Supplement on Superconductivity

The Meißner effect (left) expels a magnetic field from a superconductor, leading to levitation in type-I superconductors (centre), and to Nielsen-Olesen flux tubes in type-II superconductors (right).

Meißner-Effekt in Type-II Superconductors

Vacuum Expectation Value of Cooper-pairs $|\langle \Psi \rangle| = a \neq 0 \implies \vec{B} = 0 \implies \vec{A}(\vec{r}) = \vec{\nabla} \chi(\vec{r})$

$\Phi_{\text{mag}} = \oint d\vec{s} \cdot \vec{A}(\vec{s}) = \chi(\phi = 2\pi) - \chi(\phi = 0) \in \frac{2\pi}{Q} \mathbb{Z}$  Abrikosov-Nielsen-Olesen flux quantisation

Ginzburg-Landau theory

$$\mathcal{H}_{\text{int}} = \frac{1}{2m} \left| (i\hbar \vec{\nabla} + \frac{Q}{c} \vec{A}) \Psi \right|^2 + \lambda \left[ |\Psi|^2 - a^2 \right]^2$$

Ginzburg-Landau potential for the superconducting phase, $\lambda > 0$ (left); for the normal phase, $\lambda < 0$ (right).