Questions to Check Your Progress

(version Spring 2009)

Note: After your Electrodynamics course, you should be able to fill the following collection of concepts in Electrodynamics with life. Not all questions were necessarily detailed (enough) covered in the course; other important ones were certainly skipped; yet others received too much attention; and some questions are very vague. This sheet cannot do more than inspire you and be curious to think about and delve into the subject with the aid of a good textbook. The wrongest use of this list would be to memorise answers or formulae. In that case, you could just as well memorise Jackson's *Electrodynamics*. Rather, think whether you could arrive at (motivate/sketch a way to) an answer, based on the knowledge you have. Under no condition is this a survey of material for exams – neither maximal nor minimal. It might not even be of any use at all.

Mathematical Foundations: Div/rot/grad/nabla/Laplace/volume elements in Cartesian/cylindrical/spherical coordinates. Definition/use and utility/construction/uniqueness of Green's functions. Von-Neumann vs. Dirichlet boundary conditions. Properties of Dirac's δ-distribution. Einstein's summation convention, ε-tensor, Levi-Civitá symbol, pseudo-tensor of rank n. Theorems by Gauß, Stokes, Helmholtz. Using these theorems, derive the physical laws named after Gauß, Ampére, Faraday. What is " separation of variables"? What is a complete orthonormal basis of functions? Examples. Fourier-transformations, e.g. of Dirac's δ-distribution, Gauß' distribution function. Residues and integration over complex functions. Legendre functions and spherical harmonics. Sketch and discuss functions with pen and paper.

Special Theory of Relativity: Postulates. Definition co-/contravariant 4-vector, pseudo-metric, proper time, invariant line-element/-distance, 4-velocity/energy-momentum-vector/d'Alembert's operator. Which mathematical condition is obeyed by all Lorentz transformations? Difference proper/orthochronous/improper Lorentz transformations. Transformation properties of time- and space-coordinates when changing the inertial frame. Time dilatation, length contraction, addition of relativistic velocities. Relativistic kinematics of scattering between 2 particles/decay into 2 particles, Mandelstam variables, translation of scattering angles between lab and centre-of-mass frame. What is a time-/light-/space-like distance, light-cone? Lagrange function and Euler-Lagrange equations of a relativistic point-particle in an electromagnetic field. Definition and interpretation of minimal substitution, canonical and kinematical momentum.

Electrodynamics as Relativistic Field Theory/Fundamental Equations: What is a "field"? Euler-Lagrange formalism, Noether's theorem, energy-momentum tensor for arbitrary field. Definition field-strength tensor and its dual. Express $F^{\mu\nu}$ via $\vec{E}$ and $\vec{B}$. Covariant derivation of Lagrange density and field equations of Electrodynamics. Maxwell's equations for $\vec{E}$ and $\vec{B}$, including physical interpretation of each term. Is the continuity equation a condition which has to be imposed in addition? What would change if magnetic monopoles exist? Properties of electromagnetic fields under Lorentz transformations. Electric and magnetic field of a point charge in uniform motion. Why/how are gauge potentials (gauge fields) introduced? Discuss the gauge principle, the physical relevance of gauge fields, gauge freedom, relation to current conservation. Which gauges do you know, what are their respective advantages? Is the gauge field uniquely determined in the Lorentz gauge? Definition, derivation, interpretation of energy density and energy current density/Poynting vector of electrodynamics.

Current- and Charge-Distributions: Lorentz force on a moving charge. Force on current inside wire in magnetic field. Use δ- and θ-distributions in suitable coordinate systems to describe the charge densities of point-charges, homogeneously charged spheres/cylinders (surface or volume). When rotated about a symmetry axis, what do the corresponding currents look like? Sketch Φ, $\vec{A}$, $\vec{E}$, $\vec{B}$.

Electro-Statics: Which gauge is useful? Is giving $\vec{E}$ as solution equivalent to giving the scalar potential? Derive the Poisson equation. Scalar potential and electric field of a point-charge, dipole, continuous charge distribution $\rho(\vec{r})$. Can one uniquely determine the charge distribution inside a charged sphere from its electric field at infinity? Sketch strategies to solve the Laplace equation. Why can every function $f(\theta, \varphi)$ on the unit sphere be written as linear combination of spherical harmonics $Y_{lm}(\theta, \varphi)$? Ansatz to solve the Laplace equation in spherical coordinates. Which boundary conditions determine the solution of a

particular problem uniquely? Specify the general Cartesian/spherical multipole decomposition of the scalar potential/electric field/energy and its Cartesian/spherical multipole moments. Are all components of the quadrupole tensor independent? Rôle of symmetries. Pros and cons of Cartesian and spherical multipoles. For which (l, m) do the spherical multipoles q_{lm} describe a monopole/dipole/quadrupole? Are multipole expansions independent of the coordinate-system/-origin used? How to experimentally differentiate between multipolarities of static fields? Radial dependence of lth multipole. Under which conditions is the multipole expansion a good approximation? Characterise the force between two electric dipoles.

