after 3pm in my office. Best email what and when to discuss.

**Web-site:** <http://home.gwu.edu/~hgrie/lectures/math-methods08/math-methods08.html> for up-to-date course information, .pdf-files of Problem Sheets, suggested reading, corrections, etc.

**Prerequisites:** Advanced undergraduate Mathematical Methods, Mechanics, Electrodynamics and Quantum Mechanics. **Coordinated** with *PHYS 211: Classical Mechanics* (Haberzettl).

**Co-requisite:** *PHYS 281: Computational Physics I* (Haberzettl/Griesshammer).

**Goals:** Introduction to the mathematical methods of Theoretical Physics with many examples and applications to Physics problems. Focus on skill-building. No formal but “heuristic” proofs.

**Outline of Contents**, in thematic order only; duration is estimated only.

1. Systems of Units and Dimensional Analysis (1 lecture)

Arbitrariness of systems of units – the Natural System of Units – dimensional analysis and universal scaling laws

2. Calculus of Variations (2 lectures)

Extremisation principle – functionals and functional derivatives – fixed-endpoint and flexible-endpoint variations – Euler-Lagrange equations – Rayleigh-Ritz variational principle – invariants

3. Group Theory (4 lectures)

Matrix and abstract groups – Abelian and non-Abelian groups – discrete groups – continuous (Lie) groups and their generators –  $SO(3)$  and  $SU(2)$  – representations and irreducible tensors

4. Tensor Calculus (4 lectures)

Contra- and co-variant (dual) vectors – differentiation and integration in curvilinear coordinates – gradient, divergence, curl, Laplacean – integration theorems – Helmholtz’ fundamental theorem

5. Function Spaces and Operators (3 lectures)

Recap vector spaces and matrices – Hilbert space and its relatives – bras and kets – orthonormal bases of functions – Generalised functions – spectral decomposition – Hermitean, self-adjointed and unitary operators – pitfalls

6. Partial Differential Equations and Green’s Functions (6 lectures)

Recap Ordinary Differential Equations – Green’s functions and boundary value problems – Poisson equation – Sturm-Liouville theory – spherical harmonics and other complete orthonormal sets of functions – multipole moments – integral transforms (Fourier, Laplace, ...) – integral equations

7. Complex Analysis (6 lectures)

Theory of analytic functions – poles and cuts – Cauchy’s theorem and residue calculus – analytic continuation – special functions

8. Advanced Topics (time permitting)

**Style:** “Commenting lecture with student participation”, i.e. focus on central points to guide and assist you in exploring relevant literature. *The home-page lists strongly suggested reading to efficiently prepare and in particular follow-up on course material. I will assume you have read this material before each lecture, and will familiarised yourself with its formal aspects after each lecture.* We will *not* repeat in class the contents of textbooks, but add different perspectives, seek a deeper understanding of the underlying concepts and develop skills by addressing problems as a group. I present not all the mathematical tools you will ever need – that would be impossible –, but provide an introductory overview with a personal selection of topics which are frequently used in modern research. *Proofs will largely be deferred to books. Rigorous proofs are left to Mathematicians.* Some topics are *not covered thoroughly enough* in class, some may be hard to find in textbooks (see notes on the bibliography), and others are only addressed in the homework. The “lectures” are only a first guide to study the mathematical methods outlined more rigorously in books, e.g. in those listed below. You should ask yourselves the type of questions that lead to developing and understanding the key concepts and the skills of scientific reasoning. I as teacher can assist, guide, motivate, trigger and speed up your studies, but learning is an active process which takes place within you more than in the lecture hall. Its difference to research is mostly that when doing research, you learn what is not yet found in textbooks.

*I encourage you strongly to ask questions and initiate discussions in class and during Surgery hours at all times. Think of lectures rather as “tutorial” or “studio” than a fixed set of hours in which I talk and you listen. If I cannot give you a satisfactory answer right away, I will come back to you, and you should continue asking until you are satisfied. If you find discussion in class or Surgery hard to follow, see me instantly!*

**Grading policy:** The course will be graded on an absolute scale. The final grade is a sum of:

- Exercises/Homework (40% of total): weekly, see below for details;
- Mid-Term Exam (30% of total): **Friday, 31st Oct (?) at t.b.a in Samson Hall 209**, 2.0 hours;
- Final Exam (30% of total): **Thursday, 18th Dec at 10:00 in Samson Hall 209**, 2.5 hours.

