Problem Sheet 11

Due date: 7 April 2009 **13:00 noon**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at *http://home.gwu.edu/˜hgrie/lectures/edyn09/edyn09.html*.

1. COMPTON SCATTERING: OFF INDUCED ELECTRIC AND MAGNETIC DIPOLES – PART II (**10P**): In the lecture, we considered the case that an incoming electro-magnetic wave induces an electric dipole, whose strength is proportional to the strength of the incoming elecric field, and which oscillates in the same direction as the incoming field. Now, we extend the same set-up to magnetic responses: An monochromatic, plane electro-magnetic wave with electric and magnetic fields $\vec{E}_{\mathrm{in}}(\omega) = \vec{E}_0\, \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}}$ and $\vec{B}_{\mathrm{in}}(\omega) = \vec{e}_k \times \vec{E}_0\, \mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}}$ induces a frequency-dependent electric and magnetic dipole moment in a charge distribution, i.e.

$$\vec{d}_{\mathrm{ind}}(\omega) = \alpha(\omega)\ \vec{E}_{\mathrm{in}}(\omega) \ \ , \ \ \vec{m}_{\mathrm{ind}}(\omega) = \beta(\omega)\ \vec{B}_{\mathrm{in}}(\omega) \ \ .$$

   Electric and magnetic polarisabilities α and β are for us at present just real constants of proportionality.

   a) (**5P**) Parallel to the steps in the lecture, derive now the differential cross-section of linearly polarised light off this system. Your answer depends on the angle θ_{scatt} between incoming and outgoing wave, and the angle ϕ_d between the scattering plane and the polarisation of the incoming wave, see lecture, via:

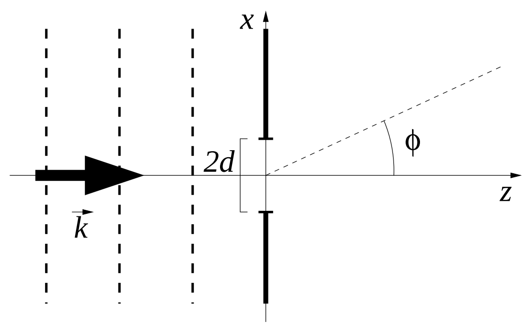
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\omega^4}{c^4}\left[(\alpha^2+\beta^2)\cos^2\theta_{\mathrm{scatt}}\cos^2\phi_d + \alpha^2\sin^2\phi_d + \beta^2\cos^2\phi_d + 2\alpha\beta\cos\theta_{\mathrm{scatt}}\right]$$

   Hint: Follow the steps in the lecture!

   b) (**5P**) Find the differential and total cross-section for an un-polarised incoming wave. Determine also the polarisation asymmetry for this case.

Please turn over.

2. Diffraction on a Slit (**15P**): An electro-magnetic field $\vec{E}(t,\vec{r})$ is perpendicular incident from the left on an infinitely long slit of width $2d$, see figure. The field is linearly polarised along the length of the slit (i.e. along the y-axis, perpendicular to the drawing plane) and has wave-vector $\vec{k} = k\vec{e}_z$. We now determine the field and intensity of a wave the right of the slit, diffracted by an angle θ. As usual, you may get inspired by a good book.



a) (**1P**) Why is $\vec{E}(t,\vec{r}) = E_0\vec{e}_y \exp -\mathrm{i}[\omega t - kz]$ on the left? Determine the electric field everywhere on the xy-plane.

b) (**8P**) Let's now try an ansatz for the wave in the right half-plane:

$$E_y^{\text{out}}(t,\vec{r}) = \int \frac{\mathrm{d}^3k'}{(2\pi)^3}\; a(\vec{k}')\; \mathrm{e}^{\mathrm{i}(\vec{k}'\cdot\vec{r}-\omega t)} \quad .$$

The function $a(\vec{k}')$ must then be determined such that the waves in the left and right half-plane agree at $z=0$ for all x,y. This can best be achieved by a Fourier transform at $z=0$. You find

$$a(\vec{k}') = 4\pi\, E_0\, \delta(k'_y)\, \frac{\sin k'_x d}{k'_x} \quad .$$

Why is $|\vec{k}| = |\vec{k}'|$? Use this to express k'_z by k'_x, k'_y and $|\vec{k}|$. You then end up with an integral which you cannot perform exactly. However, change variables

$$k'_x = k\sin\theta \;\;,\;\; k'_y = k\cos\theta$$

to implement the constraint on k'_z mentioned above. Notice that θ is not quite the angle ϕ of the above figure. What's the difference? You will then see that the integral is peaked around $\theta \approx 0$, and in that (and only that) case, $\theta \approx \phi$ (why?).

c) (**6P**) Show that the (time-averaged) intensity is

$$I(\phi) \propto <\left|E_y^{\text{out}}(t,\vec{r})\right|^2>_T \propto \mathrm{Re}\left[\int\limits_{-1}^{1}\frac{\mathrm{d}\sin\theta}{\sin\theta}\;\sin[kd\sin\theta]\;\exp \mathrm{i}kR[\sin\theta\,\sin\phi+\cos\theta\,\cos\phi]\right]^2$$

Sketch or plot it, seen on a screen far away from the slit ($\phi \ll 1$). Discuss in particular the cases $kd \gg 1$, $kd \approx 1$ and $kd \ll 1$. You may also consider the case that the screen is a very large, hollow semi-cylinder, with its axis identical with the y-axis. This may make plotting things easier and the results more instructive, but then, $\phi \not\ll 1$.