Problem Sheet 1

Special due date: Wednesday, 21st January 2009 24:00 mid-night

For full credit, you should hand in a tidy and efficiently short presentation of your results and how they come about, in a manner that can be understood and reproduced by your peers. All problems and solutions are for your personal use only. Please do not pass solutions or problems on to incoming or other students who have not taken the course (yet). Noncompliance with these rules is a breach of academic integrity.

Handwritten solutions must be on 5x5 quadrille paper; electronic solutions must be in .pdf format.

I reserve the right to award zero points for any illegible, chaotic or irreproducible section of your homework.

News and .pdf-files of Problems also at *http://home.gwu.edu/˜hgrie/lectures/edyn09/edyn09.html*.

1. WARM-UPS (**10P**):

   a) (**2P**) Calculate $\int\limits_{-\infty}^{\infty} \mathrm{d}x \; \frac{x\mathrm{e}^{\mathrm{i}x}}{x^2+a^2}$, $a>0$.

   b) (**2P**) Determine the order of and locate the singularity and evaluate the residue of $\frac{z^2\mathrm{e}^z}{1+\mathrm{e}^{2z}}$.

   c) (**2P**) Calculate with $\vec{r}=x\,\vec{\mathrm{e}}_x+y\,\vec{\mathrm{e}}_y+z\,\vec{\mathrm{e}}_z$, $r=\sqrt{x^2+y^2+z^2}$, $\vec{\mathrm{e}}_r=\frac{\vec{r}}{r}$:

   $$\vec{\nabla}\cdot\vec{r},\quad \vec{\nabla}\times\vec{r},\quad \vec{\nabla}\frac{1}{r}$$

   d) (**4P**) Do the following fields have sources/sinks or curls? Sketch or plot the vector-function in a suitable way!

   $$\vec{A}_1(\vec{x})=x\,\vec{\mathrm{e}}_y-y\,\vec{\mathrm{e}}_x \quad,\qquad \vec{A}_2(\vec{x})=x\,\vec{\mathrm{e}}_y+y\,\vec{\mathrm{e}}_x \qquad \vec{A}_3(\vec{x})=\frac{1}{1+x^2}\,\vec{r}$$

   e) (**2P**) Prove GREEN'S THEOREMS for arbitrary scalar fields Φ, Ψ (corollaries of Gauss' theorem):

   $$\int \mathrm{d}^3r\;\Big((\vec{\nabla}\Phi)\cdot(\vec{\nabla}\Psi)+\Phi\Delta\Psi\Big)=\oint \mathrm{d}^2\vec{s}\cdot(\vec{\nabla}\Psi)\,\Phi \quad,$$

   $$\int \mathrm{d}^3r\;(\Phi\Delta\Psi-\Psi\Delta\Phi)=\oint \mathrm{d}^2\vec{s}\cdot(\Phi\vec{\nabla}\Psi-\Psi\vec{\nabla}\Phi) \quad.$$

2. A TWO-DIMENSIONAL GREEN'S FUNCTION (**4P per correct, independent way**) Show that the Green's function to the *two*-dimensional Poisson equation "without boundaries at infinity" is

   $$G(\vec{r},\vec{r}\,')=\alpha\ln\frac{|\vec{r}-\vec{r}\,'|}{C}\quad,$$

   where C is an arbitrary constant. Determine the constant α. There are several ways to do this problem, given the toolchest developed in the last semester. **Each correct way gives** 4 **points.**

   You might want to recall also some Green's functions of one- and three-dimensional "empty" space.

3. SPEED OF LIGHT AS MAXIMUM VELOCITY (**2P**): A laser is mounted on a record player (these old things one used before CDs were invented, playing these huge, black vinyl waivers) such that the light it emits hits at each of the 33 rpm the moon, 400 000 km away. Determine the velocity with which the light-ray coats the moon's surface. How can you reconcile this with the axioms of Special Relativity?

Please turn over.

4. SIMULTANEÏTY AND LOCALITY (**6P**): In an inertial system **I**, two events occur at different spacetime coordinates $(ct_1, x_1, 0, 0)$ and $(ct_2, 0, 0, 0)$, where $x_1 \neq 0,\ 0 \neq t_1 \neq t_2 \neq 0$.

   a) (**3P**) Under which conditions on the invariant distance between the events can we find an inertial system **II** in which the events occur *simultaneously*, or (ii) *at the same place* in space, respectively?

   b) (**3P**) Determine for both cases the relative velocity between inertial systems **I** and **II**.

5. MUON DECAY (**2P**): A muon is an elementary particle with mass $m = 106$ MeV/c^2. In its rest-frame, it decays after $\tau \simeq 2.2 \cdot 10^{-6}$ s into an electron, a neutrino and an anti-neutrino. The collider at the Fermi National Accelerator Laboratory (Fermilab/FNAL) close to Chicago produces muons with an energy of $E = 100$ GeV. What is the life-time of these muons in the lab frame? How far can they typically travel before they decay?

6. GUESSTIMATES (**6P**): In a "guesstimate", one attempts an *order-of-magnitude* **estimate** of a quantity or effect by educated **guesswork** and as few math as possible. The trick is to see how far one can get with what is in one's head and with the information given, without resorting to books or even calculators. Guesstimates are usually made without sribbling or on a very small piece of paper, like the proverbial "back-of-the-envelope". If you come up with numbers consisting of more than 1 significant digit, you have not understood the concept. For the following guesstimates, you may find the information on essential numbers in the syllabus useful. They also also to familiarise you with the beauty of using "natural units" for nucleon masses etc.

   I have a "rechargable battery" with "2500 mAh, 1.2 V" printed on it.

   a) (**3P**) Which fraction of the electrons inside the battery needs to be dissociated from atoms/ions to comply with the specifications?

   b) (**3P**) To get a feeling how "powerful" the battery is: How much matter could you create by converting the energy stored in a charged battery (as opposed to an uncharged one), if that were possible? A gramme? A grain of dust? An amoeba's worth? A gross of atoms? Fractions of an electron?