In order to pass, you need at least 60% of all points *and* at least 50% of the points available in each of the three components *separately*. In particular, you need at least 50% of all points in all Problem sheets together (not per sheet!). 80% is an excellent score, and 90% has not been achieved yet. I do not post scores on the web. If you have questions or comments on your grade or your overall score, please see me.

For your protection, the exams are closed-book. A sheet with some possibly relevant mathematical formulae will be provided by me, several days before the exam. If you have understood the contents of formulae and practised enough examples, you will not even bother to consciously memorise anything.

**Exercises/Homework: distributed Wednesdays** in class and on the web, **due the following Tuesday at 12:00 noon** in my pigeon-hole for paper-submissions or by fax (202-994-3001) or electronically to my email (.pdf-file only). *Late homework is graded as zero points. No exceptions.*

Graded solutions are **returned and discussed during Surgery hour on Wednesday**. Typically, problem sheets contain a mix of detailed and only outlined questions, with up to 30 points per sheet. Some problems require numerics or graphics programmes (Maple, Mathematica, Fortran, Assembler, C(++), etc.). Some of the projects of *PHYS 281: Computational Physics I* will be chosen from material in this course, and hence will need some “real” programming. These do not count towards your grade in Math. Methods; see separate announcement of *PHYS 281: Computational Physics I* (Dr. Haberzettl), and later in the course.

It is a fact of life that if you score more than 60% of the homework points, you will most likely perform well in exams and the qualifying. Some exam questions will almost certainly be based on homework solutions.

While it is necessary to have the correct answer for full credit, it is not sufficient. Indeed, it may serve you only one point. What you hand in should be a tidy and efficiently short presentation of your results and how they come about, which can be understood and reproduced by your peers. Imagine it is not homework, but a research problem whose solution you are asked to explain to your peers. I neither encourage nor discourage you to submit solutions electronically. But if you do so, work with a good drawing programme like xfig or gimp (freeware) for sketches. Electronic submission is no excuse for leaving out sketches.

*I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.*

I encourage you to form study groups to discuss the reading and attack Problem sheets as team. Nothing helps you understand better than interacting with your peers. However, practise additional problems alone to make sure that you do not become dependent on the others.

You can best study and check your progress if you present results and problems with selected exercises in the seminar-style Surgery hours. Your discussion of solutions, problems and comments shape them. As integral part of the lecture, I encourage you to attend them regularly. There is no better preparation for the exams. For me, they are the prime tool to gauge our progress and revisit material which is not fully digested yet.

**Typical workload for this course:** 10 to 12 hours per week, in addition to lectures and surgery hours.

**A note on academic integrity:** You like Physics, or you would not be here. Thus, it is trivial that you will abide by the GW Code of Academic Integrity (<http://www.gwu.edu/~ntegrity/code.html>) in all graded work. You are encouraged to collaborate on your homework and even be inspired by a good textbook, but make sure you have understood what you hand in as your solution. Do *not* offend your own (and my) intelligence by copying other people's work (especially without referencing). The web-site, all problems and solutions are for your personal use only. Please do not pass solutions or problems on to any student who has not taken the course (yet). Noncompliance with these rules is a breach of integrity and will be dealt with accordingly. If you have any questions about what constitutes academic dishonesty, ask me.

**Absences and Excuses** follow standard GW policy. It is your own responsibility to make sure you fulfil the criteria for passing, in particular that you get at least 50% of all the points available in all Problem sheets together (not per sheet). The only way around this criterion is to submit in writing documentation that you were unable to perform homework for more than half the semester due to reasons out of your control, as outlined in the GW policy on absences and excuses.

There will be no make-up exams. A missed exam will be dealt with case-by-case. Bring any potential conflicts or difficulties to my attention *before* the exam. If you miss an exam for some unexpected reason, it is your responsibility to notify me in writing *within 24 hours* of the missed exam, or the grade will be zero for the missed exam. Absence for medical reasons must have formal, written documentation from the medical office providing care. DC traffic is no excuse, and no additional time will be provided for late-comers.