Magneto-Statics: Which gauge is advisable? Derive the Biot-Savart law for the $\vec{B}$-field of a localised current distribution. Definition magnetic dipole moment. What is the corresponding vector potential and $\vec{B}$-field? Force on a magnetic dipole in an external field. Relation between angular momentum and magnetic moment. Origin of Hyperfine structure in atoms.

Wave-Equation: Which gauge is advisable for radiation problems? Conditions of the Lorentz gauge. Are the gauge fields then uniquely determined? Derive the inhomogeneous wave-equation in the Lorentz gauge. Why are $\vec{E}$, $\vec{B}$ and the wave-vector perpendicular to each other in a free wave? Are there cases where this is not so? Definition of the time-dependent Green's function and its frequency-space version. Physical interpretation of the retarded/advanced Green's function. What is the physical significance of "retarded time"? In which cases does one have to use the advanced Green's function? What solutions does the wave-equation have in vacuo? Which kinds of polarisation of an electromagnetic wave do you know? How many degrees of freedom does a free electromagnetic wave have? What if the photon would have a mass? How to determine the polarisation of a wave?

Radiation and Scattering: Why does it often suffice to consider the radiation off harmonically varying charge- and current-distributions? How to compute the angular distribution/radiation characteristics of the radiated power? How to determine the spectrum/frequency distribution? How to determine which kind of multipole radiation is emitted? Differences and common grounds of dipole and quadrupole radiation. Differences and common grounds of time-dependent and static multipoles. How to differentiate between magnetic and electric dipole radiation? Which assumptions allow one to calculate the radiation field in the near-/intermediate-/far-zone? What is the long-wavelength approximation? In which parameters does one have to expand to obtain the multipole radiation? When does the multipole radiation converge? Radiation characteristics of electric dipole/quadrupole radiation. Motivate for atomic systems how strongly electric quadrupole and magnetic dipole radiation are suppressed compared to electric dipole radiation. How to treat radiation off relativistically moving charges? Sketch a derivation/application of Liénard-Wiechert potentials. Characterise spectrum and angular distribution of synchrotron/bremsstrahlung radiation. Discuss radiation loss and Larmor's formula. How to get a scattering cross-section? Thomson/Rayleigh scattering. Why is the sky blue? Can one polarise an electromagnetic wave by scattering it? Coherent vs. incoherent scattering. Sketch how Huygens' principle emerges from the solution of the free wave equation by spherical waves.

Electric and Magnetic Fields in Media: Interpret all symbols in the in-medium field equations. Sketch how and under which conditions the in-medium Maxwell equations emerge from the microscopic ones. Which charges/currents are to be used? Relation between $\vec{E}/\vec{D}/\vec{P}$, $\vec{B}/\vec{H}/\vec{M}$, resp. Are $\vec{D}$, $\vec{E}$ ($\vec{B}$, $\vec{H}$) always parallel/proportional to each other? What is the electric/magnetic polarisation of a charge/current distribution? Describe the salient features of the Lorentz-oscillator model. Estimate the electric/magnetic polarisability of the hydrogen atom. Derive conditions for the normal and tangential components of $\vec{D}$, $\vec{E}$ ($\vec{B}$, $\vec{H}$) at the interface of two isotropic media, using Gauß' and Stokes' theorems. Relation between polarisation and induced surface density. Electrostatic energy in a polarisable medium. How does the dielectric medium inside a grounded/insulated conductor react? Sketch the fields. Origins of non-trivial constants ϵ/μ, orientation-/relaxation-polarisation, para-/dia-/ferro-magnetism. Wave equation in media. Determine the index of refraction. Relations between material characteristics ϵ/μ and phase velocity/absorption of electromagnetic waves in media. Definition transmission/reflexion coefficient. Which conditions are imposed on waves at interfaces between different media? Sketch the origin of Čerenkov and transition radiation.