Additional Practise Sheet: Functional Analysis

If you want, we can discuss your solutions in the Final Question Time of the semester. No extra points are awarded – the values are only meant as grade of difficulty here.

1. Dirac’s δ-Distribution (9P): Prove all of its properties on the handout.

2. Fourier Transforms (9P): Prove all of its properties on the handout.

3. (3P) Consider a Gaussian, \( f(x) = \exp\left(-\frac{x^2}{2\sigma^2_x}\right) \), with width \( \sigma_x \). When you calculate its Fourier transform, you will find that you get another Gaussian in momentum space, but with a different width \( \sigma_k \). What is it? Make a connection to Heisenberg’s Uncertainty Relation when you interpret \( f(x) \) as a wave-packet in coordinate space.