If you see a conflict between religious observances and the class and exam schedule, you will bring them to my attention in advance, as soon as possible at the beginning of the semester.

**The University Counseling Center (UCC)** assists you in addressing personal, social, career, and study problems that can interfere with your academic progress and success.

Services for students include:

- **Crisis Consultations at 202-994-5300** open day and night, not only for emergency.
- Academic Support and Peer Tutoring Services: <http://gwired.gwu.edu/counsel/AcademicSupport>
- Podcasts and Self-Help: [gwired.gwu.edu/counsel/PodCast](http://gwired.gwu.edu/counsel/PodCast), [gwired.gwu.edu/counsel/OutreachSelfHelp](http://gwired.gwu.edu/counsel/OutreachSelfHelp)

They are also very good when you need to review your habits, like learning and exam strategies. It's never too early to get help.

## Some Suggested Reading

There is *no required reading* for this course. You will not be able to find all aspects of the lecture explained well in only one textbook. Moreover, it is an essential part of the learning process to view the same topic from different angles, i.e. using different textbooks. Here is a list of those which I found most useful. If you discover others, tell me. *The web-site lists recommended readings for each lecture.*

An asterisk \* indicates titles on Course Reserve at Gelman Library, with max. 3 days for loan. Be social.

### Books on which the course is (mostly) based

- [AW] \* G.B. Arfken and H.J. Weber: *Mathematical Methods for Physicists*; 6th ed., Academic Press 2005, ca. US\$110 (cheaper as international student edition). “Standard text” for this kind of course. Contains all aspects like an encyclopedia, but sometimes not very pedagogical. Good reference for research.
- [Sni] \* R. Snieder: *A Guided Tour of Mathematical Methods: For the Physical Sciences*; 2nd ed., Cambridge University Press 2004, ca. US\$75. Like [Sto], a new and very readable approach which is driven by physical insight. Tell me what you think of it.
- [Sto] M. Stone: *Methods of Mathematical Physics – A set of lecture notes volume I* (to be published by Cambridge University Press in 2009) and volume II (2003); online at <http://webusers.physics.uiuc.edu/~m-stone5/>, free (follow links to the two courses). Like [Sni], a new and very readable approach which is driven by physical insight. Tell me what you think of it.
- [RHB] \* K.F. Riley, M.P. Hobson and S.J. Bence: *Mathematical Methods for Physics and Engineering: A Comprehensive Guide*; 3rd ed., Cambridge University Press 2006, ca. US\$75. Many lecturers base their course on this book as pedagogical alternative to [AW]. A *Students Solutions Manual* for even-numbered solutions comes separately and is very valuable for self-study, ca. US\$25.
- [BF] \* F.W. Byron and R.W. Fuller: *Mathematics of Classical and Quantum Physics*; reprint by Dover Publications 1992, ca. US\$25. A classic which is quite readable.
- [GR] I.S.S. Gradshteyn and I.M. Ryzhik (Alan Jeffrey and Daniel Zwillinger, eds.): *Table of Integrals, Series, and Products* (with CD-ROM); 7th ed., Academic Press 2007, ca. US\$75. Not a book to study, but best reference for functions, their integrals and properties. On my desk; heavily used by theorists.

**More elementary texts** with many of the topics which are prerequisites of or covered by this course.

- [Boa] M.L. Boas: *Mathematical Methods in the Physical Sciences*; 3rd ed., John Wiley 2005, ca. US\$120.
- [Has] \* S. Hassani: *Mathematical Methods for Students of Physics and Related Fields*; Springer 2000, ca. US\$110.

**Classical texts** which are also important as resources/reference

- [MF] P.M. Morse and H. Feshbach: *Methods of Theoretical Physics* (2 volumes); priceless.
- [CH] R. Courant and D. Hilbert: *Methods of Mathematical Physics* (2 volumes); reprint by John Wiley 1991, ca. 2×US\$145.

**Reference Tables and Collections of Formulae** for reference in the University library. You will not need this in the course, you might not even need it ever – but if you do, you will need it desperately.

