Problem Sheet 5

Due date: 17th February 2009 **13:00 NEW DUE TIME!**

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.

Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.

I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.

News and .pdf-files of Problems also at *http://home.gwu.edu/˜hgrie/lectures/edyn09/edyn09.html*.

1. ELECTRIC FIELD OF THE HYDROGEN ATOM (**9P**): The charge density of the hydrogen atom (Bohr radius a) in its ground state is:

$$\rho(\vec{r}) = q\,\delta^{(3)}(\vec{r}) - \frac{q}{\pi a^3}\,\mathrm{e}^{-\frac{2r}{a}}$$

   a) (**1P**) Motivate from your QM experience that the this charge density is reasonable.
   b) (**1P**) Which symmetries should the electric field (and the electrostatic potential Φ) have?
   c) (**4P**) Calculate Φ and $\vec{E}$.
   d) (**3P**) Calculate the *numerical value* of the magnitude of the electric field at a distance of one Bohr radius, independently in SI- and Gauss' cgs-units. Check that the two results are consistent.

2. DIPOLE MOMENT OF THE HCl MOLECULE (**4P**): In a simplified model, the HCl molecule consists of a hydrogen atom (charge number $Z = 1$) with a chlorine atom, $Z = 17$, where the H-atom transfers its electron to the chlorine atom. The 18 electrons of Cl^- are then approximated by a spherically symmetric cloud around the Cl nucleus. Both nuclei are separated by $l = 1.28$ Å. We now compare the dipole moment of this molecule with the measured value, $d_{\mathrm{exp}} = 1.03 \times 10^{-18}$ esu cm (Gauß system).

   a) (**2P**) Prove that the dipole moment of a neutral object is independent of the choice of the coordinate origin.
   b) (**2P**) Calculate the dipole moment of the HCl molecule assuming that it consists of two point-like ions H^+ and Cl^- separated by a distance l. Given what you know about ionic binding, how can you explain the discrepancy to experiment?

3. MULTIPOLE MOMENTS OF AN ELLIPSOID (**4P**): A charged egg is a spheroid which is centred at the origin, fills the volume $\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} \leq 1$, and is homogeneously charged with total charge Q.

   a) (**2P**) Determine the first three Cartesian multipole moments (monopole, dipoles and quadrupoles) with respect to the coordinate system described.

      Hints: Use symmetries to avoid many integrations. In the integrations which are unavoidable, a skilful coordinate transformation maps the ellipsoid into a sphere.
   b) (**2P**) Estimate the strength of the multipole fields relative to the monopole for large and small distance from the egg. Which distance is "large", which is "small"? Interpret in particular the limit $a = b$.

Please turn over.

4. ELECTRIC DIPOLE AND PLATE (**6P**): An electric dipole $\vec{d}$ is put at a distance $\vec{a}$ from a plane-parallel, infinite, grounded metallic wall. The dipole and $\vec{a}$ point in the same direction.

   a) (**1P**) Show that the potential has the form (and determine α):

   $$\Phi(\vec{r}) = \alpha \left(\frac{\vec{d} \cdot (\vec{r} - \vec{a})}{|\vec{r} - \vec{a}|^3} + \frac{\vec{d} \cdot (\vec{r} + \vec{a})}{|\vec{r} + \vec{a}|^3} \right)$$

   b) (**3P**) Where does the induced surface charge density on the wall change sign? Based on that, draw a qualitative picture of the set-up which shows the electric field between dipole and wall, and the distribution of surface charges.

   c) (**2P**) Is the dipole attracted to or repelled by the wall? How big is the force?

5. PROBLEM WITH MULTIPLE STRATEGIES (**7P per correct strategy**): Two parallel, infinitely long, infinitesimally thin, straight wires are a distance d apart and carry stationary currents I and $-I$.

   Determine vector potential and magnetic field, as well as direction and magnitude of the force density (force per unit length of the wire) between the wires. Numerical values for $d = 1$ m, $I = 1$ A! Sketch the field.

   Hint: You know already a variety of strategies from Electro- and Magneto-statics. Although only *one* solution is required, this is a good opportunity to test your skills with as many as possible. For example, you can reduce the problem to a two-dimensional one and construct the appropriate Green's function. Or you consider the limit of two wires with finite length. If infinities should appear, recall that gauge potential are defined only up to gauge transformations.