Problem Sheet 10

Due date: 31st March 2009 **13:00 noon**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.
Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.
I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.
News and .pdf-files of Problems also at *http://home.gwu.edu/~hgrie/lectures/edyn09/edyn09.html*.

1. WHY ARE SUNSETS RED? (**5P**)

2. THE FALLING ELECTRON (**11P**): An electron is initially at rest and then falls for a few metres without friction, attracted by homogeneous gravitational field of the Earth. We consider its velocity to be non-relativistic at all times. The observer stands a few dozen metres away. You will see in a) is the wavelength of the radiation is much larger than the typical distance the electron falls.

   a) (**5P**) How does the radiation emitted depend on: height of fall; charge and mass of the electron; speed of light; acceleration factor; distance to the observer; time $T_{\max}$ over which the particle falls (non-relativistically!); weather conditions; other parameters?

      Based on that, use dimensional analysis to estimate the typical wavelength of the radiation. Give a number with a sensible length-scale attached. Under which conditions can one use the long-wavelength approximation?

   b) (**4P**) Determine the time-dependent electric field. In which directions lie the radiation maxima, at a given instant in time. Find also (by *thinking*, not by calculating) direction and value of the minima of the angular characteristics.

   c) (**2P**) After the particle has fallen some distance h, compare the energy radiated away to the gravitational potential energy lost.

3. COMPTON SCATTERING: OFF INDUCED ELECTRIC AND MAGNETIC DIPOLES; PART I – NEXT PART NEXT WEEK (**4P**): In the lecture, we considered the case that an incoming electro-magnetic wave induces an electric dipole, whose strength is proportional to the strength of the incoming elecric field, and which oscillates in the same direction as the incoming field. Now, we extend the same set-up to magnetic responses: An monochromatic, plane electro-magnetic wave with electric and magnetic fields $\vec{E}_{\text{in}}(\omega) = \vec{E}_0\,\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}}$ and $\vec{B}_{\text{in}}(\omega) = \vec{e}_k \times \vec{E}_0\,\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{r}}$ induces a frequency-dependent electric and magnetic dipole moment in a charge distribution, i.e.

   $$\vec{d}_{\text{ind}}(\omega) = \alpha(\omega)\;\vec{E}_{\text{in}}(\omega)\;\;,\;\;\vec{m}_{\text{ind}}(\omega) = \beta(\omega)\;\vec{B}_{\text{in}}(\omega)\;\;.$$

   Electric and magnetic polarisabilities α and β are for us at present just real constants of proportionality.

   a) (**1P**) Determine the electric field $\vec{E}_{\text{out}}$ and magnetic field $\vec{B}_{\text{out}}$ of the scattered wave which comes from the induced *electric* dipole $\vec{d}_{\text{ind}}$.

   b) (**2P**) Determine the electric and magnetic fields of the scattered wave which come from the induced *magnetic* dipole $\vec{m}_{\text{ind}}$.

   c) (**1P**) Find now the total electric and magnetic fields of the scattered wave: Super-impose the electric and magnetic fields coming from the electric dipole with those from the magnetic dipole.