- [AS] M. Abramowitz and I.A. Stegun (eds.): *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*; Dover 1965 and later, ca. US\$35. Started as a log-table. While descriptions and formulae are surpassed by [GR], now valuable for quick-references of plots of complicated functions. Digital version by NIST with GWU contributions in preparation, see <http://dlmf.nist.gov/>.
- [Bat] Bateman Manuscript Project (A. Erdelyi et al.): *Higher Transcendental Functions* (3 vol.); *Tables of Integral Transforms* (2 vol.). Collects properties and integrals of the weirdest functions imaginable.
- [PB] A.P. Prudnikov and Yu.A. Brychkov (and others): *Integrals and Series* (5 volumes, plus a volume *Integral Transforms of Generalized Functions*). If it’s not in there, it’s nowhere.
- [Obe] F. Oberhettinger (and others): *Tables of Indefinite Integrals, Fourier, Laplace, Bessel, Mellin Transforms* (4 volumes).

## Some Hints for a Successful Graduate Life

This is by no means a complete list, but it helps me in my teaching and research. Use your own judgement!

**Take Dr. Haberzettl's hand-out "How To Write Homework – and Related Questions" seriously.** You do not have to type your Mathematical Methods homework, but he has some very good advise how to avoid errors and which cross-checks are very valuable.

**Attack your homework early.** Don't postpone it to the last few days or even minutes.

**Make a plan of attack.** First think what solution you expect from your physical intuition. Then ponder over a good way to find the solution. This can take even an hour. Then take a deep breath. Then think again about the problem. Then solve. The time spent on first thinking about the solution is much shorter than the time wasted with abandoned attempts when you instantly start scribbling. In particular in exams.

**Form teams** (see above). Nothing helps one to understand better than discussing homework and lectures with peers. But practise additional problems alone in order not to become dependent on others.

**Put craftsmanship before ingenuity.** You will be outstanding soon enough, but for now, continuous, solid work is more reliable than occasional sparks of brilliance.

**Don't get nailed-down.** Nobody requires you to find the *best/most elegant/fastest* solution. *Any* solution will do for a start. Once you have one, you can always look for a better one – if you have the time. When stuck, discuss with your peers (and consult the lecture and books). If you get very stuck, do another problem first. It's no use to get no problem done because you wasted all your resources on the first one.

**Practise sketching and plotting.** Discussions, sketches and plots are a must! Not only because the homework is full of these words, and you will loose a lot of points if you do not discuss, sketch and plot. But human beings are visual beings: We understand and recollect much better when we see a figure.

**Scrutinise your homework when it is returned to you and reproduce a correct solution.** Clean up your notes. What did you not understand? What did you miss? Was there a faster way? Where are your strengthes and weaknesses? You should spend at least a hour on that, as soon as possible. It will help you with the next homework set.

**Work through each lecture on the day it is delivered.** If you miss that, you will have a very hard time to understand the next lecture. In that context, "Tomorrow will be another day" is a very bad motto.

**"Fill in the gaps" of the lecture.** Spell out the details of a proof, make sure the signs and factors are correct, etc. That already gives you a lot of free practise in math, and makes sure your thinking and notes are up-to-date and correct. And you have a set of notes you understand when you come back after weeks or months or years, for exams or research.

**Consult books (plural!)** after you have reviewed the lecture. It will clarify things further, show you new and different perspectives, and deepen your understanding. I usually excerpt information which I found interesting in a book, in addition to lecture notes.

**Look at the Physics behind a formula.** Does it make "sense" from your physical intuition? Do you understand what it means? What are its limits, i.e. regimes where it becomes particularly simple to understand? What are its limitations, i.e. where does it not work? Explain it and its underlying principle to a peer or to an undergraduate, using no math. You will believe your most beautiful mathematical proof only if you can also give a good intuitive argument why the formula should be right.

**Ask yourself: What is the hidden agenda** behind this topic in the lecture, homework, etc.? What can I learn that goes beyond the straightforward application? Is there a greater principle involved which I can use in different contexts? Why is e.g. a proof presented this way? In which other fields could I use similar techniques/reasoning, outside electrodynamics?

**Talk with your lecturers.** We post out office hours not out of courtesy, and we don't bite. If you don't come to me with your problems, how can I help you? I – for one – love discussing. Have no fear to overburden me. I will tell you when I have had enough.

**Have a life outside Physics.**